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*Electromechanical Dynamics*

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# Appendix A

## GLOSSARY OF COMMONLY USED SYMBOLS

Section references indicate where symbols of a given significance are introduced; grouped symbols are accompanied by their respective references. The absence of a section reference indicates that a symbol has been applied for a variety of purposes. Nomenclature used in examples is not included.

Symbol	Meaning	Section
$A$	cross-sectional area	
$A_i$	coefficient in differential equation	5.1.1
$(A_n^+, A_n^-)$	complex amplitudes of components of $n$ th mode	9.2.1
$A_w$	cross-sectional area of armature conductor	6.4.1
$a$	spacing of pole faces in magnetic circuit	8.5.1
$a, (a_c, a_s)$	phase velocity of acoustic related waves	13.2.1, 11.4.1
$a_b$	Alfvén velocity	12.2.3
$(a, b, c)$	Lagrangian coordinates	11.1
$a_i$	constant coefficient in differential equation	5.1.1
$\mathbf{a}_p$	instantaneous acceleration of point $p$ fixed in material	2.2.1c
$B, B_r, B_s$	damping constant for linear, angular and square law dampers	2.2.1b, 4.1.1, 5.2.2
$\mathbf{B}, \mathbf{B}_i, B_0$	magnetic flux density	1.1.1a, 8.1, 6.4.2
$B_i$	induced flux density	7.0
$(B_r, B_{ra}, B_{rb}, B_{rm})$	radial components of air-gap flux densities	4.1.4
$[B_{rf}, (B_{rf})_{av}]$	radial flux density due to field current	6.4.1
$b$	width of pole faces in magnetic circuit	8.5
$b$	half thickness of thin beam	11.4.2b
$C$	contour of integration	1.1.2a
$C, (C_a, C_b), C_o$	capacitance	2.1.2, 7.2.1a, 5.2.1
$C$	coefficient in boundary condition	9.1.1
$\mathbf{C}$	the curl of the displacement	11.4
$(C^+, C^-)$	designation of characteristic lines	9.1.1

Symbol	Meaning	Section
$c_p$	specific heat capacity at constant pressure	13.1.2
$c_v$	specific heat capacity at constant volume	13.1.2
<b>D</b>	electric displacement	1.1.1a
$d$	length	
$da$	elemental area	1.1.2a
$d\mathbf{f}_n$	total elemental force on material in rigid body	2.2.1c
$d\mathbf{l}$	elemental line segment	1.1.2a
$d\mathbf{T}_n$	torque on elemental volume of material	2.2.1c
$dV$	elemental volume	1.1.2b
$E$	constant of motion	5.2.1
$E$	Young's modulus or the modulus of elasticity	9.1
$\mathbf{E}, E_0$	electric field intensity	1.1.1a, 5.1.2d
$E_f$	magnitude of armature voltage generated by field current in a synchronous machine	4.1.6a
$E_i$	induced electric field intensity	7.0
$e_{11}, e_{ij}$	strain tensor	9.1, 11.2
$\dot{e}_{ij}$	strain-rate tensor	14.1.1a
$F$	magnetomotive force (mmf)	13.2.2
<b>F</b>	force density	1.1.1a
$\hat{F}$	complex amplitude of $f(t)$	5.1.1
$F_0$	amplitude of sinusoidal driving force	9.1.3
$f$	equilibrium tension of string	9.2
$f$	driving function	5.1.1
$f, \mathbf{f}, f^e, f^s, f_j, f_i, f_1$	force	2.2.1, 2.2.1c, 3.1, 5.1.2a, 3.1.2b, 8.1, 9.1
$f$	arbitrary scalar function	6.1
$f'$	scalar function in moving coordinate system	6.1
$f$	three-dimensional surface	6.2
$f$	integration constant	11.4.2a
$G$	a constant	5.1.2c
$G$	shear modulus of elasticity	11.2.2
$G$	speed coefficient	6.4.1
$G$	conductance	3.1
$g$	air-gap length	5.2.1
$g, \mathbf{g}$	acceleration of gravity	5.1.2c, 12.1.3
$(\mathbf{H}, H_x, H_y, H_z)$	magnetic field intensity	1.1.1a
$h$	specific enthalpy	13.1.2
$\mathbf{I}, I, (I_r, I_s), I_f$	electrical current	10.4.3, 12.2.1a, 4.1.2, 6.4.1
$(i, i_1, i_2, \dots, i_k), (i_{ar}, i_{as}, i_{br}, i_{bs}), i_a, (i_a, i_b, i_c), (i_f, i_t), (i_r, i_s)$	electrical current	2.1, 4.1.3, 6.4.1, 4.1.7, 6.4.1, 4.1

