COMPUTATION OF THE DISCRETE FOURIER TRANSFORM - PART 1

Solutions 18.1

The flow-graph of Fig. 9.3 of the text is based on the decomposition of X(k) in the form of equation 9.14 of text. For N = 16, the corresponding flow-graph expressing X(k) as a combination of two eight-point DFT's is shown in Figure S18.1-1 below.

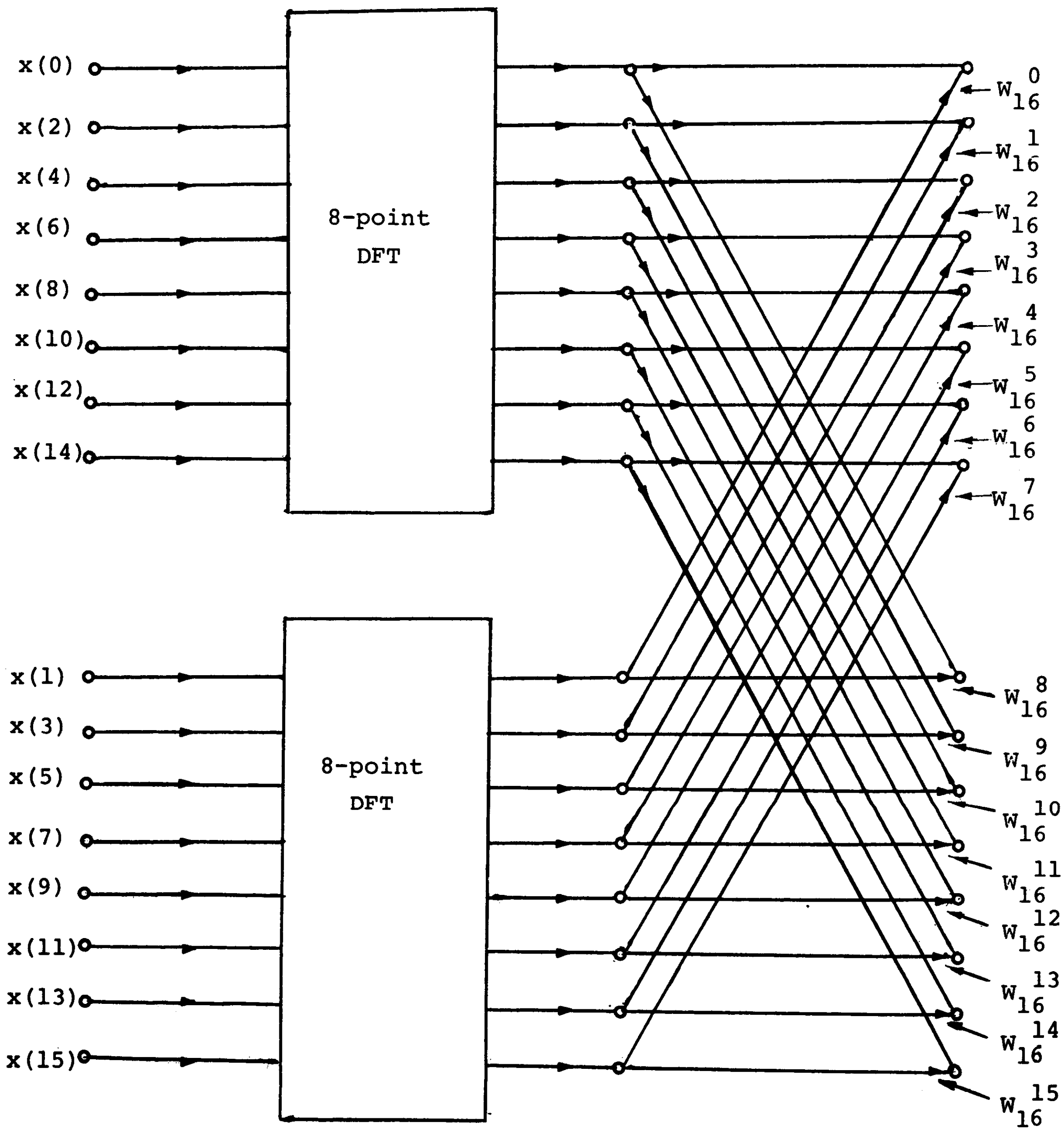

Figure S18.1-1

Solution 18.2

From the expression for the inverse DFT it follows that

$$N\, x^*(n) = \sum_{k=0}^{N-1} X^*(k)\, e^{-j(\frac{2\pi}{N})kn}$$

Thus by using as the input to the DFT program the complex conjugate of X(k), the output sequence will be N times the complex conjugate of x(n).

Solution 18.3

(a) $\frac{N}{2}$

(b) In proceeding from array (m - 1) to array m we are combining $2^{(m-1)}$ point DFT's to form 2^m point DFT's. Thus the coefficient are successive powers of W_M where $M = 2^m$. Thus these coefficients are W_M^k where $k = 0,1,2,\ldots(\frac{M}{2} - 1)$ or, since

$$W_M = (W_N)^{N/M}$$

The powers of W_N involved in computing the $m^{\underline{th}}$ array from the (m - 1)st array are

$$W_N^{Nk/M} \qquad k = 0,1,2,\ldots(M/2 - 1)$$

(c) $2^{(m-1)}$

(d) 2^m for $1 \le m \le (\log_2 N)-1$. For the last array $(m=\log_2 N)$ there are no two butterflies utilizing the same coefficients.

Solution 18.4

With $g(n) = x_1(n) + j\, x_2(n)$,

$$G(k) = X_1(k) + j\, X_2(k)$$

With $X_1(k)$ and $X_2(k)$ expressed in terms of their real and imaginary parts,

$$X_1(k) = X_{1R}(k) + j\, X_{1I}(k)$$

$$X_2(k) = X_{2R}(k) + j\, X_{2I}(k)$$

G(k) can be written as

$$G(k) = [X_{1R}(k) - X_{2I}(k)] + j\, [X_{2R}(k) + X_{1I}(k)]$$

Now, since $x_1(n)$ and $x_2(n)$ are real, $X_{1R}(k)$ and $X_{2R}(k)$ are even and $X_{1I}(k)$ and $X_{2I}(k)$ are odd. Thus,

$$G_{ER}(k) = X_{1R}(k)$$

$$G_{OR}(k) = -X_{2I}(k)$$

$$G_{EI}(k) = X_{2R}(k)$$

$$G_{OI}(k) = X_{1I}(k)$$