For Section 4.3:

Prob. 4.3.1 The cross section of a "double-sided machine" is shown in Fig. P4.3.1. The "rotor" is modeled as a current sheet.

(a) Find the force f_z acting in the z direction on an area A of the sheet.

(b) Now take the excitations as given by Eqs. 4.3.5a and 4.3.6a for synchronous interactions and evaluate f_z

(c) For a d-c interaction, the excitations are given by Eqs. 4.3.10a. Find f_z.

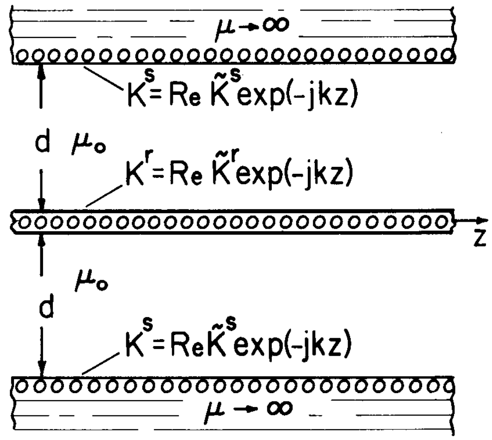

Fig. P4.3.1

Prob. 4.3.2 The developed model for a "trapped flux" synchronous machine is shown in Fig. P4.3.2. (See case 3a of Table 4.3.1). The stator surface current is specified as in Eq. 4.3.5a. The "rotor" consists of a perfectly conducting material. When t=0, the currents in this material have a pattern such that the flux normal to the rotor surface is $B_x^r = B_o^r \cos k[Ut-(z-\delta)]$, where U is the velocity of the rotor. Find f_z first in terms of $\tilde{K}^s$ and $\tilde{B}^r$ and then in terms of K_o^s and B_o^r. In practice, such a synchronous force would exist as a transient provided the initial current distribution diffused away, as described in Sec. 6.6, on a time scale long compared to that of interest.

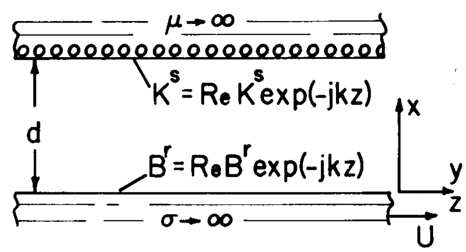

Fig. P4.3.2

Prob. 4.3.3 The moving member of an EQS device takes the form of a sheet, supporting the surface charge σ_f and moving in the z direction, as shown in Fig. P4.3.3. Electrodes on the adjacent walls constrain the potentials there.

(a) Find the force f_z on an area A of the sheet in terms of $(\hat{\Phi}^a, \hat{\sigma}_f, \hat{\Phi}^b)$.

(b) For a synchronous interaction, $\omega/k = U$. The surface charge is given by $-\sigma_o \cos[\omega t - k(z-\delta)]$ and $\Phi^a = V_o \cos(\omega t - kz)$. For even excitations $\Phi^b = \Phi^a$. Find f_z.

(c) An example of a d-c interaction is the Van de Graaff machine taken up in Sec. 4.14. With the excitations $\Phi^a = \Phi^b = -V_o \cos kz$ and $\sigma_f = \sigma_o \sin kz$, find f_z.

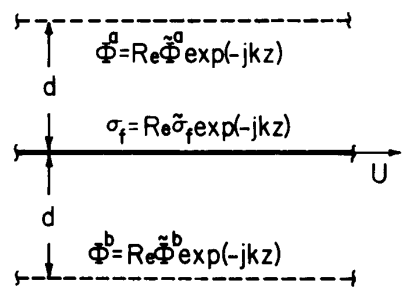

Fig. P4.3.3

For Section 4.4:

Prob. 4.4.1 This problem is intended to give the opportunity to follow through the approach to developing a lumped parameter model illustrated in Sec. 4.4. However, for best efficiency in determining the electrical terminal relations, it will be helpful to use the transfer relations of Sec. 2.19, and study of Sec. 4.7 is recommended in this regard.

The cross section of a model for a permanent-magnetization rotating magnetic machine is shown in Fig. P4.4.1. The magnetization density in the rotor is uniform and of magnitude M_o. The stator is wound with a uniform turn density N, so that the surface current density over $2\theta_o$, the span of the turns, is Ni(t).

(a) Show that in the rotor volume, $\vec{B}$ is both solenoidal and irrotational so that the transfer relations of Table 2.19.1 apply provided that μH_θ is taken as B_θ.

