

Quantum Mechanics

From Moderate Confusion to Profound Obfuscation

Classical Physics Befuddled

At the end of the 19th century, physics had done an extraordinary job of unraveling the nature of Nature. Newton told us how the planets move around the Sun with his laws of motion and gravity, and Maxwell explained what light is with his theory of electromagnetism. Virtually all known phenomena at the time seemed to be describable by the current physics.

However, critical problems lurked in the theories which needed to be fixed somehow. For instance, if one applied the physics to the motion of an electron around an atom, one would find that the electron would very quickly (in less than a nanosecond) spiral into the nucleus, which would mean that all atoms in the universe are unstable! Another problem was the so-called *ultraviolet catastrophe*, which I discussed in the first lecture. This implied, for example, that all microwave ovens contain an infinite amount of energy, which is, of course, ridiculous.

German physicist Max Planck found a way out of the catastrophe. What he did was to modify the theory of electromagnetism, such that the energy of light would come in *chunks*. This got rid of the problem of infinite energy, but it was merely a mathematical trick, as far as Planck was concerned. It was actually Einstein who attached a physical meaning to this “trick.” He proposed that light really does come in chunks, and nowadays we refer to these chunks as **photons**. So Einstein was really postulating that light consists of *particles*. In doing so, he was able to explain the **photoelectric effect**, whereby shining light at a metal causes electrons to get kicked off the metal. (He actually won the Nobel Prize for this; he didn’t get it for relativity!)

The prevailing theory of light – which held that light is a wave – was unable to explain the photoelectric effect, but it was able to explain some phenomena that any particle theory of light seemed unable to explain. Perhaps the most important experiment which seemed to favor the wave theory of light was the **double-slit experiment**.

In this experiment, monochromatic light (that is, light of a single color), is passed through two slits and, a little farther on, is shown onto a screen. This is shown in the figure below.

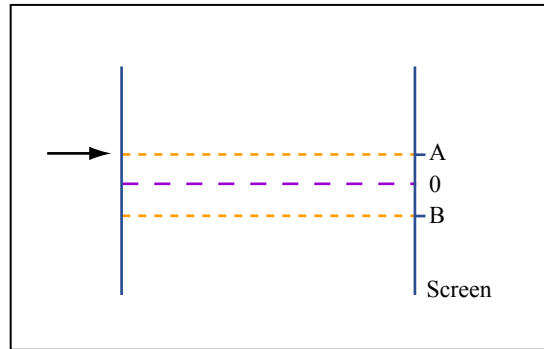


Figure by MIT OCW.

Now, there are essentially two main types of predictions you can make about what you'll observe on the screen. First, if light consists of particles, then you'd simply expect the particles composing the light to pass through the slits in a relatively straight path and then emerge on the two spots directly in front of the slits, as shown below.

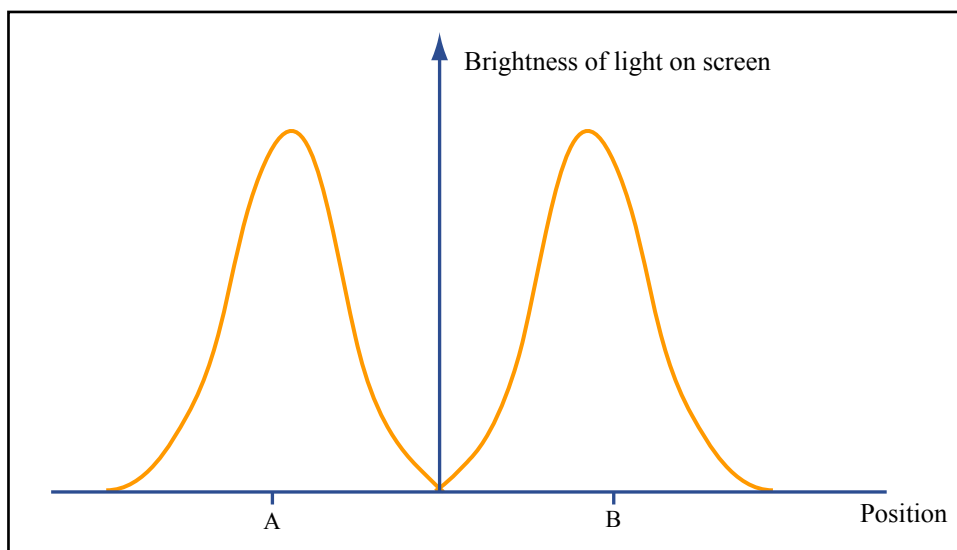


Figure by MIT OCW.

