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Electromechanical Dynamics

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PROBLEMS

13.1. A static compressible fluid is subject to a gravitational force per unit volume $-\rho g$ (Fig. 13P.1). Under the assumption that the fluid has a constant temperature T_0 and that the fluid is a perfect gas so that $p = \rho RT$ find the distribution of density $\rho(x_1)$. The density at $x_1 = 0$ is ρ_0.
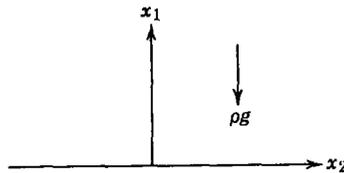

Fig. 13P.1

13.2. The MHD generator illustrated in Fig. 13P.2 uses a gas with constant specific heat capacities c_p and c_v and constant scalar conductivity σ. The dimensions are defined in the figure and it is assumed that the inlet values of all quantities are known. The loading factor K is to be held constant and the magnetic flux density is adjusted to satisfy the relation $B^2(z) A(z) = B_i^2 A_i$. For the constraint that the pressure be constant along the channel determine how the velocity v varies as a function of z.
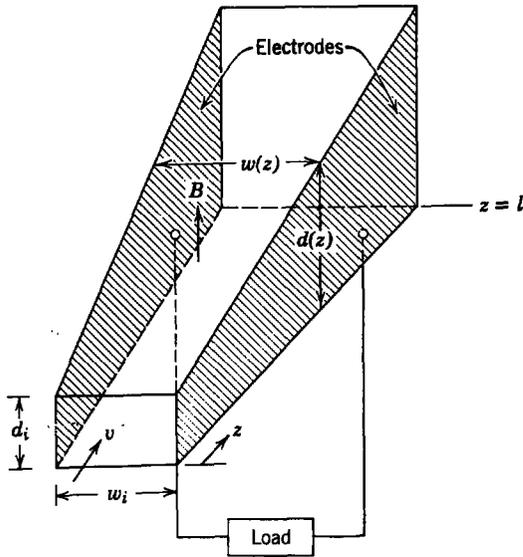
Fig. 13P.2

13.3. The dc MHD generator of Fig. 13P.3 has constant width w between the electrodes, and the magnetic flux density B_0 is constant along the length of the channel. The gas is assumed ideal with constant specific heats and with constant electrical conductivity. The inlet quantities ρ_i, p_i, T_i, and v_i are assumed known. The electrodes are short circuited together.

(a) Find the channel depth $d(z)$ necessary to maintain the temperature constant along the channel.

(b) Find the mass density variation along the channel for the channel geometry found in part (a).

13.4. For the MHD generator of Fig. 13P.4 assume that all inlet quantities are known $(p_i, \rho_i, T_i, d_i, w_i, A_i)$ and that the working gas has constant, scalar conductivity σ and ratio of specific heats γ. We apply a magnetic flux density B_0, which is constant, over the length of the channel. We now specify that we wish to design a channel such that the loading factor K will be constant along the length of the channel.

(a) For maintaining constant Mach number M along the channel find the following functions of z that satisfy the given conditions:

$$\frac{A(z)}{A_i}, \frac{d(z)}{d_i}, \frac{w(z)}{w_i}, \frac{v(z)}{v_i}, \frac{p(z)}{p_i}, \frac{T(z)}{T_i}.$$

(b) Repeat part (a) for maintaining constant velocity v along the channel.

(c) Assume that the given data is

$$\gamma = 1.4, \qquad \sigma = 40 \text{ mhos/m}, \quad R = 240 \text{ J/kg}^\circ\text{K},$$

$$p_i = 5 \times 10^5 \text{ N/m}^2, \quad \rho_i = 0.7 \text{ kg/m}^3, \quad T_i = 3000^\circ\text{K},$$

$$v_i = 700 \text{ m/sec}, \qquad w_i = 0.4 \text{ m} \qquad d_i = 0.2 \text{ m}.$$

