Dimensional Analysis

Problem Solving Series

Instructor's Guide

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Developed by the Teaching and Learning Laboratory at MIT
for the Singapore University of Technology and Design

Introduction

Key Information

Duration: 18:16
Narrator: Prof. Ken Kamrin
Materials Needed:
• Paper
• Pencil

When to Use this Video

- At the beginning of a design or capstone course, as homework or during lecture.
- Prior knowledge: unit analyisis, and the difference between dependent and independent variables

Learning Objectives

After watching this video students will be able to use dimensional analysis to estimate the size of a parachute canopy that can slow a rover to 90 m/s on its descent to Mars.

Motivation

- Dimensional analysis is an important and versatile problem solving tool. Using minimal physical knowledge, basic intuition for variable dependencies, and knowledge of the linear independence of dimension, dimensional analysis can help students solve a variety of problems.
- This video shows students how to apply Buckingham's Theorem to solve real world problems.

Student Experience

It is highly recommended that the video is paused when prompted so that students are able to attempt the activities on their own and then check their solutions against the video.

During the video, students will:

- Determine the dimension of energy.
- Identify independent variables for the parachute and rover system.
- Discuss why only diameter or surface area of the parachute canopy can be specified as an independent variable (and not both).
- Determine the dimensions of the variables in the parachute and rover system.
- Create dimensionless versions of the independent variables.
- Create expressions involving independent variables that have the same dimension as velocity. Then, determine how to define the real value function, phi, in the formula obtained through dimensional analysis in order to obtain those expressions.

Video Highlights

This table outlines a collection of activities and important ideas from the video.

Time	Feature	Comments
0:00	Mars Rover: Entry, Descent, and Landing sequence	Description of how the Mars Rover landed on Mars. Poses question: How do you know that a parachute-rover system designed on Earth will successfully land on Mars?
1:24	Prerequisite Knowledge and Learning Objectives	
1:44	Chapter 1: Dimension Defined	Dimension defined. The problem for determining parachute diameter for the Mars rover is posed.
255	Student Activity	Determine the dimension of energy in terms of the fundamental dimensions length, mass, and time.
4:39	Chapter 2: Indentifying the Variables	
6:00	Student Activity	Students consider why canopy surface area was not included in the list of independent variables.
8:13	Chapter 3: Dimensional Analysis, the Process	
8:48	Student Activity	Determine the dimension of each of the variables identified in our problem.
10:32	Student Activity	Determine dimensionless versions of variables.
13:10	Student Activity	Find expressions with correct dimension by defining real valued function in different ways.
14:41	Chapter 4: Experiments and Results	
16:36	To Review	

Video Summary

This video leads students through the problem solving method of dimensional analysis. Students use dimensional analysis to determine the diameter of a parachute needed to slow a rover to 90m/s in order to safely land on Mars. Students see how the general formula found using dimensional analysis applies on any planet, allowing for the use of experimental data collected on Earth to Mars.

Course Materials

Pre-Video Materials

When appropriate, this guide is accompanied by additional materials to aid in the delivery of some of the following activities and discussions.

1. Consider the follwing list of variables that can describe a spherical solid:

- density P
- diameter D
- mass M
- radius R
- surface area S
- volume V

(a) Which variables are independent of mass?

(b) Which variables are independent of radius?

(c) Which quantities are dependent on radius?

(d) What is the greatest number of independent variables you can form from this list and why?

2. Suppose you are told that acceleration is a function of velocity. Why is it necessary that acceleration be a function not only of velocity, but of velocity and time?

(Hint: think about units.)

Post-Video Materials

1. In the video, we used the variable *d,* for diameter of the parachute, instead of the surface area, *s*, of the parachute. Run through the process of dimensional analysis for the parachute problem, this time solving for the surface area, and show that the formula you find at the end is equivalent to the one we found in the video that solves for the diameter.

2. Use dimensional analysis to determine the period of a pendulum.

(a) Find the dependent and independent variables.

(b) Find dimensionless versions of the variables found in part (a).

(c) Find a formula for the period of the pendulum.

(d) Discuss which variables were important, and which were not. Does this match with your understanding of the period pendulum and the quantities on which it depends?

3. When we create the dimensionless versions of the variables in the dimensionless analysis process, the formulas we get are sometimes referred to as "scaling relationships". What does "scaling" mean? And why does it make sense to refer to these expressions as scaling relationships?

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Going Further

Dimensional analysis uses linear independence of dimensions to find formulas for unknown quantities in a system. Dimensional analysis is the problem solving tool of choice when creating scaled experiements. The power of this tool comes from the fact that it uses mathematics rather than physical knowledge to derive these formulas. This makes it an ideal first step will dealing with complex or unfamiliar systems. These formulas can then be confirmed experiementally, and used to predict system behavior in a variety of scaled environments.

References

The following resources contain problems, examples, and methods for using dimensional analysis.

- Mahajan, S. (2010). *Streetfighting Mathematics: the art of educated guessing and opportunistic problem solving*, Cambridge MA: MIT Press.
- Misic, T., Najdanovic-Lukic, M., & Nesic, L., (2010). Dimensional analysis in physics and the Buckingham theorem, *Eur. J. Phys.31,* 893-906.

The follwing video reference introduces dimension and dimensional anlaysis.

- Lewin, W., *8.01 Classical Mechanics*, Fall 1999. (Massachusetts Institute of Technology: MIT OpenCouseWare), http://ocw.mit.edu (Accessed November 27, 2012). License: Creative Commons BY-NC-SA
  -Lecture 1: Units and Dimensional Analysis

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