18

Discrete-Time Processing of Continuous-Time Signals

One very important application of the concept of sampling is its role in processing continuous-time signals using discrete-time systems. Specifically, the continuous-time signal, which either is assumed to be bandlimited or is forced to be bandlimited by first processing with an anti-aliasing filter, is sampled and the samples are converted to a discrete-time representation. The discrete-time signal may, for example, represent values in successive locations in a digital memory. After being processed with a discrete-time system, the sequence is "desampled"; that is, a continuous-time signal is reconstructed, ideally through bandlimited interpolation, by converting the sequence to an impulse (or pulse) train followed by lowpass filtering. In our discussions of discrete-time signals and systems, we have made a point of indexing them on an integer variable without reference to a "sampling period" since discrete-time signals arise in a wide variety of ways besides periodic time sampling.

In converting the impulse train of samples to a sequence of samples, we are in effect normalizing the time axis. In the previous lecture we discussed the effect of this in the Fourier domain, concluding that the discrete-time Fourier transform of the sequence of samples is basically the same as the continuous-time Fourier transform of the impulse train resulting after the sampler, with the exception that the frequency axis is normalized. The discrete-time sequence is processed by a discrete-time LTI system whose frequency response is likewise represented on a normalized frequency axis. Converting the filtered output sequence back to a continuous-time signal can be interpreted as "unnormalizing" the frequency axis, with the eventual conclusion that the overall system is equivalent to a continuous-time filter whose frequency response is basically the same as the frequency response of the discrete-time filter with a linear scaling of the frequency axis. Thus, for example, if the cutoff frequency of the discrete-time filter is one-tenth of 2π, then the

equivalent continuous-time filter will have a cutoff frequency that is one-tenth of the sampling frequency. Because of this dependence on the sampling frequency, with a fixed cutoff frequency for the discrete-time filter, the cutoff frequency for the equivalent continuous-time filter can be varied by varying the sampling frequency.

In this lecture a number of the points mentioned above are illustrated through a demonstration. The system demonstrated involves sampling a continuous-time signal and filtering the resulting sequences with a lowpass filter with an approximate cutoff frequency that is one-tenth of 2π. The resulting output sequence is then used to reconstruct a continuous-time signal using (approximately) bandlimited interpolation. In the first part of the demonstration, we show the impulse response of the system. In particular, the discrete-time filter corresponds to a linear-phase filter with a finite impulse response.

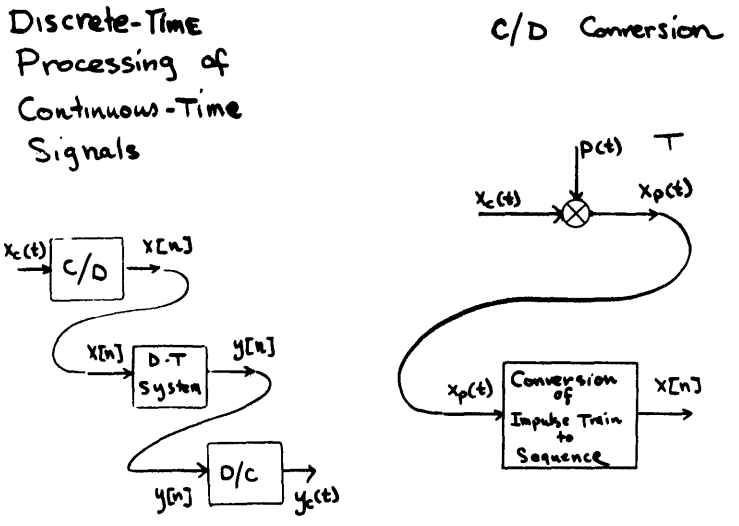
In the second part of the demonstration, we illustrate the frequency response of the equivalent continuous-time filter by putting a sinusoidal signal at the input and observing the response both as a function of time and as a function of input frequency. It is important to note that the system demonstrated does not have an anti-aliasing filter. Consequently, as the input sinusoidal frequency increases past half the sampling frequency, aliasing results and the input sequence to the discrete-time filter begins to decrease in frequency even though the frequency of the continuous-time signal is increasing. Thus, as we observe the overall output as the input sinusoid sweeps from zero to half the sampling frequency, the output moves through the passband into the stopband. As the input frequency increases further, the resulting output will be associated not with the input frequency but with the aliased frequency; thus, as the input frequency continues to increase we will see the output behave as though the passband is repeated. The two consistent interpretations of this periodicity in the frequency response are (1) that it is a consequence of aliasing on the input and (2) that it is a consequence of the periodicity of the discrete-time filter. With either interpretation, if an anti-aliasing filter had been present at the input, all frequencies above half the sampling frequency would be rejected before the sampler and this periodic repetition of the passband would not occur.

