

# Conditional Probability

## Probability and Statistics Series

### *Instructor's Guide*

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# Introduction

## When to Use this Video

- In EPDE 101, at home, in class, or in recitation, before or during Lecture 5: Conditional Probabilities.
- Prior knowledge: Basic probability vocabulary and the definition of conditional probability

## Learning Objectives

After watching this video students will be able to:

- Calculate the conditional probability of a given event using tables and trees.
- Understand how conditional probability can be used to interpret medical diagnoses.

## Motivation

- Students often learn to calculate conditional probabilities using equations taught in class, without deep understanding.
- Students have difficulty transferring their understanding to different examples.
- It is important for students to realize that in some situations, we must consider only probabilities within a specific and much smaller subset.

## Student Experience

It is highly recommended that the video is paused when prompted so that students are able to attempt the activities on their own and then check their solutions against the video.

During the video, students will:

- Be introduced to basic conditional probability notation.
- Calculate the probability of an event given certain conditions.
- Calculate conditional probabilities using both two-way tables and tree diagrams.
- Determine the likelihood of cancer from a given medical diagnosis.

## Key Information

*Duration:* 12:28

*Narrator:* Samuel S. Watson

*Materials Needed:*

- paper
- pencil

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## Video Highlights

This table outlines a collection of activities and important ideas from the video.

Time	Feature	Comments
00:00	Cancer diagnosis example	
00:38	Chapter 1: Basic Conditional Probability	Introduction to conditional probability, its proper notation, and how to use a two-way table to calculate probabilities.
01:05	Marbles in bowl example	The distribution of marbles in two bowls, bowl A and bowl B is described, and expressed in the form of a table.
01:25	Student Activity	Calculate the probability of drawing a particular color of marble from two bowls. Introduce basic definition of probability.
02:08	Student Activity	Calculate the probability of drawing a blue marble given a restricted sample space (Bowl A).
03:10	Conditional probability notation introduced	
03:35	Student Activity	Calculate the conditional probability of drawing a marble from Bowl A, given its color.
04:40	Chapter 2: More on Conditional Probability	Use tree diagrams to visualize and calculate the probabilities of drawing marbles in succession.
05:43	Student Activity	Complete tree diagram with probability of color for a second marble, given the color of the first marble.
06:54	Student Activity	Calculate the probability of drawing various combinations of two marbles successively.
07:40	Student Activity	Calculate the conditional probability of drawing a blue marble first, given that the second is yellow.
08:43	Chapter 3: Understanding Medical Diagnoses	Revisit cancer example in more detail and use conditional probability to make sense of it.
10:36	Student Activity	Calculate the probability that one actually has cancer, given a positive test result.

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### Video Summary

This video provides an introduction to conditional probability and its calculations. The first chapter reviews basic probability terminology and introduces standard conditional probability notation using a simple marble drawing example. The second chapter introduces the use of tree diagrams to help visualize the sample space and allow for more complex probability calculations. In the last chapter, we see how conditional probability can and must be used to make sense of medical diagnoses.

# EPDE 101 Materials

## Pre-Video Materials

When appropriate, this guide is accompanied by additional materials to aid in the delivery of some of the following activities and discussions.



1. Probability tells us the chance that something will happen. A basic knowledge of probability helps us to understand the events and decisions that we face in our everyday lives. Discuss specific examples of how probability is used in the real world. (E.g., weather report, chances of being struck by lightning, statistical significance of experiments, etc.)



2. Basic probability and conditional probability questions that introduce the use of two-way tables.

Below is a two-way table, showing U.S. Senators from the 113th Congress, classified by party affiliation and by gender.

	Male	Female
Democrats	36	16
Republicans	42	4
Independents	2	0

*Basic probability questions.* Students should be able to answer these without prompting. If we select a Senator at random, what is the probability that we choose:

- (a) A male?
- (b) A female?
- (c) A Democrat?
- (d) A Republican?

*Basic conditional probability questions.* Students should be able to answer these easily. However, ask them what they did differently from the above calculations. If necessary, help students to realize that the sample space has changed due to a restrictive condition.

- (a) What fraction of women in the Senate are Democrats?
- (b) What percent of women in the Senate are Democrats?
- (c) What fraction of Democrats in the Senate are women?
- (d) What percent of Democrats in the Senate are women?

Finally, pose these two questions to students and have them do the calculations. They should realize that these questions seem similar, but are in fact very different!

- (a) Is it true that most women in the Senate are Democrats?
- (b) Is it true that most Democrats in the Senate are women?

## Post-Video Materials



1. Derive  $P(A \text{ and } B) = P(B|A) \times P(A)$  from the probability tree.



2. Re-evaluate the medical test diagnosis example from the video, using a two-way frequency table instead of a tree diagram. Explain that different methods can be used to solve the same problem.

	cancer	no cancer
tests +	990	9990
tests -	10	990,010
total	1000	999,000



3. When would one use a tree diagram? How about a two-way table? What are the advantages of one representation method over another? What are some other methods that students may use to solve probability problems?

# Additional Resources

## Going Further

Students use their understanding of basic probability to make sense of their everyday lives, but often overlook the importance of conditional information. Now that students have a better understanding of conditional probability, they should seek to reinforce and apply this information in both their personal and professional lives. Introduce more advanced problems with implicit conditioning events to give students practice in recognizing when conditional probabilities should be applied. Discuss how professionals (e.g. researchers, executives, engineers, etc.) might use conditional probability knowledge in each particular field of work.

## References

The following resources provide an excellent explanation of conditional probability and contain problems, examples, and methods for using conditional probability:

- Grinstead, C.M., Snell, J.L. (1998). Introduction to probability. *Americal Mathematical Society*. (Chapter 4: Conditional Probability)
- Seier, E., Joplin, K.H. (2011). Introduction to Statistics in a Biological Context, *CreateSpace Independent Publishing Platform* (Chapter 6: Conditional Probability)

The following resources contains example probability problems and teasers:

- Be an Actuary Probability Exam - <http://www.casact.org/admissions/syllabus/index.cfm?fa=1syllabi>
- Bar-Hillel, M., and Falk, R. (1982). Some teasers concerning conditional probability. *Cognition*, 11, 109–122.

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