Chapter 10

Contracrostipunctus

More layered than layered cake.

10.1   Abstract

This dialogue is central to the book, for it contains a set of paraphrases of Gödel's self-referential construction and of his Incompleteness Theorem. One of the paraphrases of the Theorem says, "For each record player there is a record which cannot play." The Dialogue's title is a cross between the word "acrostic" and the word "contrapunctus", a Latin word which Bach used to denote the many fugues and canons making up his *Art of the Fugue.* Some explicit references to the *Art of the Fugue* are made. The Dialogue itself conceals some acrostic tricks. *GEB pp. ix*

10.2   Secrets

Although the chapter will have lots more to say, the background for this dialogue requires telling.

Around the end of the 19th century all sorts of paradoxes and counter-intuitive results were popping up in logic and mathematics. Both Zermelo and Russell stumbled upon variants of Russell's paradox, and the grand-daddy of mathematics at the time, David Hilbert, really wanted to stomp out these pathologies of set theory. Hilbert very much believed in the idea that every mathematical problem was solvable and that mathematics could actually be proved to be consistent and complete. He viewed mathematics as the paragon of truth, which rested on completely self-evident ideas that were true externally of what humans think to be true. Hilbert championed this cause until 1931 when a young mathematician named Kurt Gödel proved that:

1. Any consistent formal theory capable of producing arithmetical truths cannot be both consistent and complete.

2. Any formal theory containing the truths of arithmetic and truths about provability has a statement of its own consistency if and only if the theory is inconsistent.

In proving these results, Gödel shattered Hilbert's dreams of securing mathematics sure-footing.

10.3 Questions

1. Pay careful attention to the roles of the Tortoise and the Crab: Who is Hilbert? Who is Gödel?

2. What is the equivalence between low and high fidelity records and the formal theories mentioned above in Gödel's two incompleteness theorems?

3. *GEB pp. 81 – For instance, Lewis Carroll often hid words and names in the first letters (or characters) of the successive lines in poems he wrote. Poems which conceal messages that way are called "acrostics".* Might this quote apply to this dialogue?