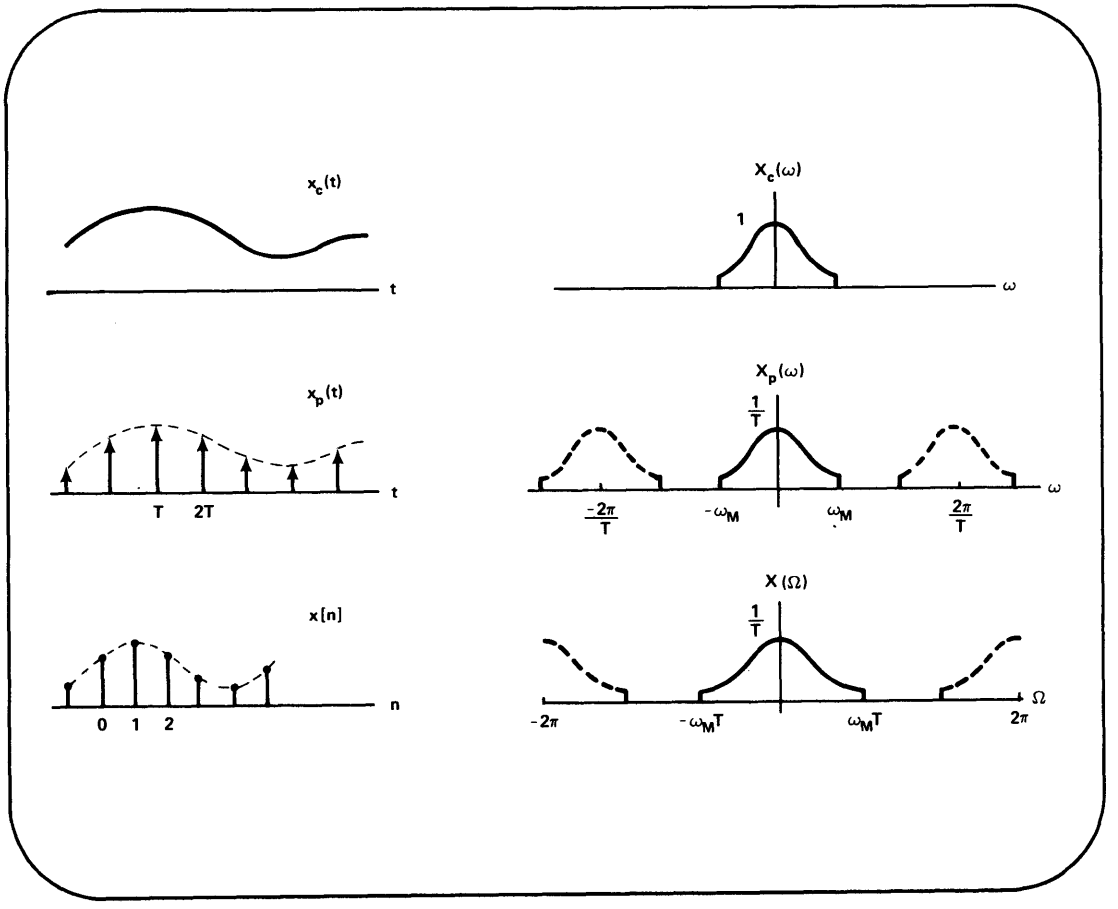
In the third part of the demonstration, we illustrate the way in which the cutoff frequency of the overall continuous-time filter is dependent on the sampling frequency. Specifically, since the equivalent cutoff frequency is one-tenth of the sampling frequency, as the sampling frequency increases or decreases, the equivalent cutoff frequency of the continuous-time filter will also.

