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Big Picture

Calculus connects Function (1) with Function (2) = **rate of change** of (1)

Function (1) Distance traveled $f(t)$ Function (2) Speed $s(t)$

Function (1) Height of graph $y(x)$ Function (2) Slope $s(x)$

Function (2) tells how quickly Function (1) is changing

KEY Constant speed $s = \dfrac{\text{Distance } f}{\text{Time } t}$ Constant slope $s = \dfrac{y}{x} = \dfrac{\text{Distance up}}{\text{Distance across}}$

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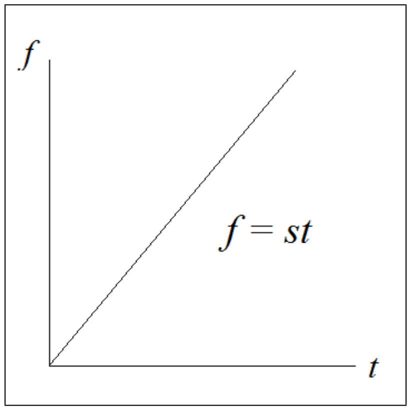
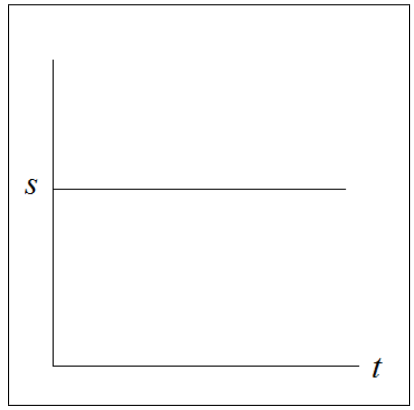
Graphs of (1) and (2)

f = increasing distance

s = constant speed





Slope of f-graph $= \dfrac{\text{up}}{\text{across}} = \dfrac{st}{t} = s$

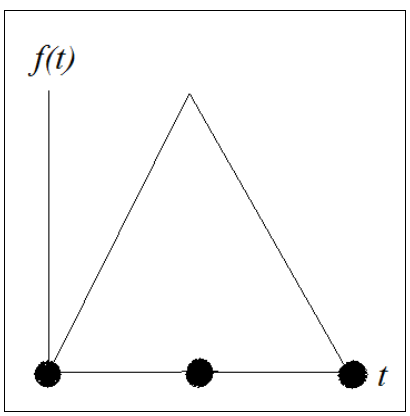
Area under s-graph = area of rectangle $= st = f$

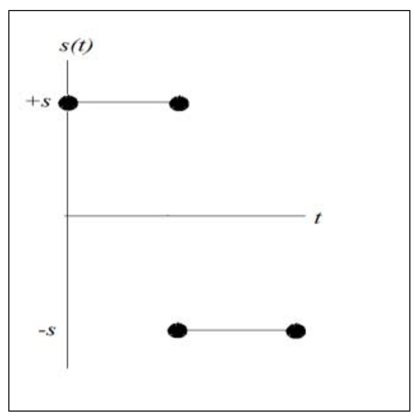
Now run the car backwards.

Speed is negative

Distance goes down to 0

Area "under" $s(t)$ is zero





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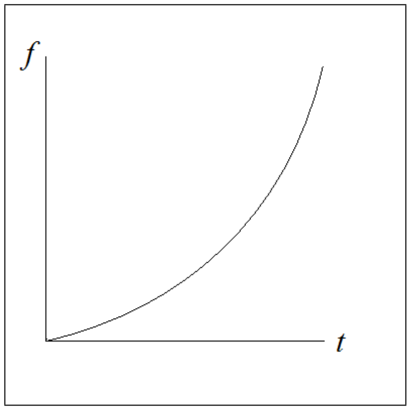
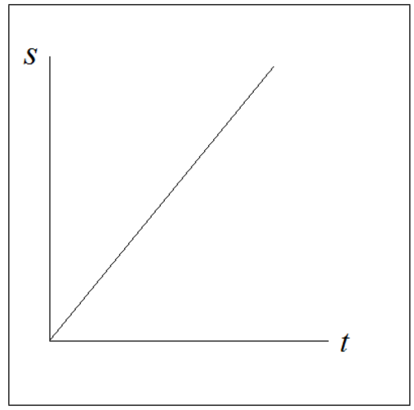
Example with increasing speed Then distance has steeper slope

$$\boxed{\begin{array}{l} f = 10t^2 \\ s = 20t \end{array}}$$





When speed is changing, algebra is not enough $s = \dfrac{f}{t}$ is wrong

Still true that area under s = triangle area $= \dfrac{1}{2}(t)(20t) = 10t^2 = f$

Still true that s = slope of $f = \dfrac{df}{dt}$ = "derivative" of f

When f is increasing, the slope s is **positive**

When f is decreasing, the slope s is **negative**

When f is at its maximum or minimum, the slope s is **zero**

The graphs of any $f(t)$ and $F(t) = f(t) + 10$ have the same slope at every t

To recover f = Function (1) from $\dfrac{df}{dt}$, good to know a starting height $f(0)$

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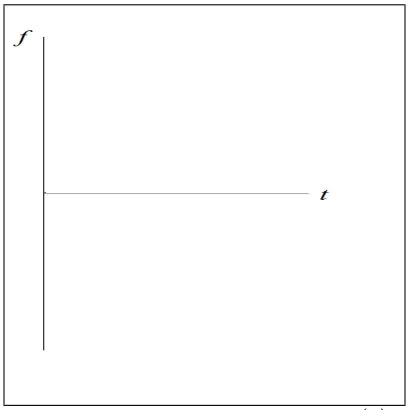
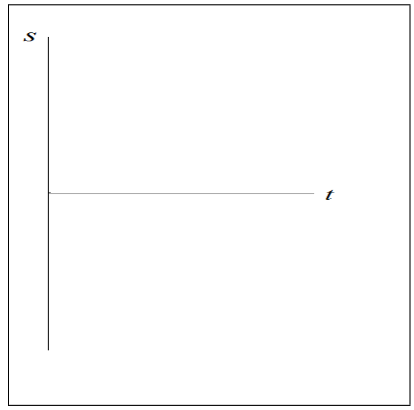
Practice Questions

1. Draw a graph of $f(t)$ that goes up and down and up again.

Then draw a reasonable graph of its slope.





2. The world population $f(t)$ increased slowly at first, now quickly, later slowly again (we hope and expect). Maybe a limit ≈ 12 or 14 billion.

Draw a graph for $f(t)$ and its slope $s(t) = \dfrac{df}{dt}$

3. Suppose $f(t) = 2t$ for $t \leq 1$ and then $f(t) = 3t + 2$ for $t \geq 1$

Describe the slope graph $s(t)$. Compare its area out to $t = 3$ with $f(3)$

4. Draw a graph of $f(t) = \cos t$. Then sketch a graph of its slope. At what angles t is the slope zero (slope $= 0$ when $f(t)$ is "flat").

5. The graph of $f(t)$ is shaped like the capital letter **W**. Describe the graph of $s(t) = \dfrac{df}{dt}$. What is the total area "under" the graph of s?

6. A train goes a distance f at constant speed s. Inside the train, a passenger walks forward a distance F at walking speed S.

What distance does the passenger go? At what speed? (Measure from train station).

Resource: Highlights of Calculus
Gilbert Strang