DESIGN OF FIR DIGITAL FILTERS

1. Lesson 17 -39 minutes

DESIGN OF FIR DIGITAL FILTERS

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

FOR LINEAR PHASE $h(n) = h(N-1-n)$

BASIC DESIGN METHODS:

① WINDOWS

② FREQUENCY SAMPLING

③ EQUIRIPPLE DESIGN

Basic design methods for FIR digital filters

a.

DESIGN OF FIR FILTERS USING WINDOWS

DESIRED UNIT SAMPLE RESPONSE: $h_d(n)$

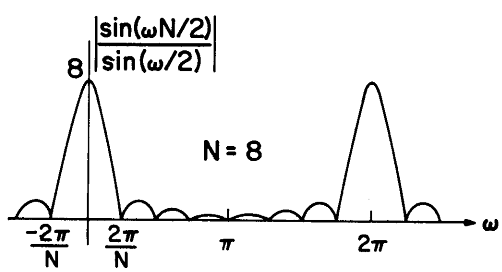
$$h(n) = w(n) h_d(n)$$

$$w(n) = 0 \qquad n < 0,\ n > (N-1)$$

$$H(e^{j\omega}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\theta})\, W\left[e^{j(\omega-\theta)}\right] d\theta$$

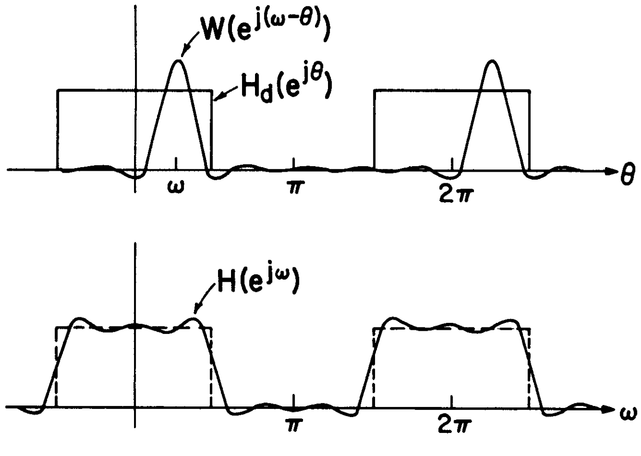
Design of FIR filters using the window method.

b.



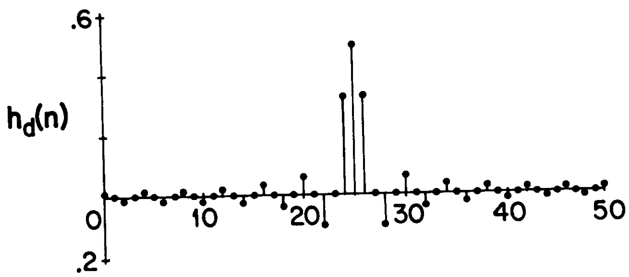
Magnitude of the Fourier transform for an eight point rectangular window.

c.



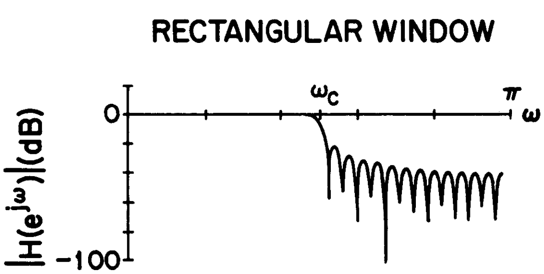
Effect of convolving the Fourier transform of a rectangular window with an ideal low pass filter characteristic.

d.



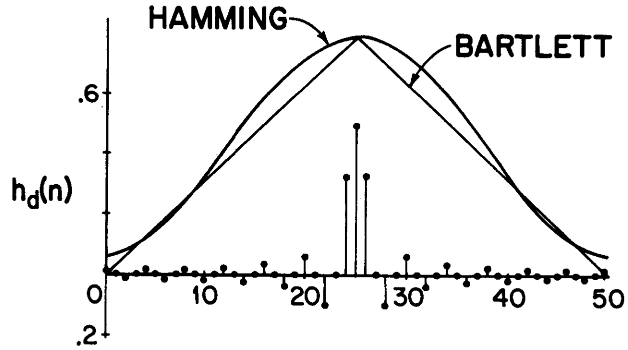
Unit-sample response of an ideal low-pass filter truncated by a 51-point rectangular window.

e.