(b) Show that boundary conditions at the rotor interface implied by the divergence condition on $\vec{B}$ and Ampere's law are

$$\vec{n} \cdot [\![\vec{B}]\!] = 0 \quad ; \quad \vec{n} \times [\![B]\!] = \mu_o \vec{K}_f + \mu_o \vec{n} \times [\![\vec{M}]\!]$$

Prob. 4.4.1 (continued)

(c) Find the instantaneous torque on the rotor as a function of (θ_r, i). (Your result should be analogous to Eq. 4.4.11.)

(d) Find the electrical terminal relation $\lambda(\theta_r, i, M_o)$. (This result is analogous to Eq. 4.4.14.)

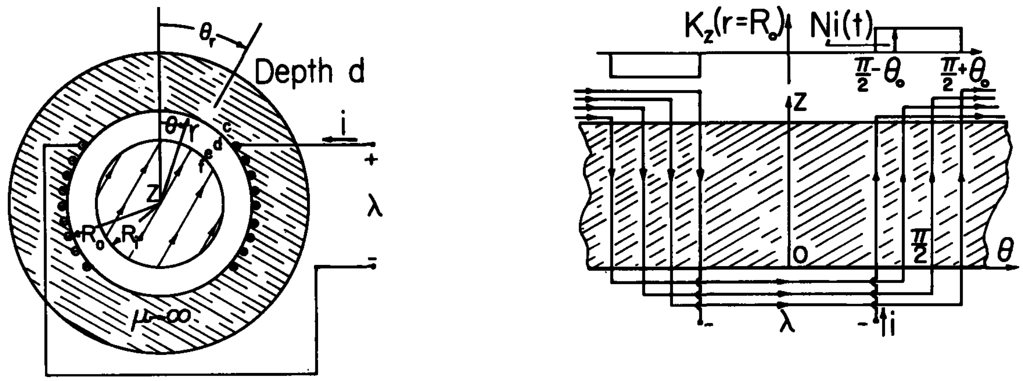

Fig. P4.4.1

For Section 4.6:

Prob. 4.6.1 A charged particle beam takes the form of a planar layer moving in the z direction with the velocity U, as shown in Fig. P4.6.1. The charge density within the beam is

$$\rho = \text{Re}\, \tilde{\rho}_o\, e^{-jkz}$$

Thus the density is uniform in the x direction within the beam, i.e., in the region $-b/2 < x < b/2$. The walls, which are constrained in potential as shown, are separated from the beam by planar regions of free space of thickness d.

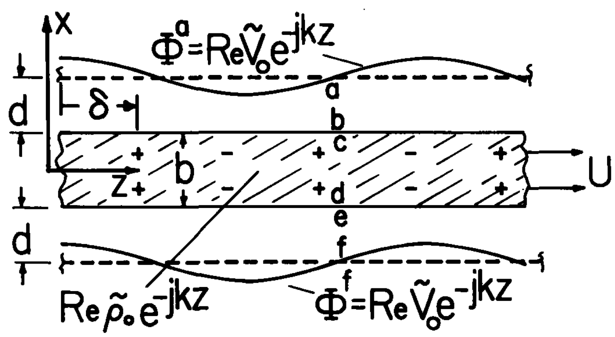

Fig. P4.6.1

(a) In terms of the complex functions of time $\tilde{V}_o$ and $\tilde{\rho}_o$, find the electrical force acting on an area A (in the y-z plane) of the beam in the z direction.

(b) Now, specialize the analysis by letting

$$\Phi^a = \Phi^f = V_o \cos(\omega t - kz)$$

$$\rho = -\rho_o \cos[\omega t - k(z-\delta)]$$

Given that the charged particles comprising the beam move with velocity U, and that k is specified what is ω? Evaluate the force found in (a) in terms of the phase displacement δ and the amplitudes V_o and ρ_o.

(c) Now consider the same problem from another viewpoint. Consider the entire region $-(d+\frac{b}{2}) < x < (d+\frac{b}{2})$ as one region and find alternative expressions for parts (a) and (b).

For Section 4.8:

Prob. 4.8.1 Transfer relations are developed here that are the Cartesian coordinate analogues of those in Sec. 4.8.