Suggested Reading

Section 8.4, Discrete-Time Processing of Continuous-Time Signals, pages 531–539

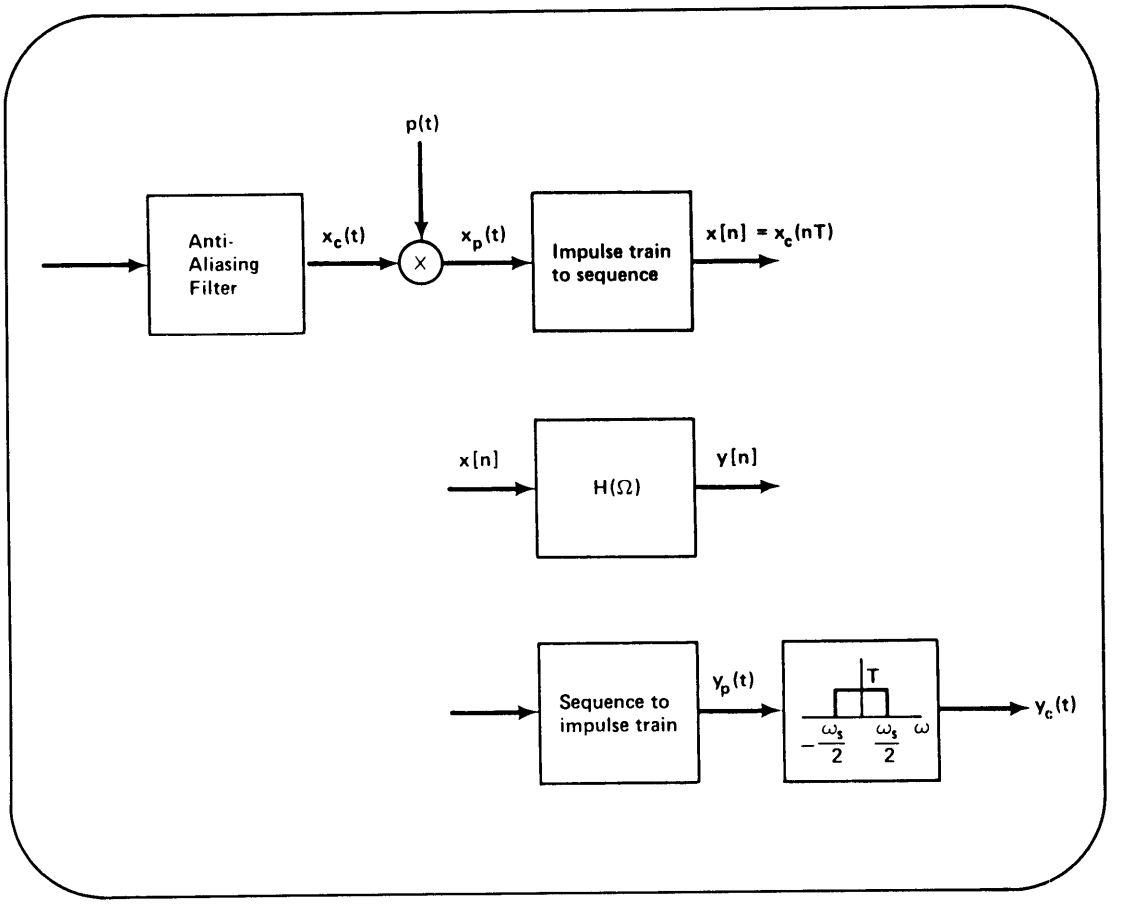
MARKERBOARD
18.1



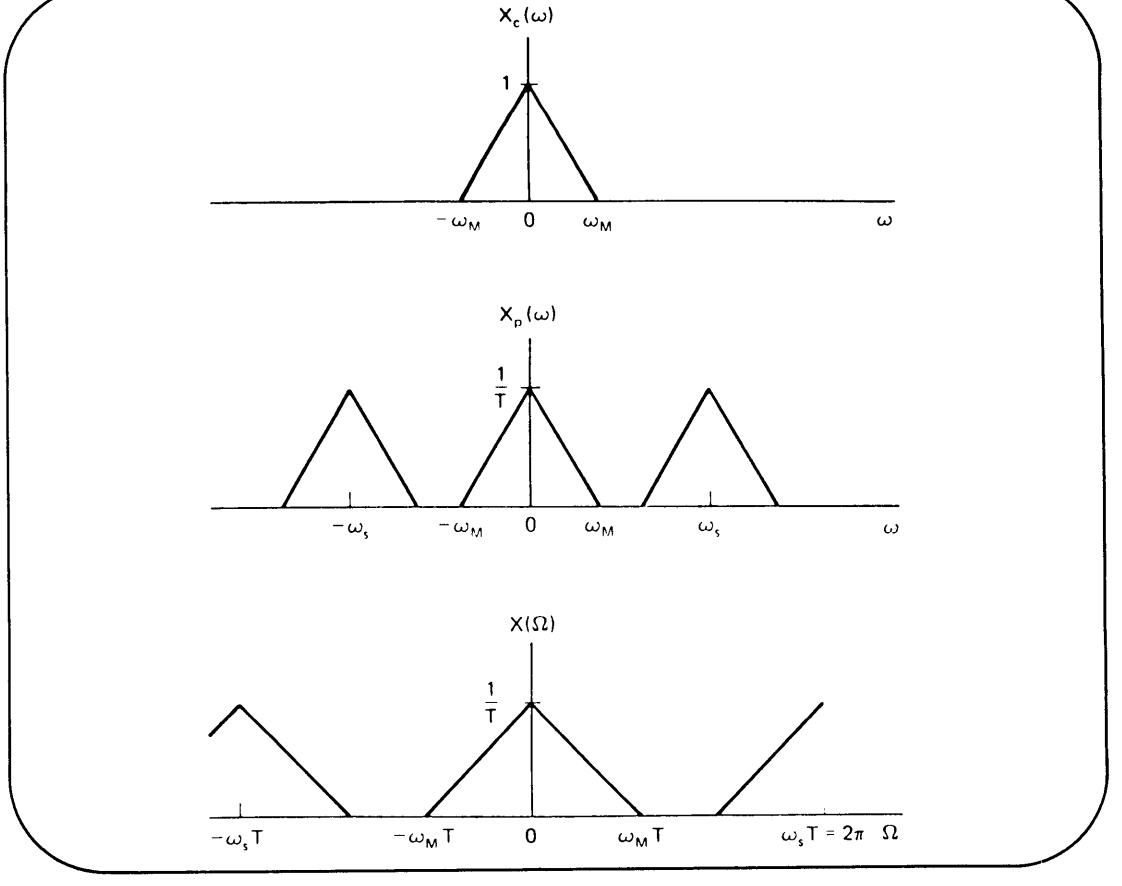