$$K = 0.5, \qquad\qquad B_0 = 4 \text{ Wb/m}^2$$

For the generators of parts (a) and (b) find the length of generator necessary to reduce the total enthalpy per unit mass $(c_p T + \frac{1}{2}v^2)$ by 10%.
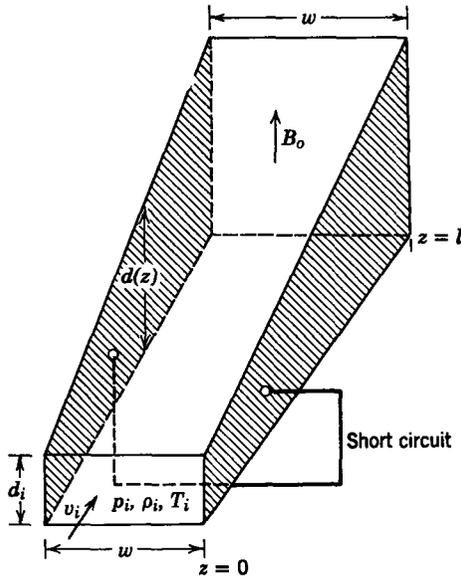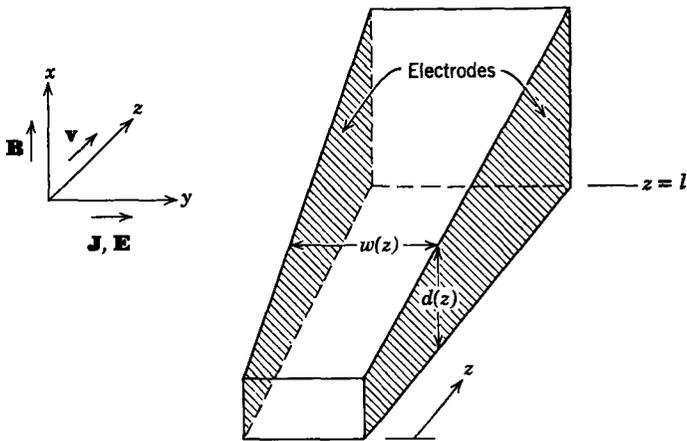
Fig. 13P.3

Fig. 13P.4

13.5. An MHD generator (Fig. 13P.4) uses an ideal gas with a constant ratio of specific heats $\gamma = 1.4$ and a constant, scalar, electrical conductivity $\sigma = 50$ mhos/m. At the inlet to the MHD generator channel the parameters and variables are adjusted to have the values

$$B_i = 4 \text{ Wb/m}^2, \qquad v_i = 700 \text{ m/sec},$$
$$\rho_i = 0.7 \text{ kg/m}^3, \qquad M_i{}^2 = 0.5,$$
$$p_i = 5 \times 10^5 \text{ N/m}^2, \qquad d_i = 0.5 \text{ m}$$
$$w_i = 1 \text{ m},$$

The aspect ratio of the channel is to remain constant so that the electric and magnetic fields will vary as

$$\frac{B(z)}{B_i} = \frac{E(z)}{E_i} = \left[\frac{w(z)}{w_i}\right]^{-1} = \left[\frac{d(z)}{d_i}\right]^{-1} = \left[\frac{A(z)}{A_i}\right]^{-\frac{1}{2}},$$

where A is the channel cross sectional area. For the conditions specified and for a loading factor $K = 0.5$ complete the following:

  (a) Find the area as a function of z necessary to keep the gas velocity constant.
  (b) For the channel of part (a) specify the length l necessary to reduce the gas temperature by 20 percent; that is, find l such that $T(l)/T_i = 0.8$.
  (c) Calculate and plot curves of $p(z)$, $\rho(z)$, $T(z)$ and $M^2(z)$ over the length of the channel.
  (d) Calculate the total electrical power drawn from this generator under the conditions given.

13.6. An ion propulsion device is represented schematically in Fig. 13P.6. The lateral dimensions are much larger than the separation of the accelerator electrodes so that fringing fields can be neglected. Ions, each having a charge q and mass m are injected with negligible initial velocity at $x = 0$. The system operates in the steady state in the space-charge-limited mode, in which case the solution for the electric field between the electrodes is

$$\mathbf{E} = \mathbf{i}_x \frac{4}{3} \frac{V_0}{L^{4/3}} x^{1/3},$$

and the solution for the charge density between the electrodes is

$$\rho_e = \frac{4}{9} \frac{\epsilon_0 V_0}{L^{4/3} x^{2/3}}$$

The charge on each ion does not change during the acceleration process; that is,

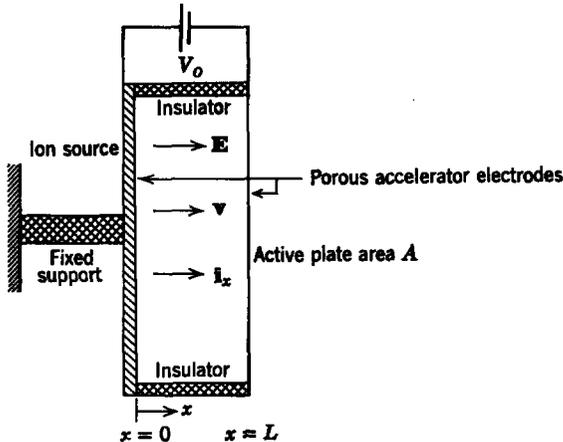$$\frac{\rho_e}{\rho_m} = \frac{q}{m},$$

V_0

Insulator

Ion source
$\longrightarrow$ **E**

Porous accelerator electrodes

$\longrightarrow$ **v**

Fixed
support

$\longrightarrow$ **i**$_x$

Active plate area A

Insulator

$\longmapsto x$

$x = 0$ $x = L$

Fig. 13P.6

where ρ_m is mass density. The pressure in the ion gas is negligibly small. For steady-state operation complete the following:

(a) Find the velocity of the ion gas as a function of x between the electrodes.

