Probability: Random Isn't So Random
Summer 2008

Probability Axioms, Conditional Probability

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HSSP – July 6, 2008

Administrative things

- Late registration
- Claroline class server

Review of last class

- What are the two types of probability?

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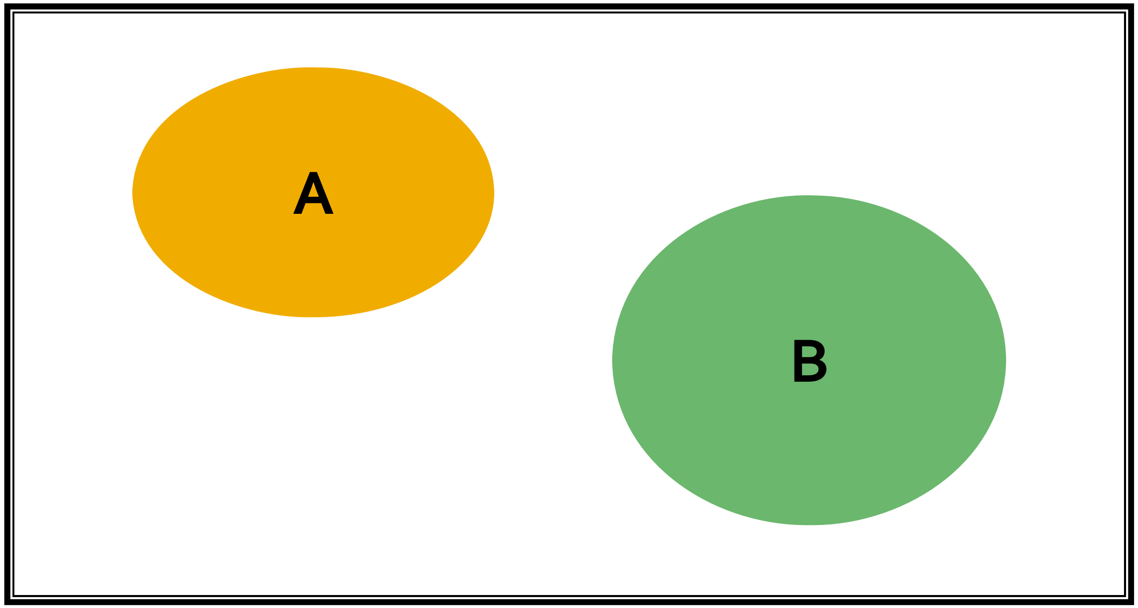
- What does "U" stand for?
- What does $P(A^C)$ mean?

