S O L U T I O N S

Feedback Compensation of an Operational Amplifier 12

Note: All references to Figures and Equations whose numbers are *not* preceded by an "S" refer to the textbook.

Solution 12.1 (P13.5)

This approximate $a(s)$ is identical to the approximate $a(s)$ given by Equation 13.20 of the textbook with $K = 2 \times 10^{-4}$ mho. If a single capacitor is used for the compensating element, and, as is the case here, the crossover frequency is low relative to higher-order singularities, then Equation 13.26 applies. Here, $f_o = 1$, thus the closed-loop crossover frequency is given by $\omega_h = \frac{K}{C_c}$. This can be set to 10^6 rad/sec by choosing $C_c = \frac{2 \times 10^{-4}}{10^6} = 200$ pF.

Solution 12.2 (P13.6)

The loop transmission for the log circuit of Figure 13.9*a* is given by Equation 13.19 of the textbook. At room temperature, $\frac{q}{kT} \simeq 40$, thus the loop transmission is $L(s) = -40\, a(s)v_I$. The loop transmission varies from 0 to $-400\, a(s)$ as the input varies from 0 to $+10$ volts. Thus, we need to maintain adequate phase margin over a wide range of frequencies. This requires single-pole compensation.

With single-pole compensation, if the system is stable for the largest loop-transmission magnitude, then it will be stable for all smaller loop-transmission magnitudes. Thus, we force crossover at 1 MHz when $v_I = +10$ volts, and the crossover will occur at lower frequencies with adequate phase margin for all $0 \leq v_I < 10$ volts. Letting $Y_c(s) = Cs$, and evaluating the magnitude at 1 MHz, we have

$$\left| L(j\omega) \right| \Big|_{\omega = 6.28 \times 10^6 \text{ rad/sec}} = 400 \times \frac{2 \times 10^{-4}}{C \times 6.28 \times 10^6} = \frac{1.27 \times 10^{-8}}{C} \qquad \textbf{(S12.1)}$$

Unity-gain crossover at 1 MHz is set by choosing $C = 1.27 \times 10^{-8} \simeq 0.013\ \mu\text{F}$.

Now, with $V_I = 0.1$ volt, the loop-transmission magnitude is reduced by a factor of 100 from the value when $V_I = 10$ volts, and crossover occurs at 10 kHz. The circuit step response will be first order with a time constant $\tau = \dfrac{1}{2\pi \times 10^4} = 16\ \mu\text{sec}$. Settling to within 1% of final value requires that $e^{-t/\tau} = 0.01$, which is solved by $t = 4.6\tau = 73\ \mu\text{sec}$.

Solution 12.3 (P13.7)

The closed-loop response of a unity-gain inverting amplifier is given by $A(s) = -\frac{1}{2}\dfrac{a(s)}{1 + \frac{1}{2}a(s)}$. As is apparent from this expression, the closed-loop steady-state response to a sinusoid at 10 kHz is determined entirely by the magnitude and phase of $a(s)$ at 10 kHz. We shall see that the two-pole compensation yields better phase accuracy, with slightly less closed-loop gain accuracy than the single-pole compensation, due to the differing magnitude and phase of $a(s)$ under the two compensation schemes.

The loop transmission is $-\frac{1}{2}a(s)$. Thus, for single-pole compensation, to cross over at 1 MHz, we must choose $a'(s) = \dfrac{10^6}{\dfrac{0.5s}{2\pi} + 1}$. At 10 kHz, $a'(s)$ is

$$a'(j2\pi 10^4) = \frac{10^6}{j5 \times 10^3 + 1} = 199.99999\ e^{-j1.5705963} \quad \textbf{(S12.2)}$$

Thus, the single-pole compensated closed-loop response at 10 kHz is

$$\begin{aligned} A'(j2\pi 10^4) &= -\frac{1}{2}\frac{199.99999\ e^{-j1.5705963}}{1 + \frac{1}{2} \times 199.99999\ e^{-j1.5705963}} \\ &= 0.99994800 \measuredangle 179.427^\circ \end{aligned} \quad \textbf{(S12.3)}$$

where phasor notation has been used to indicate the angle in degrees.

For two-pole compensation, many choices are possible; however, for simplicity, we choose to place the compensating zero at 100 kHz, a factor of 10 below crossover. The double poles must then be placed at 447 Hz to set crossover at 1 MHz. That is, $a''(s)$ is given by

$$a''(s) = \frac{10^6\left(\dfrac{s}{2\pi \times 10^5} + 1\right)}{\left(\dfrac{s}{2\pi \times 447} + 1\right)^2} \tag{S12.4}$$

At 10 kHz, $a''(s)$ is

$$a''(j2\pi 10^4) = \frac{10^6(0.1j + 1)}{\left(\dfrac{j10^4}{447} + 1\right)^2} = 2004.0513\, e^{-j2.9525834} \tag{S12.5}$$

Thus, the two-pole compensated closed-loop response is

$$\begin{aligned} A''(j2\pi 10^4) &= -\frac{1}{2}\,\frac{2004.0513\, e^{-j2.9525834}}{1 + \frac{1}{2} \times 2004.0513\, e^{-j2.9525834}} \\ &= 1.0009811 \angle 179.989^\circ \end{aligned} \tag{S12.6}$$

again in phasor notation.