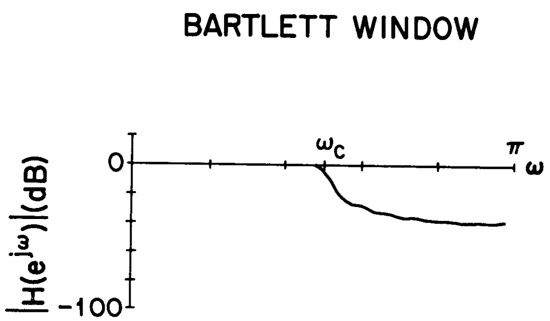
Frequency response corresponding to the unit-sample response in viewgraph e.

f.



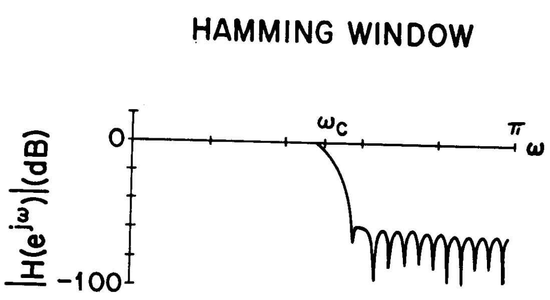
The Hamming and Bartlett windows.

g.



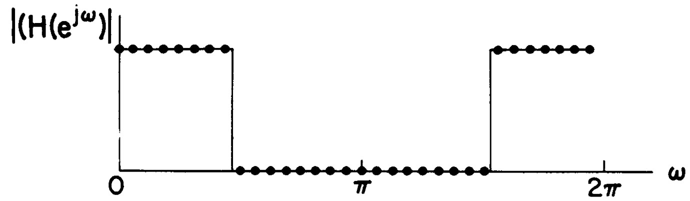
Frequency response of an FIR lowpass filter obtained by multiplying the unit-sample response of an ideal low pass filter by a Bartlett window.

h.



Frequency response of an FIR lowpass filter filter obtained by multiplying the unit sample response of an ideal lowpass filter by a Hamming window. (Note that the stopband attenuation is approximately 65 db <u>not</u> 30 db as stated in the lecture.)

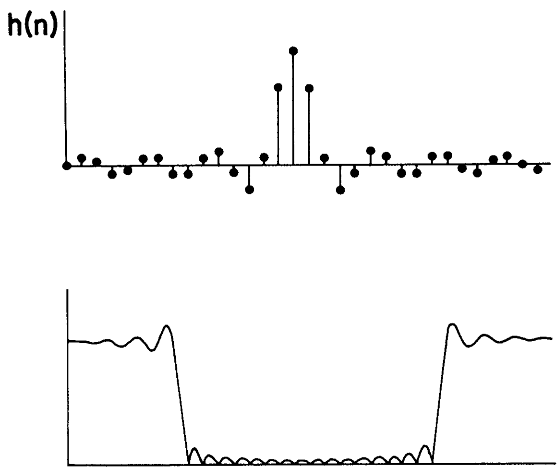
i.



$$H(e^{j\omega_k}) = H_D(e^{j\omega_k}) \quad \omega_k = \frac{2\pi}{N}k \quad k = 0,1,\ldots,(N-1)$$

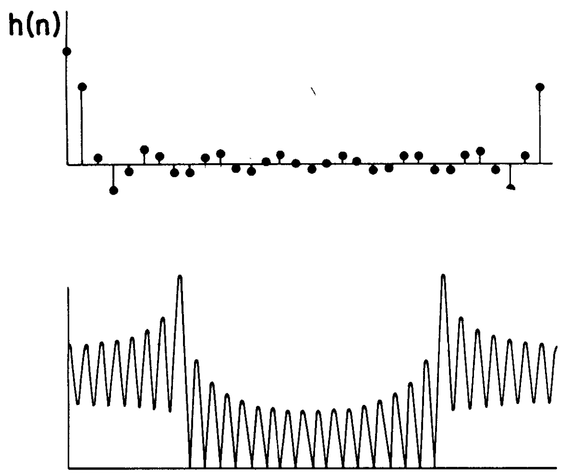
Equally spaced frequency samples of an ideal low pass filter.

j.



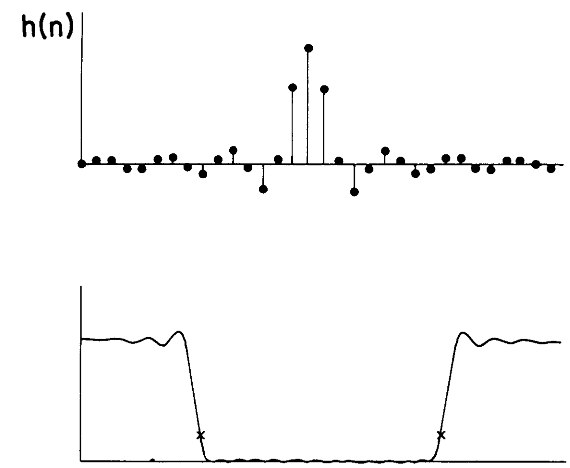
A unit-sample response the magnitude of whose DFT is equal to the frequency samples in viewgraph j.. The bottom trace is the magnitude of the Fourier transform of this unit-sample response.

k.