Symbol	Meaning	Section
$\mathbf{i}_n$	unit vector perpendicular to area of integration	6.2.1
$\mathbf{i}_s$	unit vector normal to surface of integration	6.2.1
$(\mathbf{i}_x, \mathbf{i}_y, \mathbf{i}_z), (\mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_3)$	unit vectors in coordinate directions	2.2.1c
$J, \mathbf{J}_f$	current density	7.0, 1.1.1a
$J, J_x, (J_x, J_y, J_z)$	moment of inertia	5.1.2b, 4.1.1, 2.2.1c
$J_{xx}, J_{yz}$	products of inertia	2.2.1c
$j$	$\sqrt{-1}$	4.1.6a
$K$	loading factor	13.2.2
$K, \mathbf{K}_f$	surface current density	7.0, 1.1.1a
$K$	linear or torsional spring constant	2.2.1a
$K_t$	induced surface current density	7.0
$k, k_o, (k_r, k_t)$	wavenumber	7.1.3, 10.1.3, 10.0
$k$	summation index	2.1.1
$k$	maximum coefficient of coupling	4.1.6b
$k_n$	$n$ th eigenvalue	9.2
$(L, L_1, L_2), (L_a, L_f),$ $L_m, (L_0, L_2),$ $(L_r, L_g, L_{gr}), L_{gs}$	inductance	2.1.1, 6.4.1, 2.1.1, 4.2.1, 4.1.1, 4.2.4
$L$	length of incremental line segment	6.2.1
$l$	value of relative displacement for which spring force is zero	2.2.1a
$l, l_w, l_y$	length	
$M$	Hartmann number	14.2.2
$M$	mass of one mole of gas in kilograms	13.1.2
$M$	Mach number	13.2.1
$M$	mass	2.2.1c
$M$	number of mechanical terminal pairs	2.1.1
$M, M_s$	mutual inductance	4.1.1, 4.2.4
$M$	magnetization density	1.1.1a
$m$	mass/unit length of string	9.2
$N$	number of electrical terminal pairs	2.1.1
$N$	number of turns	5.2.2
$n$	number density of ions	12.3.1
$n$	integer	7.1.1
$\mathbf{n}$	unit normal vector	1.1.2
$\mathbf{P}$	polarization density	1.1.1a
$P$	power	12.2.1a
$p$	number of pole pairs in a machine	4.1.8
$P$	power per unit area	14.2.1
$p$	pressure	5.1.2d and 12.1.4
$P_o, P_g, P_m, P_r$	power	4.1.6a, 4.1.6b, 4.1.2, 4.1.6b
$Q$	electric charge	7.2.1a
$q, q_i, q_k$	electric charge	1.1.3 and 2.1.2, 8.1, 2.1.2
$R, R_t, R_o$	radius	

Symbol	Meaning	Section
$R, R_a, R_b, R_f, R_r, R_s$	resistance	
$(R, R_g)$	gas constant	13.1.2
$R_e$	electric Reynolds number	7.0
$R_m$	magnetic Reynolds number	7.0
$r$	radial coordinate	
$\mathbf{r}$	position vector of material	2.2.1c
$\mathbf{r}'$	position vector in moving reference frame	6.1
$\mathbf{r}_m$	center of mass of rigid body	2.2.1c
$S$	reciprocal modulus of elasticity	11.5.2c
$S$	surface of integration	1.1.2a
$S$	normalized frequency	7.2.4
$S$	membrane tension	9.2
$S_z$	transverse force/unit length acting on string	9.2
$s$	complex frequency	5.1.1
$(s, s_{mT})$	slip	4.1.6b
$s_i$	$i$ th root of characteristic equation, a natural frequency	5.1.1
$T$	period of oscillation	5.2.1
$T$	temperature	13.1.2
$\mathbf{T}, T, T^e, T_{em}, T_m,$ $T_0, T_1$	torque	2.2.1c, 5.1.2b, 3.1.1, 4.1.6b, 4.1.1, 6.4.1, 6.4.1
$\mathbf{T}$	surface force	8.4
$T_{ij}^m$	mechanical stress tensor	13.1.2
$T_{mn}$	the component of the stress-tensor in the $m$ th-direction on a cartesian surface with a normal vector in the $n$ th-direction	8.1
$T_{or}$	constant of coulomb damping	4.1.1
$T_0$	initial stress distribution on thin rod	9.1.1
$T$	longitudinal stress on a thin rod	9.1.1
$T_z$	transverse force per unit area on membrane	9.2
$T_2$	transverse force per unit area acting on thin beam	11.4.2b
$t$	time	1.1.1
$t'$	time measured in moving reference frame	6.1
$U$	gravitational potential	12.1.3
$U$	longitudinal steady velocity of string or membrane	10.2
$u$	internal energy per unit mass	13.1.1
$u$	surface coordinate	11.3
$u_0(x - x_0)$	unit impulse at $x = x_0$	9.2.1
$u$	transverse deflection of wire in $x$ -direction	10.4.3
$u_{-1}(t)$	unit step occurring at $t = 0$	5.1.2b
$V, V_m$	velocity	7.0, 13.2.3
$V$	volume	1.1.2
$V, V_a, V_f, V_o, V_s$	voltage	
$V$	potential energy	5.2.1