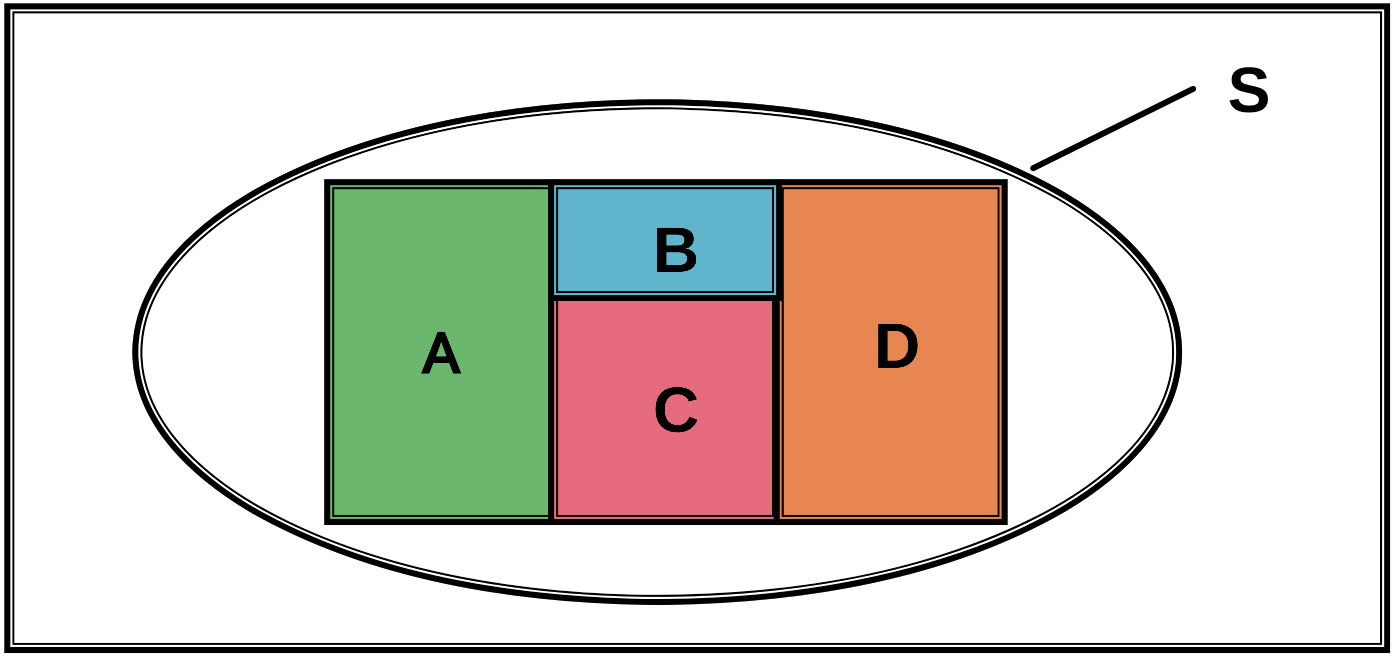
Two More Set Terms

- Disjoint sets
  - No common elements



Two More Set Terms

- Partition (of set S)
  - A collection of disjoint sets whose union is S



Probability Axioms

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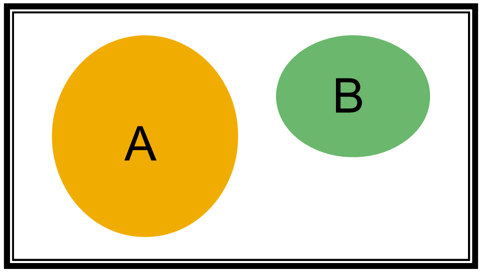
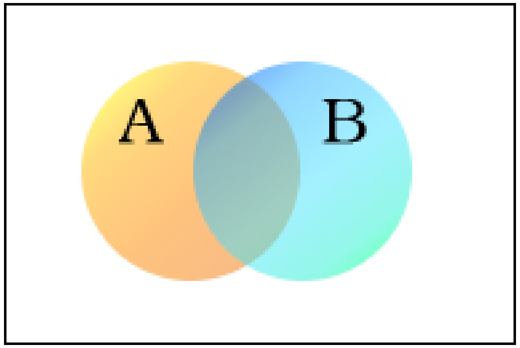
- Normalization
  - $P(\Omega) = 1$

What about overlapping events?

- If A and B are disjoint
  - P (A U B) = P(A) + P(B)



- What if A and B are not disjoint?
  - What is P(A U B)?



Second image from http://commons.wikimedia.org/wiki/Image:Venn-diagram-AB.png

Discrete vs. Continuous

- Discrete: **finite** number of possible outcomes
  - Number on a die roll
  - Possible letter grades on a test
- Continuous: **infinite** number of possible outcomes
  - How long you have to wait for a bus
  - How tall someone can be

Discrete Probability Laws

- The probability of any event $\{s_1, s_2, s_3, \ldots, s_n\}$ is the sum of the probabilities of its elements

$$P(\{s_1, s_2, \ldots, s_n\}) = P(s_1)+P(s_2)+\ldots+P(s_n)$$

Discrete Probability Laws

- If the sample space consists of n possible and equally likely outcomes, then the probability of any event A is

$$P(A) = \frac{\text{number of elements in A}}{n}$$

Conditional Probability

- Probability of an event based on partial information
- "Conditional probability of A given B"
- $P(A \mid B)$

Example: Die Roll

- Assume all six possible outcomes of a fair die are equally likely
- What is the probability that we rolled a 6, given that the outcome is even?
- P(outcome is 6 | outcome is even)

Example: Die Roll

- P(outcome = 6 | outcome is even) = ?