(b) Find the magnitude and direction of the force that must be supplied by the fixed support to keep the accelerator system at rest.

13.7. In Fig. 13P.7 a liquid is placed between rigid walls and a movable piston. The liquid has a speed of sound a. The system is uniform in the x_2- and x_3-directions ($\partial/\partial x_2 = \partial/\partial x_3 = 0$). Assuming that the piston is moved by a velocity source $V = V_o \cos \omega t$ at $x_1 = -L$,
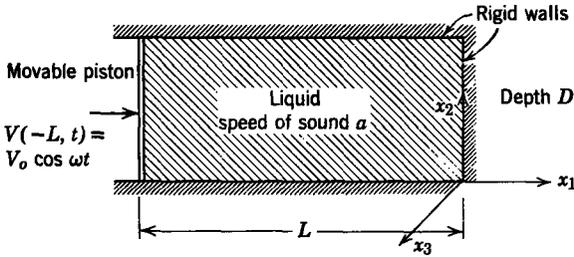

Fig. 13P.7

complete the following:

(a) Find the pressure in the liquid at $x_1 = -L$. Take the equilibrium pressure inside and outside the liquid as p_o.

(b) At what frequencies will there be resonances in the pressure, $p(-L, t)$?

13.8. A perfectly conducting compressible inviscid fluid fills the region $0 < x_2 < d$ as shown in Fig. 13P.8. When the fluid is static, it is permeated by a magnetic field $\mathbf{H} = H_o \mathbf{i}_3$. The fluid is set in motion by a piston at $x_1 = -L$ having velocity as shown, and is constrained at $x_1 = 0$ by a piston having mass M.

(a) *Derive* a differential equation (one equation in one unknown) for the velocity $v_1(x_1, t)$. Use a linearized analysis.

(b) Find the velocity of the piston, which in equilibrium is at $x_1 = 0$, under the conditions shown in Fig. 13P.8.
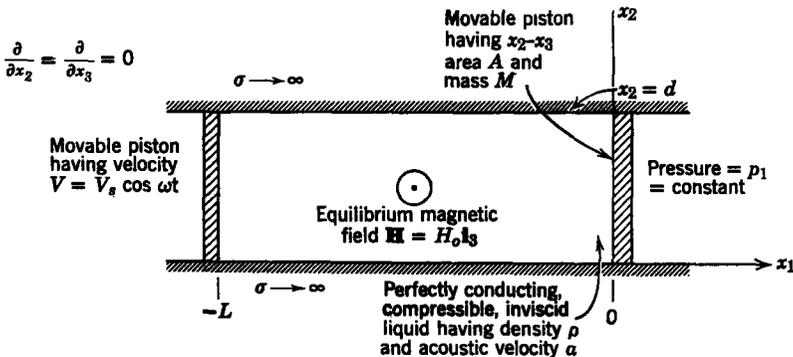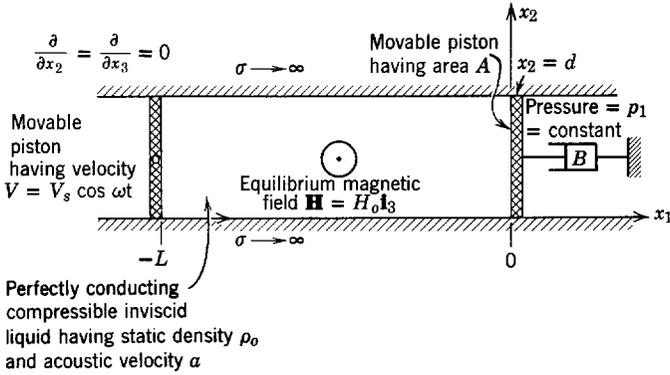

Fig. 13P.8

Fig. 13P.9

13.9. The region $0 < x_2 < d$ in Fig. 13P.9 is filled with a perfectly conducting inviscid compressible fluid. When the fluid is static, it is permeated by a magnetic field $H = H_o i_3$. The fluid is excited by a piston at $x_1 = -L$ with velocity $v(t)$, as shown, and constrained at $x_1 = 0$ by a movable massless insulating piston connected to a dashpot.

