Rotating Frames of Reference

Representations Series

Instructor's Guide

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Developed by the Teaching and Learning Laboratory at MIT
for the Singapore University of Technology and Design

Introduction

When to Use this Video

- In EPDM 301, in recitation, after Lecture 4: Magic and super-magic formulae
- Prior knowledge: how to define basis vectors, inertial and non-inertial reference frames; and the representation of rotation rates as vector cross products.

Key Information

Duration: 15:18
Narrator: Prof. Sanjay Sarma
Materials Needed:
• Paper
• Pencil

Learning Objectives

After watching this video students will be able to:

- Explain why centripetal and Coriolis accelerations arise in rotating frames of reference.
- Apply their understanding of the Coriolis acceleration to determine the direction of rotation of hurricanes.

Motivation

- This video aims to disambiguate the "fictitious forces" that arise in rotating frames of reference by explictly tying them to the unobserved acceleration of the rotating frame.

Student Experience

It is highly recommended that the video is paused when prompted so that students are able to attempt the activities on their own and then check their solutions against the video.

During the video, students will:

- Determine whether or not a force opposite an applied tension force on a disk on a rotating turntable exists, and where it comes from.
- Determine the velocity of the disk in the ground frame, given that it is not moving in the turntable frame.
- Compute a general formula that finds the acceleration of an object in one frame in terms of the accleration in another, given that one frame is rotating but not translating with respect to the other.
- Apply the formula to determine the accelerations of objects in two different examples.
- Discuss why the observed motion of a rolled ball is curved in a rotating frame of reference.
- Predict the motion of a rolled ball as observed from the ground frame.
- Use the Coriolis effect to explain why hurricanes in the Northern hemisphere rotate counterclockwise.

Video Highlights

This table outlines a collection of activities and important ideas from the video.

Time	Feature	Comments
0:00	Hurricanes in the Northern Hemisphere	Footage from NASA shows hurricanes with counterclockwise rotation.
0:56	Prerequisites and Learning Objectives	
1:23	Chapter 1: Rotating Frames of Reference	
3:12	Activity	What is the apparent force acting on the orange disk in the turntable frame of reference?
4:30	Activity	Determine the velocity of the orange disk in the ground frame of reference.
5:50	Activity	Obtain a general formula for the acceleration of an object in the ground frame with respect to related terms in the turntable frame of reference.
7:04	Activity	Determine the ground frame acceleration of the orange disk.
8:32	Chapter 2: Coriolis Effect	
8:51	Activity	Describe curved motion of the ball in the turntable frame of reference.
9:09	Activity	Predict the motion of the ball as observed from the ground frame of reference.
10:10	Activity	Determine turntable frame acceleration of the ball.
12:05	Chapter 3: Bringing it back to Earth	
13:09	Activity	Explain why hurricanes in the Northern Hemisphere rotate counterclockwise.
14:08	To Review	
14:53	Activity	Use Coriolis Effect to explain direction of rotation of hurricanes in the Southern Hemisphere, and why hurricanes cannot form at the equator.

Video Summary

This video leads students through descriptions of the motion of two objects observed from two frames of reference: a rotating turntable, and the relatively stationary ground frame. The centripetal and Coriolis accelerations that arise in rotating frames of reference are explored. Finally, we use the Coriolis effect to explain the rotational direction of hurricanes in the Northern Hemisphere.

EPDM 301 Materials

Pre-Video Materials

When appropriate, this guide is accompanied by additional materials to aid in the delivery of some of the following activities and discussions.

1. Suppose that a spider is walking along a frisbee, that is being thrown through the air, on board a ship that is rocking in the ocean, which is on Earth, which is rotating about its axis, revolving about the sun, and accelerating through the universe.

(a) Determine all of the frames of reference.

(b) Which are inertial, if any? Explain.

2. Discussion question: Is there a universal reference frame? Why or why not.

Post-Video Materials

1. Suppose the reference frame M is translating, but not rotating, with respect to the G frame.

(a) Find a formula for the acceleration in the G frame in terms of the M frame acceleration.

(b) Consider that T is a frame that is both translating and rotating with respect to G. The third frame of reference, M, is translating, but not rotating with respect to G. The frame T is rotating, but not translating with respect to M. Find a formula for the G frame acceleration in terms of the T frame acceleration.