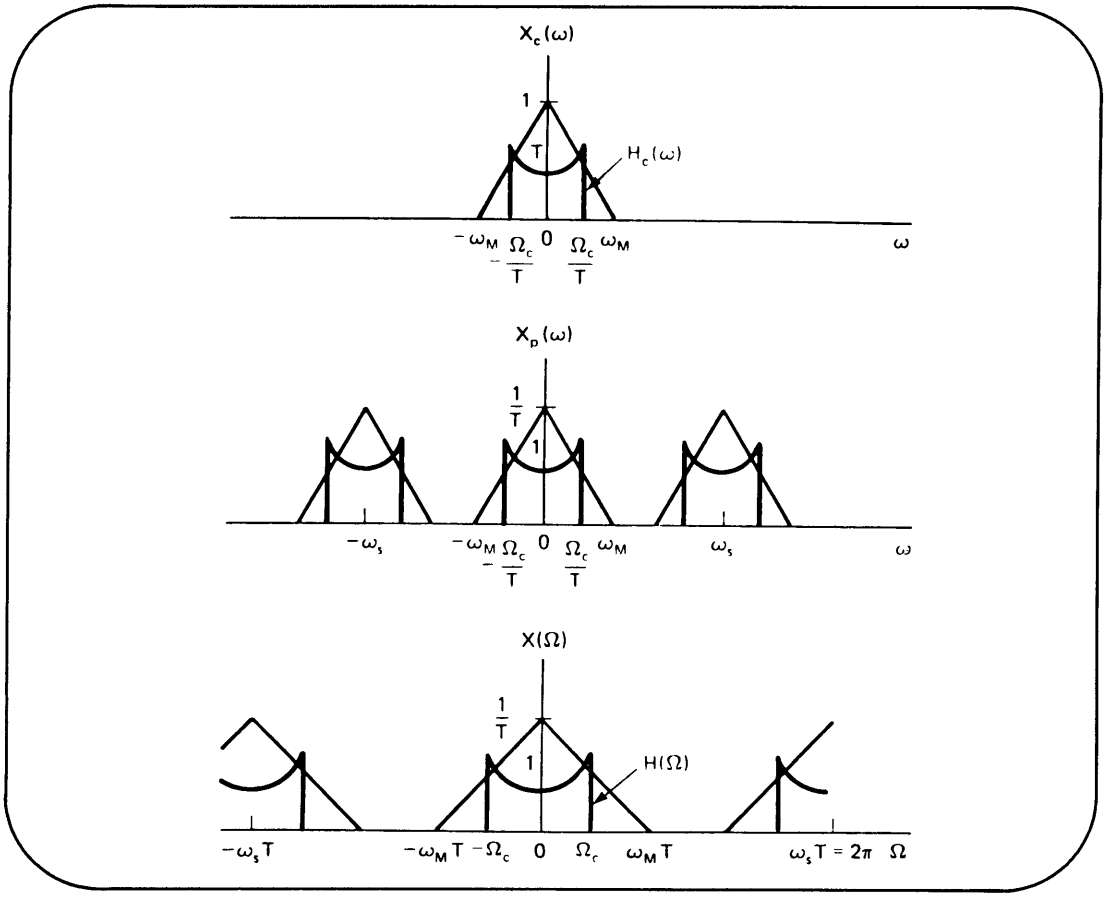
TRANSPARENCY
18.1
Illustration of the time normalization and associated frequency normalization inherent in the conversion of a continuous-time signal to a discrete-time sequence.