(a) With variables taking the form $A = \text{Re}\, \tilde{A}(x,t)e^{-jky}$ and $H_y = \text{Re}\, \tilde{H}_y(x,t)e^{-jky}$ and a volume current density (in the z direction) $J = \text{Re}\, \tilde{J}(x,t)e^{-jky}$, start with Eq. (b) of Table 2.19.1 and show

Prob. 4.8.1 (continued)

that the transfer relations take the form

$$\begin{bmatrix} \tilde{A}^\alpha \\ \\ \tilde{A}^\beta \end{bmatrix} = \frac{\mu}{k}\begin{bmatrix} -\coth k\Delta & \dfrac{1}{\sinh k\Delta} \\ \\ \dfrac{-1}{\sinh k\Delta} & \coth k\Delta \end{bmatrix}\begin{bmatrix} \tilde{H}_y^\alpha \\ \\ \tilde{H}_y^\beta \end{bmatrix} - \frac{\mu}{k}\begin{bmatrix} -\coth k\Delta & \dfrac{1}{\sinh k\Delta} \\ \\ \dfrac{-1}{\sinh k\Delta} & \coth k\Delta \end{bmatrix}\begin{bmatrix} \tilde{H}_{yp}^\alpha \\ \\ \tilde{H}_{yp}^\beta \end{bmatrix} + \begin{bmatrix} \tilde{A}_p^\alpha \\ \\ \tilde{A}_p^\beta \end{bmatrix}$$

(b) The bulk current density and particular solution for A are represented in terms of modes $\Pi_i(x)$:

$$J = \text{Re}\sum_{i=0}^{\infty} \tilde{J}_i(t)\Pi_i(x)e^{-jky} \quad ; \qquad A_p = \text{Re}\sum_{i=0}^{\infty} \tilde{A}_i(t)\Pi_i(x)e^{-jky}$$

Show that if the modes are required to have zero derivatives at the surfaces, the transfer relations become

$$\begin{bmatrix} \tilde{A}^\alpha \\ \\ \tilde{A}^\beta \end{bmatrix} = \frac{\mu}{k}\begin{bmatrix} -\coth k\Delta & \dfrac{1}{\sinh k\Delta} \\ \\ \dfrac{-1}{\sinh k\Delta} & \coth k\Delta \end{bmatrix}\begin{bmatrix} \tilde{H}_y^\alpha \\ \\ \tilde{H}_y^\beta \end{bmatrix} + \sum_{i=0}^{\infty}\frac{\mu\tilde{J}_i}{(\frac{i\pi}{\Delta})^2 + k^2}\begin{bmatrix} (-1)^i \\ \\ 1 \end{bmatrix}$$

For Section 4.9:

Prob. 4.9.1 A developed model for an exposed winding machine is shown in Fig. P4.9.1. The infinitely permeable stator structure has a winding that is modeled by the surface current $K^s = \text{Re}\,\tilde{K}^s e^{-jky}$. The rotor consists of a winding that completely fills the air gap and is backed by an infinitely permeable material. At a given instant, the current distribution in the rotor windings is uniform over the cross section of the gap; it is a square wave in the y direction, as shown. That is, the winding density (n wires per unit area) is uniform. Use the result of Prob. 4.8.1 to find the force per unit y-z area in the y direction acting on the rotor (note Eq. 2.15.17). Express this force for the synchronous interaction in which $K^s = K_o^s\cos(\omega t - \frac{2\pi}{\ell}y)$.

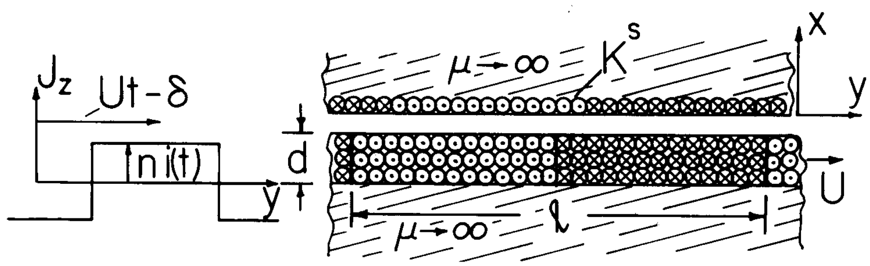

Fig. P4.9.1

For Section 4.10:

Prob. 4.10.1 A developed model for a d-c machine is shown in Fig. P4.10.1. The field winding is represented by a surface current distribution at x = b that is a positive impulse at z = 0 and a negative one at $z = \ell$, each of magnitude $n_f i_f$ as shown. Following the outline given in Sec. 4.10, develop the mechanical and electrical terminal relations analogous to Eqs. 4.10.6, 4.10.17 and 4.10.21. (See

