

18.01 Exam 2

Tuesday, Oct. 17, 2006

Problem 1. (15 pts). Estimate the following to two decimal places (show work)

a. (8 pts). $\sin(\pi + 1/100)$

$$\sin(\pi + \frac{1}{100}) \approx \sin(\pi) + \cos(\pi) \frac{1}{100} = 0 - 1 \frac{1}{100} = -0.01$$

SOLUTIONS

Name: _____

Put a check next to your recitation:

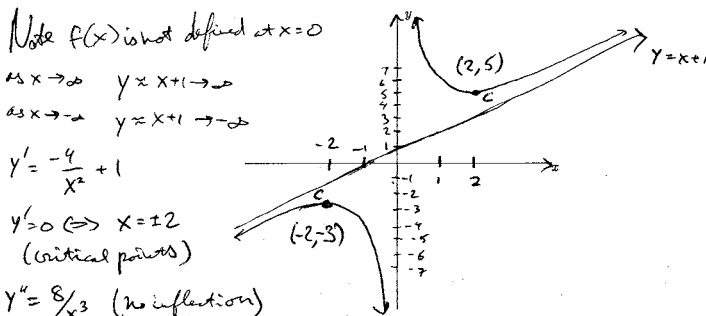
Rec. 1	Ilya Elson	10am	
Rec. 2	Kobi Kremnizer	10am	
Rec. 3	Liat Kessler	12pm	
Rec. 4	Matthew Hedden	1pm	
Rec. 5	Jérôme Waldispühl	2pm	
Rec. 6	Liat Kessler	2pm	
Rec. 7	Matthew Hedden	2pm	
Rec. 8	Jérôme Waldispühl	3pm	

Problem 1	15 points	
Problem 2	20 points	
Problem 3	20 points	
Problem 4	15 points	
Problem 5	20 points	
Problem 6	10 points	

b. (7 pts). $\sqrt{101}$

$$\sqrt{101} = \sqrt{100+1} = \sqrt{100(1+\frac{1}{100})} = 10\sqrt{1+\frac{1}{100}} \approx 10(1+\frac{1}{200}) = 10 + \frac{1}{20} = 10.05$$

Problem 2. (20 pts). Sketch the graph of $y = \frac{4}{x} + x + 1$ on $-\infty < x < \infty$ and label all critical points and inflection points with their coordinates on the graph along with the letter "C" or "I".



Write down on which intervals the function is:

increasing: $y' > 0$

$$-\infty < x \leq -2$$

$$2 \leq x < \infty$$

decreasing: $y' < 0$

$-2 \leq x < 0$ } NOT $-2 < x < 2$
 $0 < x \leq 2$ } because the function is undefined at $x=0$.

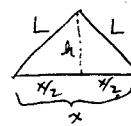
concave up: $y'' > 0$

$$0 < x < \infty$$

concave down: $y'' < 0$

$$-\infty < x < 0$$

Problem 3. (20 pts.) An architect plans to build a triangular enclosure with a fence on two sides and a wall on the third side. Each of the fence segments has fixed length L . What is the length x of the third side if the region enclosed has the largest possible area? Show work and include an argument to show that your answer really gives the maximum area.



$$A = \frac{1}{2} x h, \quad h^2 + (x/2)^2 = L^2, \quad 0 \leq x \leq 2L.$$

METHOD 1 (Substitution) $h = \sqrt{L^2 - x^2/4}$, $A = \frac{1}{2} x \sqrt{L^2 - x^2/4}$

$$A' = \frac{1}{2} (\sqrt{L^2 - x^2/4} + \frac{1}{2} x (-\frac{x}{4}) / \sqrt{L^2 - x^2/4}) = \frac{1}{2\sqrt{L^2 - x^2/4}} (L^2 - \frac{x^2}{4} - \frac{x^2}{4}) = 0$$

Hence $L^2 = x^2/2$, and $x = \sqrt{2} L$

MAX BECAUSE: $A=0$ at both ends $x=0$ and $x=2L$

and $A > 0$ at the unique critical pt. in between.

So this crit pt must be where max is achieved. (2nd deriv test is longer, e.g., too.)

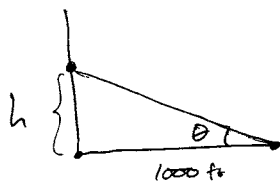
METHOD 2 (Implicit Diff.) $2h h' + x/2 = 0 \Rightarrow h' = -\frac{x}{4h}$

$$0 = A' = \frac{1}{2} (h + x h') = \frac{1}{2} (h + x (-\frac{x}{4h})) \Rightarrow h = x^2/4h$$

$$\Rightarrow h^2 = x^2/4 \Rightarrow \frac{x^2}{4} + \frac{x^2}{4} = L^2 \Rightarrow x^2 = 2L^2 \Rightarrow x = \sqrt{2} L$$

(REASONING FOR MAX IS THE SAME.)

Problem 4. (15 pts.) A rocket is launched straight up, and its altitude is $h = 10t^2$ feet after t seconds. You are on the ground 1000 feet from the launch site. The line of sight from you to the rocket makes an angle θ with the horizontal. By how many radians per second is θ changing ten seconds after the launch?



$$\tan \theta = \frac{h}{1000} = \frac{10t^2}{1000} = \frac{1}{100} t^2$$

$$\sec^2 \theta \frac{d\theta}{dt} = \frac{1}{50} t$$

$$\text{when } t = 10, h(10) = 10 \cdot 10^2 = 1000, \text{ so}$$

$$\tan \theta = 1 \text{ so } \theta = \pi/4$$

$$\frac{1}{\cos^2(\pi/4)} \frac{d\theta}{dt} \Big|_{t=10} = \frac{1}{50} \cdot 10$$

$$\frac{d\theta}{dt} \Big|_{t=10} = \frac{1}{5} \cos^2 \frac{\pi}{4} = \frac{1}{5} \left(\frac{\sqrt{2}}{2} \right)^2 = \boxed{\frac{1}{10} \text{ rad/s}}$$

Problem 5. a. (10 pts) Evaluate the following indefinite integrals

$$\text{i. } \int \cos(3x) dx = \frac{1}{3} \sin(3x) + C$$

$$\text{ii. } \int x e^{x^2} dx = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + C$$

$$u = x^2$$

$$du = 2x dx$$

$$= \frac{1}{2} e^{x^2} + C$$

b. (10 pts) Find $y(x)$ such that $y' = \frac{1}{y^3}$ and $y(0) = 1$

$$\frac{dy}{dx} = \frac{1}{y^3}$$

$$y^3 dy = dx$$

$$\frac{1}{4} y^4 = x + C$$

$$\frac{1}{4} = C$$

$$y = (4x+1)^{1/4}$$

Problem 6. (10 pts.) Suppose that $f'(x) = e^{e^x}$, and $f(0) = 10$. One can conclude from the mean value theorem that

$$A < f(1) < B$$

for which numbers A and B ?

$$\frac{f(1) - f(0)}{1 - 0} = f'(c) \text{ for some } c, 0 < c < 1.$$

$$\text{Or } f(1) = f(0) + f'(c) = 10 + e^{e^c}, 0 < c < 1.$$

$$\text{If } 0 < c < 1, \text{ then } e^{e^c} < e \text{ and } 1 < e^{e^c}.$$

$$\text{So } f(1) = 10 + e^{e^c} < 10 + e$$

$$\text{and } f(1) = 10 + e^{e^c} > 10 + 1 = 11$$

$$\boxed{A = 11}$$

$$\boxed{B = 10 + e}$$