TRANSPARENCY 18.2
Overall block diagram for discrete-time processing of continuous-time signals.

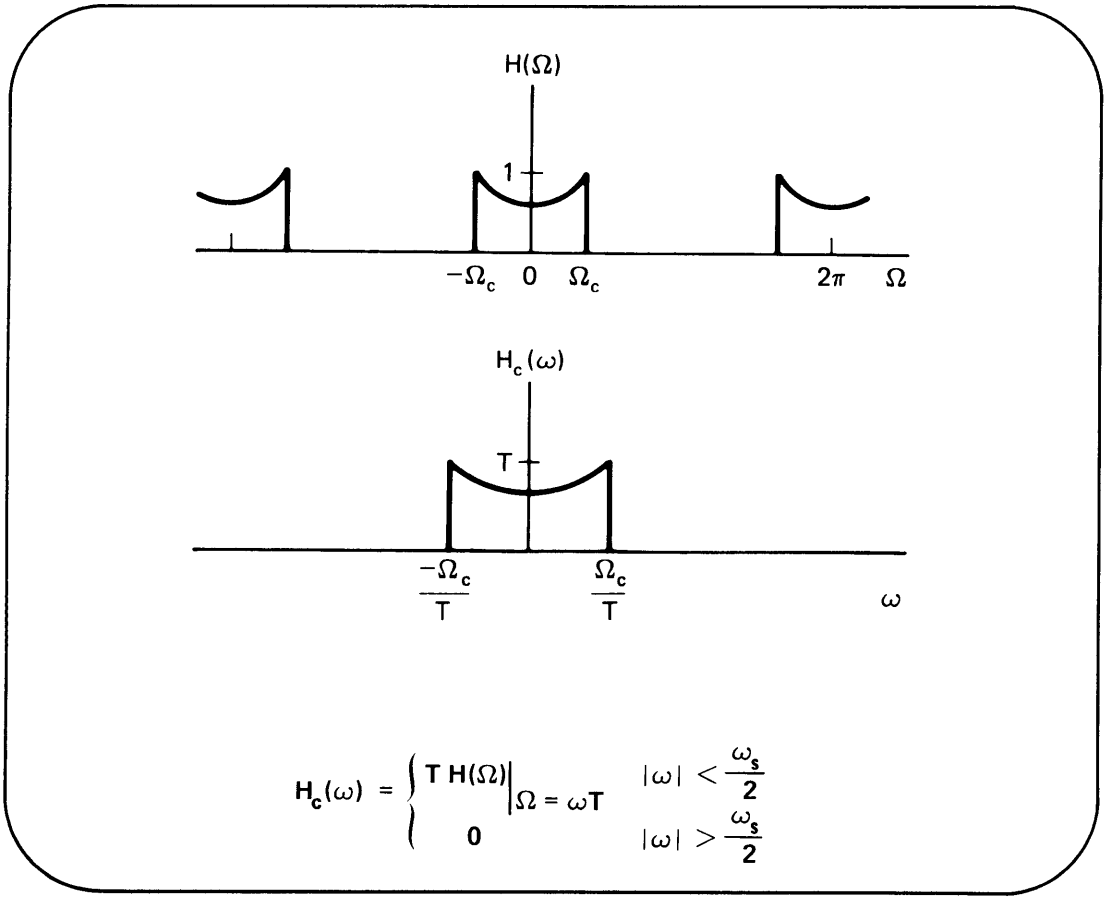


TRANSPARENCY 18.3
Illustration of spectra associated with conversion from continuous time to discrete time.