Conditional Probability

(Assuming $P(B) > 0$) ← **Can't divide by zero!**

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

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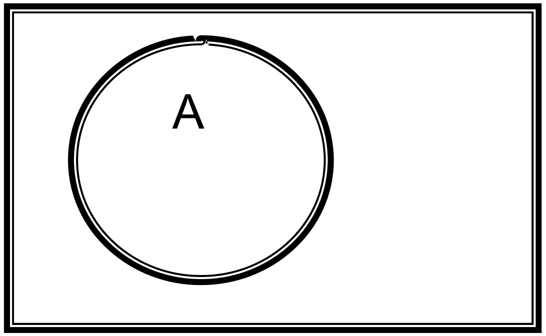
discrete! → (Assuming finite, equally likely outcomes)

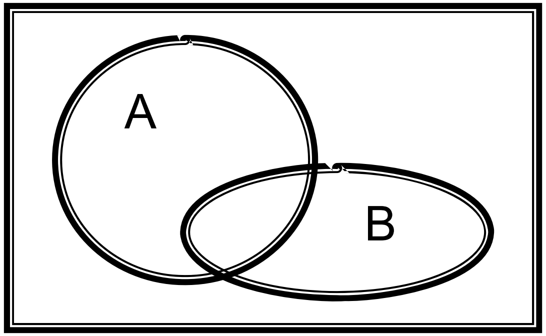
$$P(A|B) = \frac{\text{number of elements of } A \cap B}{\text{number of elements of } B}$$

Conditional Probability

- Probability
  - $P(A) = \frac{P(A \cap \Omega)}{P(\Omega)} = \frac{P(A)}{1} = P(A)$



- Conditional Probability
  - $P(A|B) = \frac{P(A \cap B)}{P(B)}$



Example: Radar Detection

- If an airplane is present in a certain area, the radar correctly registers its presence with 0.99 probability
- If it's not present, the radar falsely registers it anyway with 0.10 probability
- Assume the airplane is present with probability 0.05

Example: Radar Detection

- What is the probability of false alarm?
  - radar registers presence even though airplane is not there
- What is the probability of missed detection?
  - radar does not register, but airplane is there

Example: Radar Detection

- What is our sample space?
- How are we going to represent it?

Example: Radar Detection

- What are the probabilities?

Multiplication Rule

- P(sequence of events) =
  - P(event 1) x P(event 2 | event 1) x P(event 3 | event 1 and event 2) ….

- $P\ (A_{1\text{-}n}) = P(A_1)P(A_2|A_1)P(A_3|A_1 \cap A_2)\ldots.$

[tree]

Problem #1

- Three cards are drawn from an ordinary 52-card decks without replacement (drawn cards do not go back into the deck).
- What's the probability that none of the three cards is a heart?

Problem #2

- There are 4 boys and 12 girls in a class. They are randomly divided into 4 groups of 4.
- What is the probability that each group includes 1 boy?

Monty Hall Problem

- Game show: there are three doors: one has $1 million behind it, the other two have nothing
- You pick one but it remains unclosed
- The host opens one door that reveals nothing (he knows which door has the prize)
- Before he opens your door (you only can pick one door), he gives you the choice of staying with your door or switching to the third door

http://www.imaph.tu-bs.de/qi/monty/puzz.gif

Monty Hall Problem

Switch or Stay?

Summary

- More set terms: disjoint, partition
- Probability axioms
- Discrete vs. continuous
- Conditional probability
- Multiplication rule

Card Deck (for your reference)

Image removed due to copyright restrictions. To see an image of entire deck of cards, please click on the link below.
http://commons.wikimedia.org/wiki/Image:Cards.jpg