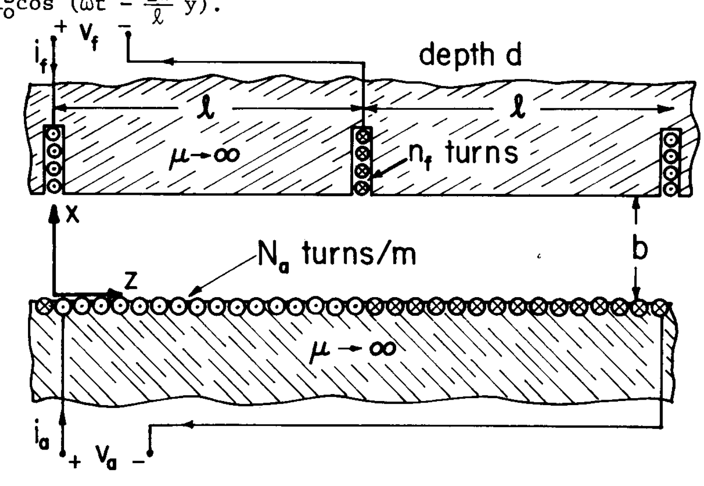

Fig. P4.10.1

Prob. 4.14.1 for a different approach with results that suggest simplification of those found here.)

For Section 4.12:

Prob. 4.12.1 The potential along the axis of a cylindrical coordinate system is $\Phi(z)$. The system is axisymmetric, so that $E_r = 0$ along the z axis. Show that fields in the vicinity of the z axis can be approximated in terms of $\Phi(z)$ by $E_z = -d\Phi/dz$ and

$$E_r = -\frac{r}{2}\frac{d^2\Phi}{dz^2}$$

For Section 4.13:

Prob. 4.13.1 An alternative to the quasi-one-dimensional model developed in this section is a "linearized" model, based on the stator and rotor amplitudes being small compared to the mean spacing d. In the context of a salient-pole machine, this approach is illustrated in Sec. 4.3. Assume at the outset that $\xi_r/d \ll 1$ and $\xi_s/d \ll 1$ but that the wavelength λ is arbitrary compared to d. Find the time-average force acting on one wavelength of the rotor. Take the limit $2\pi d/\lambda \ll 1$, and show that this force reduces to Eq. 4.13.12.

Prob. 4.13.2 A developed model for a salient pole magnetic machine is shown in Fig. P4.13.2. A set of distributed windings on the stator surface impose the surface current

$$K_y = K_o^s \sin(\omega t - kz)$$

and the geometry of the rotor surface is described by

$$\xi = \xi_o \cos 2k[Ut-(z-\delta)]$$

Both the rotor and stator are infinitely permeable.

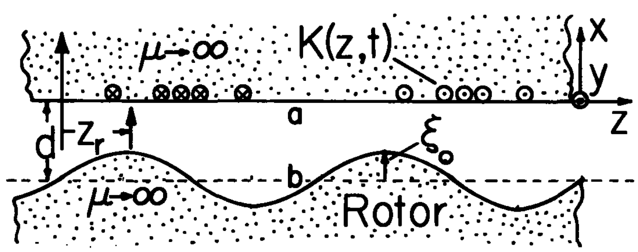

Fig. P4.13.2

(a) What are the lowest order H_x and H_z in a quasi-one-dimensional model?

(b) Find the average force f_z on one wavelength in the form of Eq. 4.13.8.

(c) Compare your result to that of Sec. 4.3, Eq. 4.3.27.

For Section 4.14:

Prob. 4.14.1

(a) For the magnetic d-c machine described in Prob. 4.10.1, show that the quasi-one-dimensional fields in the gap (based on $\ell \gg d$) are
b

$$H_x = \pm\frac{N_a i_a}{b}\left(z - \begin{matrix}\ell/2\\ 3\ell/2\end{matrix}\right) \pm \frac{n_f i_f}{2b} \qquad 0 < z < \ell \qquad (1)$$

$$H_z = \pm N_a i_a \left(\frac{x}{b} - 1\right) \qquad \ell < z < 2\ell \qquad (2)$$

(b) Based on these fields, what is the force on a length, 2ℓ, of the armature written in the form $f_z = -G_m i_f i_a$?

(c) Write the electrical terminal relations in the form of Eqs. 4.10.17 and 4.10.21.