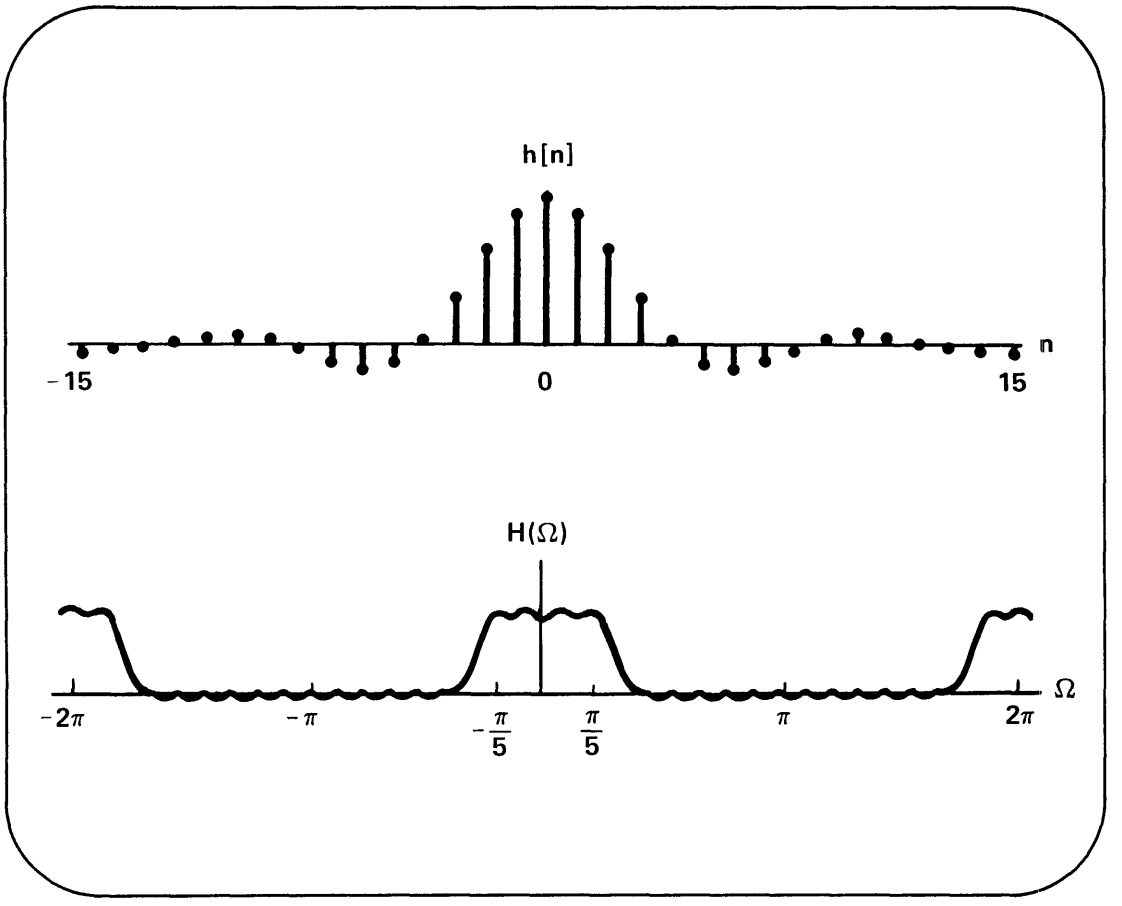


TRANSPARENCY 18.4
Illustration of spectra associated with applying a discrete-time frequency response to a sequence obtained by sampling a continuous-time signal followed by conversion back from discrete time to continuous time.