Thus, for the single-pole compensation the closed-loop gain is accurate to within about 0.005%, with a phase error of about 0.57°. For the two-pole compensation, the closed-loop gain is accurate to within about 0.1%, with a phase error of about 0.01°. So in terms of phase accuracy, the two-pole compensation is far superior. For gain accuracy, the single-pole scheme is better.

A good approximation to the above results can be derived with far less computational effort by using the asymptotic values for $a'(s)$ and $a''(s)$. That is, assume that at 10 kHz, $a'(j2\pi 10^4) \simeq 200\, e^{-j\pi/2}$ and $a''(j2\pi 10^4) \simeq 2000\, e^{-j\pi}$. Then, plug these approximations into the expressions for $A'(j2\pi 10^4)$ and $A''(j2\pi 10^4)$. The results will be essentially the same as the detailed analysis above.

Solution 12.4 (P13.8)

(a) With the given input, the output of the amplifier will be a ramp with a slope of 10^5 volts per second. The input to the amplifier is 10 mV. Following the discussion of Section 13.3.3, we assume that the amplifier functions as an integrator on an open-loop basis, that is, $a(s) \simeq \frac{k}{s}$. The constant k is in volts per second per volt and is given by $k = \frac{10^5 \text{ V/sec}}{10 \text{ mV}} = 10^7$. Thus $a(s) \simeq \frac{10^7}{s}$.

(b) Following the discussion of Sections 5.3, 9.2.3, and 13.3, the magnitude of the open-loop response of the amplifier will follow the lower of the single-pole approximation and the uncompensated transfer function. Thus, we can use the transfer function given in the problem statement to refine the transfer-function estimate. The two magnitude curves are shown in Figure S12.1. We have omitted the corresponding phase curves because they are unnecessary for this problem. The more accurate approximation is shown as the darkened line indicating the lower of the two curves.

(c) For an LM301A, from Equation 13.20, $a(s) \simeq \frac{K}{Y_c(s)}$, where $K = 2 \times 10^{-4}$ mho. From part a, we have $a(s) \simeq \frac{10^7}{s}$. Thus, $Y_c(s) = \frac{Ks}{10^7} = 2 \times 10^{-11}s$. This is the admittance for a 20 pF capacitance, which is therefore the compensating element.

(d) As described in Section 13.3.3, for essentially zero steady-state ramp error, we select two-pole compensation. From Equation 13.36, for two-pole compensation,

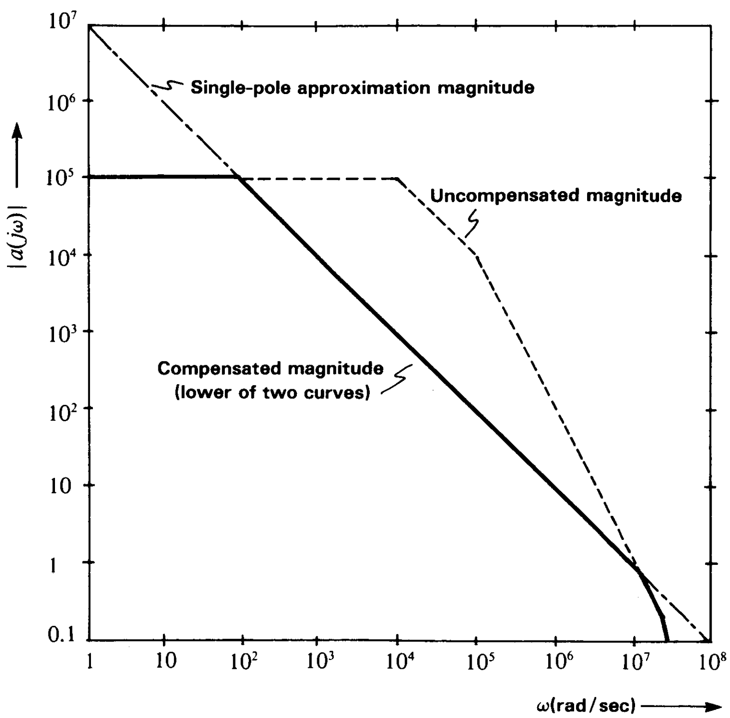

Figure S12.1 Open-loop transfer functions for amplifier of Problem 12.4 (P13.8).

$$a(s) \simeq \frac{K'(\tau s + 1)}{s^2} \qquad \textbf{(S12.7)}$$

where $\tau = R(C_1 + C_2)$ and $K' = K/RC_1C_2$. As usual, $K = 2 \times 10^{-4}$ mho. The compensating network topology is shown in Figure 13.19 of the text. The results of part b indicate that the single pole compensated gain-of-ten amplifier loop transmission will cross over at 10^6 rad/sec, with about 90° of phase margin. We now design the two-pole compensator to have the same crossover frequency. We locate the zero a decade below crossover to guarantee adequate phase margin. Thus, $\tau = 10^{-5}$ sec. Then, to set crossover at 10^6 rad/sec, we must have $K'\tau = 10^7$, which gives $K' = 10^{12}$. One more constraint is required to solve for the compensating element values. This represents an extraneous degree of freedom, which we eliminate by choosing $C_1 = C_2$. Then, the above equations in τ and K' allow us to solve for $C_1 = C_2 = 40$ pF, and $R = 125$ kΩ. The resulting phase margin is about 84°.