Symbol	Meaning	Section
$v, \mathbf{v}$	velocity	
$(v, v_1, \dots, v_k)$	voltage	2.1.1
$v', (v_a, v_b, v_c),$ $v_f, v_{oc}, v_l$	voltage	
$v_n$	velocity of surface in normal direction	6.2.1
$v_o$	initial velocity distribution on thin rod	9.1.1
$v_p$	phase velocity	9.1.1 and 10.2
$\mathbf{v}^r$	relative velocity of inertial reference frames	6.1
$v_s$	$\sqrt{f/m}$ for a string under tension $f$ and having mass/unit length $m$	10.1.1
$v$	longitudinal material velocity on thin rod	9.1.1
$v$	transverse deflection of wire in $y$ -direction	10.4.3
$(W_e, W_m)$	energy stored in electromechanical coupling	3.1.1
$(W'_e, W'_m, W')$	coenergy stored in electromechanical coupling	3.1.2b
$W''$	hybrid energy function	5.2.1
$w$	width	5.2.2
$w$	energy density	11.5.2c
$w'$	coenergy density	8.5
$X$	equilibrium position	5.1.2a
$(x, x_1, x_2, \dots, x_k)$	displacement of mechanical node	2.1.1
$x$	dependent variable	5.1.1
$x_p$	particular solution of differential equation	5.1.1
$(x_1, x_2, x_3), (x, y, z)$	cartesian coordinates	8.1, 6.1
$(x', y', z')$	cartesian coordinates of moving frame	6.1
$(\alpha, \beta)$	constants along $C^+$ and $C^-$ characteristics, respectively	9.1.1
$(\alpha, \beta)$	see (10.2.20) or (10.2.27)	
$\alpha$	transverse wavenumber	11.4.3
$(\alpha, \beta)$	angles used to define shear strain	11.2
$(\alpha, \beta)$	constant angles	4.1.6b
$\alpha$	space decay parameter	7.1.4
$\alpha$	damping constant	5.1.2b
$\alpha$	equilibrium angle of torsional spring	2.2.1a
$\gamma$	ratio of specific heats	13.1.2
$\gamma$	piezoelectric constant	11.5.2c
$\gamma, \gamma_0, \gamma'$	angular position	
$\Delta_d(t)$	slope excitation of string	10.2.1b
$\Delta_0$	amplitude of sinusoidal slope excitation	10.2.1b
$\Delta \mathbf{r}$	distance between unstressed material points	11.2.1a
$\Delta \mathbf{s}$	distance between stressed positions of material points	11.2.1a
$\delta( )$	incremental change in ( )	8.5
$\delta, \delta_1, \delta_0$	displacement of elastic material	11.1, 9.1, 11.4.2a
$\delta$	thickness of incremental volume element	6.2.1
$\delta$	torque angle	4.1.6a

