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Probability: Random Isn't So Random
Summer 2008

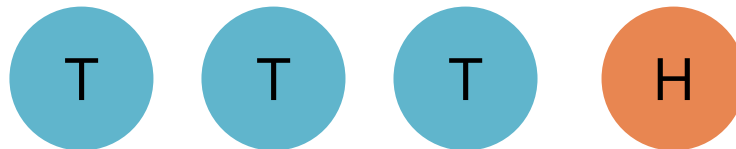# RV II: More Random Variables, Expectation, Variance

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Catchup from last class

- Random Variables Intro (see last lecture)

Geometric Random Variable

- X = number of tosses needed for a head to come up the first time
- p = probability that a head is tossed

- $p_x(k) = (1-p)^{k-1}p$ $k = [1, 2, \ldots]$

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k = 4

Geometric RV

- PMF of the geometric RV

Geometric RV

- Applications
  - Passing a test in a given try
  - Finding a missing item in a given search

Poisson Random Variable

- $p_x(k) = e^{-\lambda} \dfrac{\lambda^k}{k!}$ $k = 0, 1, 2\ldots$

Poisson RV

- Good approximation for binomial RV if
  - $\lambda = np$ [n is large, p is small]

- Example: # of typos in a book with
  - n = total words
  - p = small probability of words being misspelled
- Example: # of cars involved in an accident on a given day

Example

- If n = 100, p=0.01
- Result using the binomial PMF:

- Result using the Poisson PMF:

- Poisson RV is easier to calculate

Problem

- You go to a party with 500 guests. What is the probability that exactly one other guest has the same birthday as you? Calculate this exactly and also approximately using the Poisson PMF. (Exclude birthdays on February 29.)

List of RVs so far

- Bernoulli
- Binomial
- Geometric
- Poisson

Quantities Associated with RVs

- Expectation
  - Mean
  - Expected Value
- Variance
  - Measure of how spread out a distribution is

Example

- Consider 2 independent coin tosses, each with a ¾ probability of a head, and let X be the # of heads obtained.

- This is a binomial random variable with parameters n=2, p=3/4

Example

- What is the PMF?
- What is its mean?

$$E[X] = ?$$

Expectation

- $E[X] = \sum x\, p_x(x)$

- Sum of all possible values of x multiplied by their probabilities

Variance

- Var(X) = expected value of $(X-E[X])^2$
  - $E[(X - E[X])^2]$

- Related quantity: Standard Deviation
  - $StdDev(X) = \sqrt{Var(X)}$