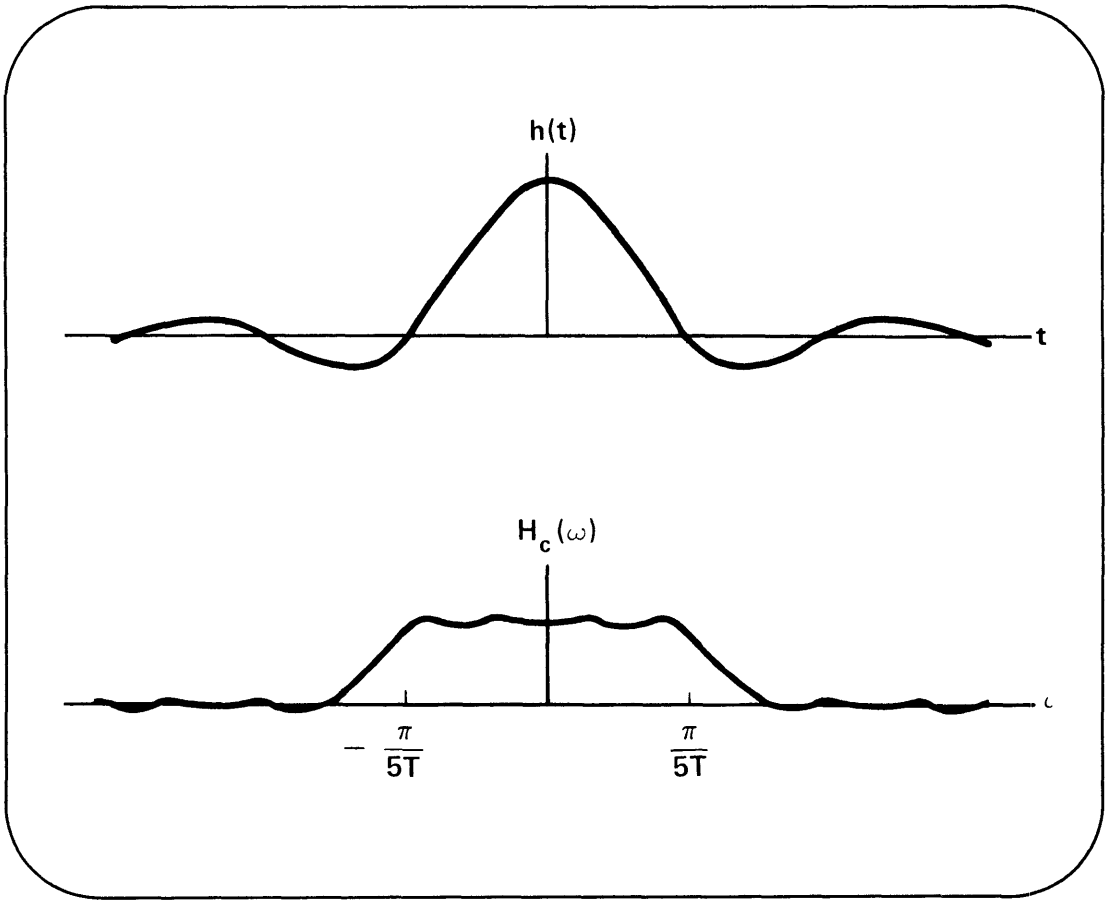


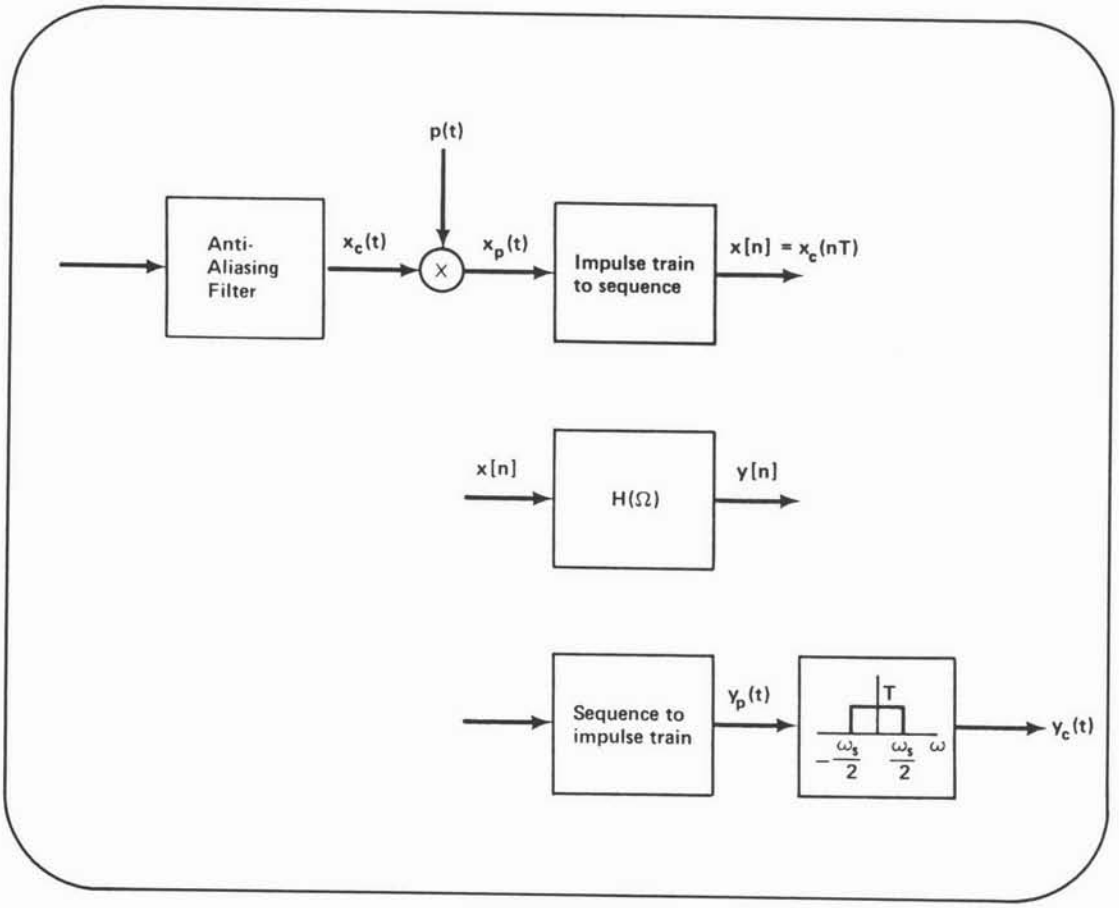
TRANSPARENCY 18.5
Relationship between the frequency response of the discrete-time filter and the frequency response of an equivalent continuous-time filter.