2. Is the direction of rotation of a toilet flush determined by the Coriolis effect? Why or Why not? Use approximations to determine the size effect of the Coriolis effect based on the radius of a rotating object.

3. Consider another model of the Earth where at any point on the sphere, the Earth can be modeled as the plane tangent to the sphere at that point. Determine the equation for the plane based on the latitude. Remind yourself of the "right-hand rule" for taking cross products.

Use this model to determine the direction of the centripetal and Coriolis accelerations for an object moving due north...

(a) At the equator.

(b) In the northern hemisphere, at a latitude 30 degrees north of the equator.

(c) In the southern hemisphere, at latitude of -30 degrees.

4. The following problem is adapted from Prof. Phil Marcus of UC Berkeley.

A hurricane has several important forces and accelerations that control its dynamics. One is the Coriolis acceleration, which from physics you know is the cross product $v \times 2\Omega$, where v is the velocity (of the wind of the hurricane) and Ω is the angular velocity vector of the rotating earth. We can make the crude approximation that the earth is flat in which case

$$\Omega = \Omega \sin(t)e_3 + \Omega \cos(t)\, e_2$$

where Ω is the scalar angular velocity of the earth, t is the latitude on earth where the hurricane is located, e_3 is the unit vector pointing straight upward, e_2 is the unit vector in the north direction (in our flat world), and e_1 is the unit vector in the eastern direction (in our flat world).

From problem 3, you found that the Coriolis acceleration has two components. One component is proportional to sin(t) and points radial outward from the center of the hurricane. The second component of the Coriolis acceleration is proportional to cos(t) and points in the z, or up-down direction. We can ignore the up-down component of the Coriolis force, and consider the air/water of the hurricane to be moving tangent to the plane of our flat-world model.

(a) Hurricanes always have low pressures at their centers compared to the high pressure of the air surrounding a hurricane. The acceleration due to this pressure difference is radially inward, and this acceleration balances the outward acceleration from the Coriolis acceleration.

When a hurricane matures, it stops growing in strength. The reason it stops growing is because another acceleration acting on the hurricane becomes comparable in magnitude to the pressure and Coriolis accelerations. This new acceleration is the centrifugal acceleration.

You know that the centrifugal acceleration on an object (such as a parcel of air orbiting around the center of a hurricane) is $|v|2/r$, where v is the azimuthal velocity around the center (in this case the eye of the hurricane) of the parcel's orbit and r is the radius of the orbit. A mature hurricane has Coriolis and centrifugal accelerations that are approximately the same; using this fact find the relationship between the magnitude $|v|$ of the winds of a hurricane and the hurricane's radius r.

Note that the scalar angular velocity Ω of the earth is $2\pi/T$, where T is the period of the rotating body (and the period for the earth is 1 day). The dimension of Ω is 1/time.

Write the relationship between the r and $|v|$ of the hurricane, t, and T. Check to see if your equation has the correct dimensions.

(b) Find images of hurricanes online. Estimate the latitude and the radius of the hurricane. Based on your estimates for the location and size, what is $|v|$?

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Additional Resources

Going Further

This video and the first reference below from the Physical Science Study Committee provide excellent foundations for students interested in pursuing study of relativtiy or more advanced kinematics.

References

The following video created by The Physical Science Study Committee in 1960 is an excellent introduction to frames of reference. This video provides an excellent "prequel" to the Rotating Frames of Reference video.

- Leacock, R. (Directors), Smith, K. & The Physical Science Study Committee (Producers), (1960). *Frames of Reference*, [Film].
  -Part 1 of 2 available from http://www.youtube.com/watch?v=pyBNImQkRuk
  -Part 2 of 2 available from http://www.youtube.com/watch?v=053nYXFfjZE

The following article is a reference for problem 2 in the Post-Video Materials subsection. It discusses myths and misconceptions surrounding the Coriolis effect.

- DiSpezio, M. A., (2011). A different spin on Coriolis: Applying frames of reference. *Science Scope*, *34*(5), 8–10.

The following text is the original reference for the frames of reference notation used with derivatives in this video.

- Kane, T. R. (1985). *Dynamics, theory and applications.* New York, NY: McGraw–Hill.