However, if light is *wave*, then you'd expect to see something quite different on the screen. In fact, one would expect something like the following:

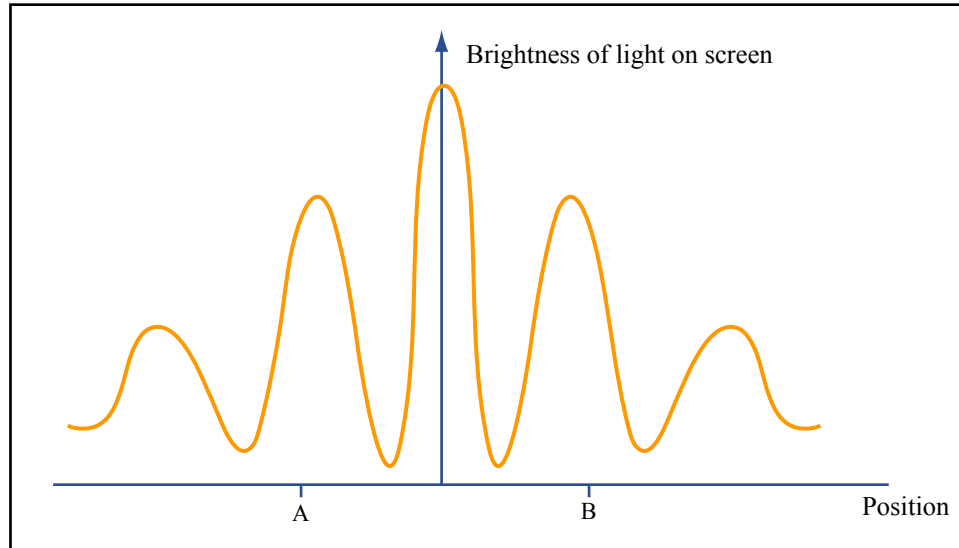


Figure by MIT OCW.

Here's why. Waves, as you've surely heard at some point in your life, have what are called "crests," which are the top points of the wave, and "troughs," which are the low points. You can think of water waves in the back of your mind. The height of the water varies throughout to form a characteristic wave shape, the crest corresponding to where the height is at a maximum and the trough corresponding to the lowest height.

Now suppose two waves are headed towards each other. There are two extreme cases to consider. First, the two waves could be headed such that, when they finally meet, the crest of one wave meets with the trough of the other. As a result, the two waves totally cancel each other out – this is called **destructive interference**. In the other case, the two waves could be headed towards each other such that, when they finally meet, crest meets crest or trough meets trough. If this happens, then the two waves will *amplify* one another: crest meeting crest results in the height being even higher than initially, and trough meeting trough results in the height being even lower than initially. This is known as **constructive interference**.

You can use these ideas to cook up many interesting scenarios with waves. You can make waves interfere neither constructively nor destructively but somewhere in between, for instance. Or you can make many waves interfere with each other. The precise details behind how the pattern produced on the screen in Figure 3 would take a bit of explaining, and they aren't incredibly important; you can at least imagine how constructive and destructive (and partial) interference could give rise to it. The main point is that waves are *very* different from particles.

As I've said, the *wave* prediction is actually what's observed on the screen.*
[*Footnote. Strictly speaking, the particle prediction is sometimes correct, and you might have observed this yourself. In fact, it turns out that you can arrange the experiment in such a way that sometimes the wave and particle predictions agree with each other. But, whenever the particle prediction is right, so is the wave prediction. And, while the particle theory often doesn't agree with the experiment, the wave theory always does. This lends overwhelming support to the wave theory.]

So, here's the big question: is light a particle or a wave?