TRANSPARENCY 18.6
Discrete-time impulse response and frequency response for a filter used in discrete-time processing of continuous-time signals.

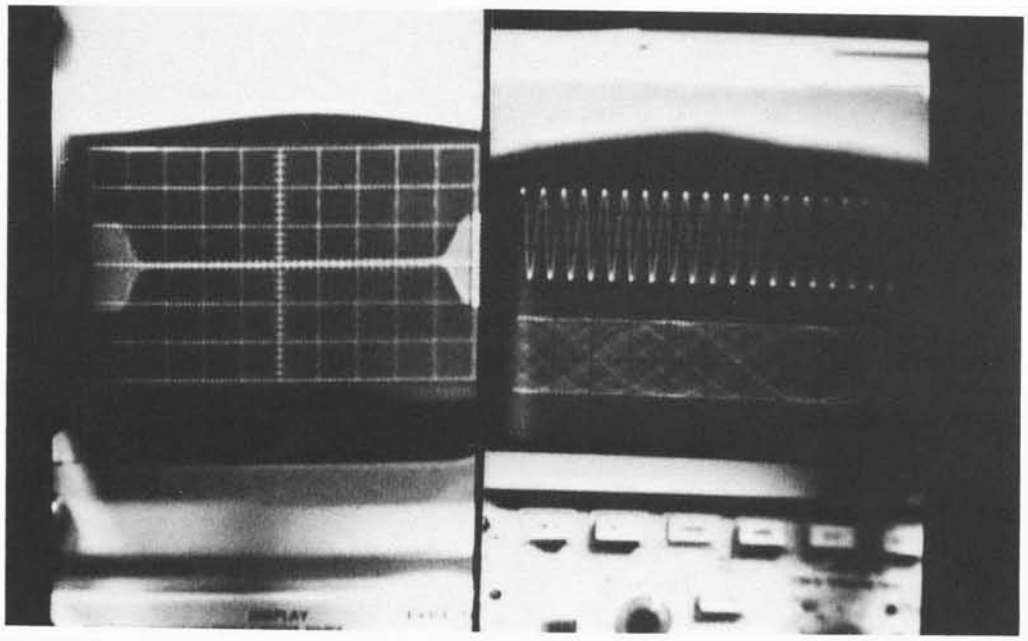


TRANSPARENCY 18.7
Equivalent continuous-time impulse response and frequency response.





TRANSPARENCY 18.8
Overall block diagram for discrete-time processing of continuous-time signals. [Transparency 18.2 repeated]



DEMONSTRATION 18.1
Frequency-domain (left) and time-domain (right) illustration of a continuous-time sinusoidal signal sampled and filtered with a discrete-time lowpass filter.

MIT OpenCourseWare
http://ocw.mit.edu

Resource: Signals and Systems

Professor Alan V. Oppenheim

The following may not correspond to a particular course on MIT OpenCourseWare, but has been provided by the author as an individual learning resource.

For information about citing these materials or our Terms of Use, visit: http://ocw.mit.edu/terms.