

18.01 Practice Questions for Exam 1

Solutions will be posted on the 18.01 website.

No books, notes, or calculators will be allowed at the exam.

1. Evaluate each of the following, simplifying where possible; for (b) indicate reasoning. The letters a and k represent constants.

$$a) \left. \frac{d}{dt} \left(\frac{3t}{\ln t} \right) \right|_{e^2} \quad b) \lim_{u \rightarrow 0} \frac{3u}{\tan 2u} \quad c) \frac{d^3}{dx^3} \sin kx \quad d) \frac{d}{d\theta} \sqrt[3]{a + k \sin^2 \theta}$$

2. Derive the formula for $\frac{d}{dx} x^3$ at the point $x = x_0$ directly from the definition of derivative.

3. Find $\lim_{h \rightarrow 0} \frac{1 - \sqrt[3]{1+h}}{h}$ by relating it to a derivative. (Indicate reasoning.)

4. Sketch the curve $y = \sin^{-1} x$, $-1 \leq x \leq 1$, and derive the formula for its derivative from that for the derivative of $\sin x$.

5. For the function

$$f(x) = \begin{cases} ax + b, & x > 0 \\ 1 - x + x^2, & x \leq 0, \end{cases} \quad a \text{ and } b \text{ constants,}$$

a) find all values of a and b for which the function will be continuous;

b) find all values of a and b for which the function will be differentiable.

6. For the curve given by the equation

$$x^2 y + y^3 + x^2 = 8,$$

find all points on the curve where its tangent line is horizontal.

7. Where does the tangent line to the graph of $y = f(x)$ at the point (x_0, y_0) intersect the x -axis?

8. The volume of a spherical balloon is decreasing at the instantaneous rate of $-10 \text{ cm}^3/\text{sec}$, at the moment when its radius is 20 cm. At that moment, how rapidly is its radius decreasing?

9. Where are the following functions discontinuous?

$$a) \sec x \quad b) \frac{1+x^2}{1-x^2} \quad c) \frac{d}{dx} |x|$$

10. A radioactive substance decays according to a law $A = A_0 e^{-rt}$, where $A(t)$ is the amount in present at time t , and r is a positive constant.

a) Derive an expression in terms of r for the time it takes for the amount to fall to one-quarter of the initial amount A_0 .

b) At the moment when the amount has fallen to 1/4 the initial amount, how rapidly is the amount falling? (Units: grams, seconds.)