Symbol	Meaning	Section
$\delta_{ij}$	Kronecker delta	8.1
$(\delta_+, \delta_-)$	wave components traveling in the $\pm x$ -directions	9.1.1
$\epsilon$	linear permittivity	1.1.1b
$\epsilon_0$	permittivity of free space	1.1.1a
$\eta$	efficiency of an induction motor	4.1.6b
$\eta$	second coefficient of viscosity	14.1.1c
$\theta, \theta_i, \theta_m$	angular displacement	2.1.1, 3.1.1, 5.2.1
$\theta$	power factor angle; phase angle between current and voltage	4.1.6a
$\theta$	equilibrium angle	5.2.1
$\dot{\theta}$	angular velocity of armature	6.4.1
$\theta_m$	maximum angular deflection	5.2.1
$(\lambda, \lambda_1, \lambda_2, \dots, \lambda_k)$	magnetic flux linkage	2.1.1, 6.4.1, 4.1.7,
$\lambda_a$		4.1.3, 4.1
$(\lambda_a, \lambda_b, \lambda_c)$		
$(\lambda_{ar}, \lambda_{as}, \lambda_{br}, \lambda_{bs})$		
$(\lambda_r, \lambda_s)$		
$\lambda$	Lamé constant for elastic material	11.2.3
$\lambda$	wavelength	7.1.4
$\mu$	linear permeability	1.1.1a
$\mu, (\mu_+, \mu_-)$	mobility	12.3.1, 1.1.1b
$\mu$	coefficient of viscosity	14.1.1
$\mu_d$	coefficient of dynamic friction	2.2.1b
$\mu_0$	permeability of free space	1.1.1a
$\mu_s$	coefficient of static friction	2.2.1b
$\nu$	Poisson's ratio for elastic material	11.2.2
$\nu$	damping frequency	10.1.4
$(\xi, \xi)$	continuum displacement	8.5
$\xi_0$	initial deflection of string	9.2
$\xi_d$	amplitude of sinusoidal driving deflection	9.2
$(\xi_n(x), \hat{\xi}_n(x))$	$n$ th eigenfunctions	9.2.1b
$(\xi_+, \xi_-)$	amplitudes of forward and backward traveling waves	9.2
$\dot{\xi}_0(x)$	initial velocity of string	9.2
$\rho$	mass density	2.2.1c
$\rho_f$	free charge density	1.1.1a
$\rho_s$	surface mass density	11.3
$\Sigma$	surface of discontinuity	6.2
$\sigma$	conductivity	1.1.1a
$\sigma_f$	free surface charge density	1.1.1a
$\sigma_m$	surface mass density of membrane	9.2
$\sigma_o$	surface charge density	7.2.3
$\sigma_s$	surface conductivity	1.1.1a
$\sigma_u$	surface charge density	7.2.3
$\tau$	surface traction	8.2.1
$\tau, \tau_d$	diffusion time constant	7.1.1, 7.1.2a
$\tau$	relaxation time	7.2.1a

Symbol	Meaning	Section
$\tau_e$	electrical time constant	5.2.2
$\tau_m$	time for air gap to close	5.2.2
$\tau_o$	time constant	5.1.3
$\tau_t$	traversal time	7.1.2a
$\phi$	electric potential	7.2
$\phi$	magnetic flux	2.1.1
$\phi$	cylindrical coordinate	2.1.1
$\phi$	potential for $\mathbf{H}$ when $\mathbf{J}_f = 0$	8.5.2
$\phi$	flow potential	12.2
$\chi_e$	electric susceptibility	1.1.1b
$\chi_m$	magnetic susceptibility	1.1.1a
$\psi$	the divergence of the material displacement	11.4
$\psi$	angle defined in Fig. 6.4.2	6.4.1
$\psi$	angular position in the air gap measured from stator winding ( <i>a</i> ) magnetic axis	4.1.4
$\psi$	electromagnetic force potential	12.2
$\psi$	angular deflection of wire	10.4.3
$\Omega$	equilibrium rotational speed	5.1.2b
$\mathbf{\Omega}$	rotation vector in elastic material	11.2.1a
$\Omega_n$	real part of eigenfrequency (10.1.47)	10.1.4
$\omega, (\omega_r, \omega_s)$	radian frequency of electrical excitation	4.1.6a, 4.1.2
$\omega$	natural angular frequency (Im $s$ )	5.1.2b
$\boldsymbol{\omega}, \omega_m$	angular velocity	2.2.1c, 4.1.2
$\omega_c$	cutoff frequency for evanescent waves	10.1.2
$\omega_d$	driving frequency	9.2
$\omega_n$	$n$ th eigenfrequency	9.2
$\omega_o$	natural angular frequency	5.1.3
$(\omega_r, \omega_i)$	real and imaginary parts of $\omega$	10.0
$\nabla$	nabla	6.1
$\nabla_\Sigma$	surface divergence	6.2.1