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**Summary: Big Picture - Derivatives**

Function (1) Start with a known function $y(x)$

This tells us the height $y$ above each point $x$

The problem is to find the "instant slope" at $x$

This slope $s(x)$ is written $\dfrac{dy}{dx}$ It is Function (2)

KEY: $\dfrac{\Delta y}{\Delta x} = \dfrac{y(x+\Delta x) - y(x)}{\Delta x} = \dfrac{\text{up}}{\text{across}}$ approaches $\dfrac{dy}{dx}$

Instant slope $\dfrac{dy}{dx}$ for a new function $y = x^3$

First find average slope between $x$ and $x + \Delta x$

Average slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{(x+\Delta x)^3 - x^3}{\Delta x}$

Write $(x+\Delta x)^3 = x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3$

Subtract $x^3$ and divide by $\Delta x$

$$\frac{\Delta y}{\Delta x} = \frac{3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3}{\Delta x} = 3x^2 + 3x\Delta x + (\Delta x)^2$$

When $\Delta x \to 0$, this becomes $\dfrac{dy}{dx} = 3x^2$

$y = x^3$ fits the pattern for $y = x^n$

The slope is $\dfrac{dy}{dx} = nx^{n-1}$ (one lower power)

NOTICE:

$y = Cx^n$ has slope $Cnx^{n-1}$

Multiply $y$ by $C \to$ Multiply $\Delta y$ by $C \to$ Multiply $\dfrac{dy}{dx}$ by $C$

The slope of $y = 7x^2$ is $\dfrac{dy}{dx} = 14x$

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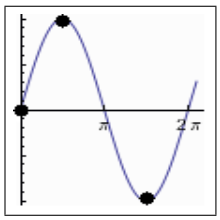
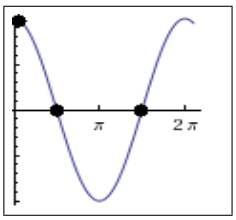
**Neat Fact:** The slope of $y = \sin x$ is $\dfrac{dy}{dx} = \cos x$

The graphs show this is reasonable

Slope at the start is 1 (to find later)




$y = \sin x$
slope $= \cos x$




Sine curve climbing → Cosine curve $> 0$

Top of sine curve (flat) → Cosine is zero

Sine curve falling → Cosine curve $< 0$

Bottom of sine curve (flat) → Cosine back to zero

**Practice Questions**

1. For $y = 2x^3$, what is the average slope $= \dfrac{\Delta y}{\Delta x}$ from $x = 1$ to $x = 2$?

2. What is the instant slope of $y = 2x^3$ at $x = 1$?

3. $y = x^n$ has $\dfrac{dy}{dx} = nx^{n-1}$. What is $\dfrac{dy}{dx}$ when $y(x) = \dfrac{1}{x} = x^{-1}$?

4. For $y = x^{-1}$, what is the average slope $\dfrac{\Delta y}{\Delta x}$ from $x = \dfrac{1}{2}$ to $x = 1$?

5. What is the instant slope of $y = x^{-1}$ at $x = \dfrac{1}{2}$?

6. Suppose the graph of $y(x)$ climbs up to its maximum at $x = 1$
Then it goes downward for $x > 1$

6A. What is the sign of $\dfrac{dy}{dx}$ for $x < 1$ and then for $x > 1$?

6B. What is the instant slope at $x = 1$?

7. If $y = \sin x$, write an expression for $\dfrac{\Delta y}{\Delta x}$ at any point $x$.

We see later that this $\dfrac{\Delta y}{\Delta x}$ approaches $\cos x$

# Resource: Highlights of Calculus

Gilbert Strang