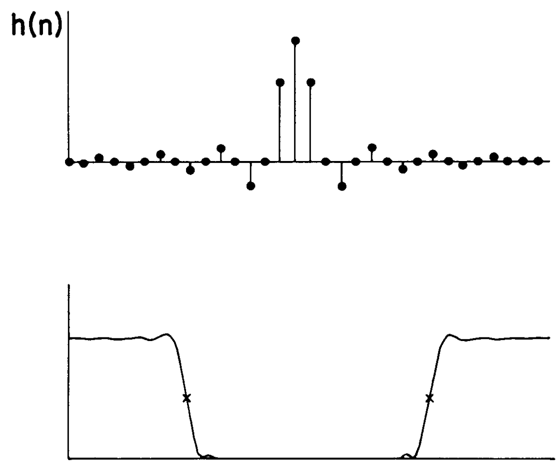
Another unit-sample response the magnitude of whose DFT is equal to the frequency samples in viewgraph j.. The bottom trace is the magnitude of the Fourier transform of this unit-sample response.

l.



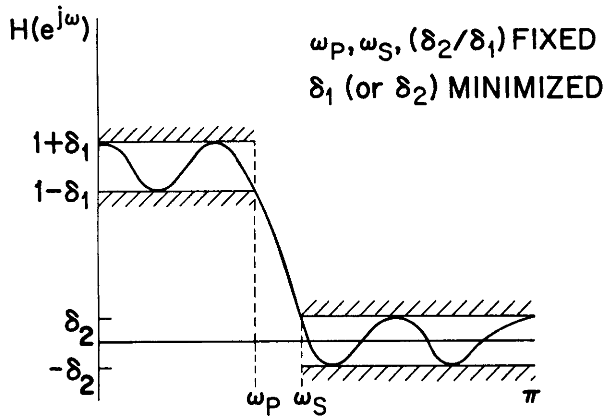
Unit sample response and frequency response when one frequency sample is moved from the stopband into the transition band.

m.



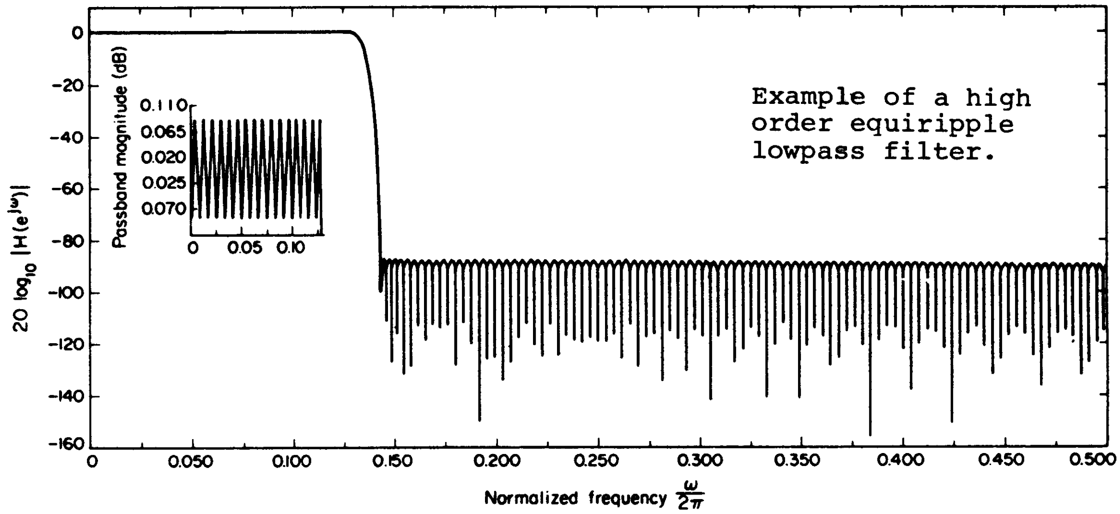
Similar to viewgraph m. with a different value of the frequency sample in the transition band.

n.



Equiripple approximation of a lowpass filter.

o.



p.

2. Comments

FIR filters are an important class of digital filters, and in contrast with continuous-time FIR filters, the implementation of digital filters of this type is relatively straightforward.