  (a) *Derive* a differential equation (one equation in one unknown) for the velocity $v_1(x_1, t)$. Use a linearized analysis.
  (b) The system is to be used as a delay line in which the object is to delay a signal by a fixed time without distorting the signal. If the signal is to be transmitted delayed and undistorted to $x_1 = 0$, the backward traveling wave must be eliminated by choosing a specific value for the dashpot coefficient B. Find this value in terms of the appropriate constants of the system.

13.10. The system shown in Fig. 13P.10 consists of a cylinder of cross-sectional area A_c containing a liquid of compressibility κ and equilibrium density ρ_l at equilibrium pressure p_o. The fluid is constrained at $x = 0$ by the closed end of the cylinder and at the other end by a rigid piston of mass M and thickness Δ. The equilibrium length of the liquid in the x-direction is L_1. The left face of the piston is connected to a thin elastic rod of cross-sectional area A_s, modulus of elasticity E, and equilibrium density ρ_s. The equilibrium length of the elastic rod is L_2. The left end of the elastic rod is driven by a stress source $T_o + T_s(t)$, where $T_s(t) \ll T_o$. For small-signal dynamic operation around an equilibrium point the general solutions are for the elastic rod: $-(L_1 + L_2 + \Delta) < x < -(L_1 + \Delta)$, $T(x, t)$ and $v_e(x, t)$; for the fluid $-L_1 < x < 0$, $p(x, t)$ and $v_l(x, t)$. Assume that except for constants to be determined by boundary conditions these solutions are known. Set up the equations that describe all the boundary conditions necessary for specializing constants in the general solutions. *Note.* You are not required to solve these equations.
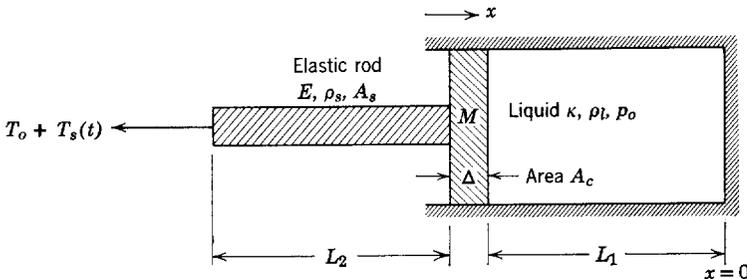

Fig. 13P.10

13.11. A slightly compressible inviscid fluid flows with a steady velocity V_o in the x-direction (Fig. 13P.11). The motions are in the x-direction and only a function of (x, t). The velocity of sound in the fluid is a_s.

   (a) Find the dispersion equation for small disturbances in the form of $p =$ Re $[\hat{p}e^{j(\omega t - kx)}]$.
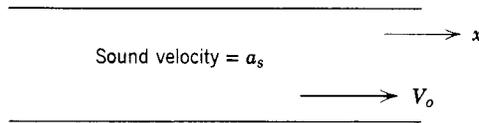   (b) Under what condition will the phases of both waves propagate in the positive x-direction?

Sound velocity $= a_s$

$\longrightarrow x$

$\longrightarrow V_o$

Fig. 13P.11

13.12. A static, inviscid fluid of conductivity σ is immersed in a uniform magnetic field H_o (Fig. 13P.12). In the limit in which $\sigma \to \infty$, it is possible for magnetoacoustic plane waves to propagate along the x-axis. In this problem investigate the consequences of having a finite conductivity σ.
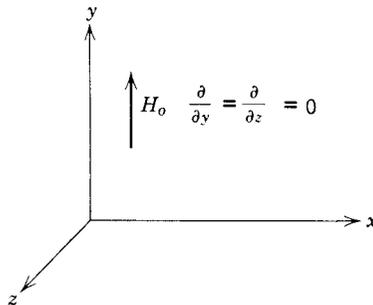

Fig. 13P.12

   (a) Write the linearized equations of motion for perturbations that are compressional (along the x-axis) and depend only on (x, t). (You need not combine these equations.)
   (b) Consider solutions of the form $v_x =$ Re $[\hat{v}e^{j(\omega t - kx)}]$ and find the dispersion relation between ω and k. Show that in the limit $\sigma \to \infty$ the lossless dispersion equation is retained.
   (c) Show that when σ is small there are two pairs of waves, each pair consisting of a forward and backward traveling wave. What would you call these waves?
   (d) Consider the case in which σ is very large but finite (slight losses) and in which the excitation is sinusoidal (ω is real). Find an approximate expression for the rate at which waves decay in space. *Hint.* Write the dispersion equation in the form

$$k^2 = \frac{\omega^2}{a^2} - j\frac{f(\omega, k)}{\sigma},$$

where a is the phase velocity of magnetoacoustic waves. When σ is large but finite, the second term can be approximated by making $k^2 \approx (\omega^2/a^2)$. (Why?)