Design techniques for FIR digital filters are generally carried out directly in the discrete-time domain. This lecture introduces the three primary design techniques, specifically the window method, the frequency sampling method, and the algorithmic design of optimum filters. The window method basically begins with a desired unit-sample response which is then truncated by means of a finite duration window. In the frequency sampling method, the frequency response of the FIR filter is specified in terms of samples of the desired frequency response. The first two design procedures generally do not result in optimum filter designs. There is available an algorithmic design procedure which generates optimum equiripple FIR filter designs. While I mention this technique only briefly in the lecture, it is developed in considerable detail in the text in sections 7.6 and 7.7

3. Reading

Text: Section 7.4 (page 444) and section 7.8.

Problem 17.1

In the window method of design for lowpass FIR filters we indicated that the transition width of the resulting filter is dependent primarily on the width of the main lobe of the Fourier transform of the window. For the purposes of this problem, we define the main lobe as the symmetric interval between the first negative and positive frequencies at which $W(e^{j\omega}) = 0$. Consider the following three windows:

Rectangular:	$w_R(n) = 1$	$\lvert n\rvert \le N-1$
	$= 0$	otherwise
Bartlett:	$w_B(n) = 1 - \frac{\lvert n\rvert}{N}$	$\lvert n\rvert \le N-1$
	$= 0$	otherwise
Raised cosine:	$w_H(n) = \alpha+\beta\cos(\pi n/(N-1))$	$\lvert n\rvert \le N-1$
	$= 0$	otherwise

(If $\alpha=\beta=0.5$ this is the Hanning window and if $\alpha=0.54$ and $\beta=0.46$ this is the Hamming window.)

Determine the Fourier transforms of each of these windows. Also, determine the width of the main lobe for each window, assuming that $N >> 1$.

Problem 17.2

Let $h_1(n)$ and $h_2(n)$ denote the unit sample responses of two FIR filters of length 8. The two are related by a four-point circular shift, i.e.,

$$h_2(n) = h_1((n-4))_8 \, R_8(n) \; .$$

The Fourier transform of $h_1(n)$ is sketched in Figure P17.2-1, and corresponds to a low pass filter with cut-off frequency $\frac{\pi}{2}$.

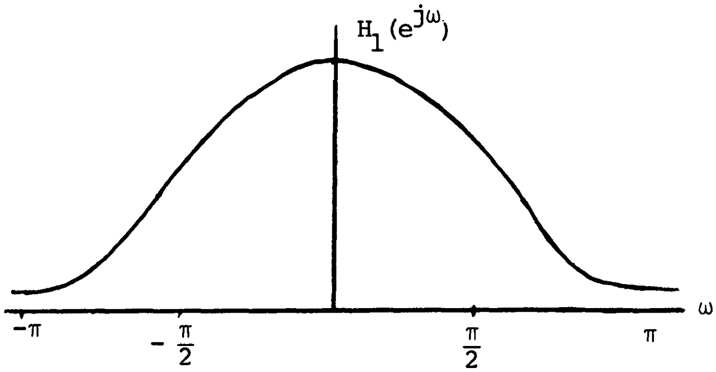


Figure P17.2-1

(a) Determine the relationship between the DFT of $h_1(n)$ and $h_2(n)$ and show in particular that the magnitude of the DFT of $h_1(n)$ and $h_2(n)$ are equal.

(b) Would it also be reasonable to consider $h_2(n)$ to correspond to a lowpass filter with cut-off frequency of $\frac{\pi}{2}$?

Problem 17.3

Let $h_d(n)$ denote the unit-sample response of an ideal desired system with frequency response $H_d(e^{j\omega})$ and let $h(n)$ denote the unit-sample response, of the length N sampled, for an FIR system with frequency response $H(e^{j\omega})$. In Sec. 5.6 it was asserted that a rectangular window of length N samples applied to $h_d(n)$ will produce a unit-sample response $H(n)$ such that the mean-square error

$$\varepsilon^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |H_d(e^{j\omega}) - H(e^{j\omega})|^2 \, d\omega$$

is minimized.

(a) The error function $E(e^{j\omega}) = H_d(e^{j\omega}) - H(e^{j\omega})$ can be expressed as the power series

$$E(e^{j\omega}) = \sum_{n=-\infty}^{\infty} e(n) e^{-j\omega n}$$

Find the coefficients $e(n)$ in terms of $h_d(n)$ and $h(n)$.

(b) Express the mean-square error ε^2 in terms of the coefficients $e(n)$.

(c) Show that for a unit-sample response $h(n)$ of length N samples, ε^2 is minimized when

$$h(n) = \begin{cases} h_d(n), & 0 \leq n \leq N - 1 \\ 0, & \text{otherwise} \end{cases}$$

That is, the rectangular window gives the best mean-square approximation to a desired frequency response for a fixed value of N.