Quantum Mechanics Un-befuddles by Befuddling More

When it was finally formulated in the 1920s, quantum mechanics definitively answered this question. However, its answer was truly disturbing: light is neither a particle nor a wave! Rather, it is simply this *thing* that, in some cases, acts wave-like and, in other cases, acts particle-like. Which way it acts depends on the situation – this is the essence of **wave-particle duality**.

What about objects which we usually call “particles,” like protons, electrons, bullets, and baseballs? Well, according to quantum mechanics, those “particles” can also act wave-like! For example, you can repeat the double-slit experiment with electrons. Provided that the slits are narrow enough, you'll actually observe interference – the electrons will really interfere with each other, just as quantum mechanics predicts. The experiment has even been done with *viruses* – which are much larger than single atoms – and, lo and behold, interference is observed!

Here's an interesting question: What if we tried doing the double-slit experiment with a human? According to quantum mechanics, interference should occur under suitable experimental conditions. But... what would it feel like? What would go through your mind?

So, what does quantum mechanics actually *say*? How does it describe Nature?

Well, according to quantum mechanics, there exist *things* in the world (like electrons, baseballs, and the President of the United States), and each of these things is in its own **state** at a given moment in time. (Note: I'll generally use the word "particles" instead of "things" in the remainder of these notes, but keep in mind that nothing is truly a particle or a wave. The notions of "particles" and "waves" are merely intuitive idealizations.)

Now, this state – the *quantum* state (also called the **wavefunction**) – is quite different from the *classical* state. According to classical physics (i.e., pre-1900 physics), a particle may be completely described by specifying its position and speed; doing so specifies the classical state. Of course, you may not know with perfect experimental certainty what a particle's position and speed are, but according to the classical description of the world, you can rest assured that the particle certainly *has* a position and a speed at all times.

The quantum description of the world is utterly different. According to quantum mechanics, a particle simply *doesn't* have a position and a speed; these notions are *meaningless*. They don't exist in reality. (At least, they don't exist until you make a measurement of the particle, but we'll get to this point in a minute. In any case, a particle never *simultaneously* has both a position and a speed, even after you make a measurement.) If this weren't bizarre enough, quantum mechanics actually says that a particle is, in a sense, in many places at the same time (not just one place), and it has many speeds at the same time (not just one speed)! This is called the **superposition principle**: prior to a measurement, a particle is in state which is a kind of mixture of what may be called "definite location" and "definite speed" states.

Suppose now that you'd like to measure, say, a particle's position. Before you do that, though, you have to be careful interpreting the previous

sentence. As I said, the particle doesn't have a position until you try to *measure* its position. More accurately, you *give* the particle a position once you try measuring its position. This process of "giving" a particle a definite property – like a position, a speed, an energy, etc. – is called **collapsing** the particle's wavefunction. So, the result of measurement on a particle is to collapse the particle's wavefunction into a "definite location" or a "definite speed" (or a "definite energy," etc.) wavefunction. Suffice it to say, measurement in quantum mechanics is a very peculiar process, and to this day it's actually still not completely understood (or, at least, not even the experts agree on what it really is).

Let's get back to measuring the particle's position. There are many possible outcomes as to what you can measure – in principle, there are infinitely many! You could measure the particle at some spot in front of you, or a millimeter away from that spot, or a foot away, or 833 light-years away... However, *where* you measure it depends on the particle's state just before measurement. Oddly, the particle's state does *not* specify with certainty where you'll find it! What the state *does* tell you is the various *probabilities* of finding it at certain locations, i.e., it tells you a *probability distribution*. It might tell you, for example, that there's a 97% probability of finding an electron inside of some box, but a 3% chance of finding it outside the box. This means that, if you prepared an electron in a certain state 100 different times, and then tried measuring its position, you'd expect to find the electron inside the box 97 times but outside just 3 times. Another example: probability distribution of electrons in an atom.

Thus, as far as results of measurements go, quantum mechanics is a *non-deterministic* theory; classical mechanics, by contrast, is a deterministic theory, since one could, in principle, make perfect predictions given a completely specified classical state.* [Footnote. *You might wonder, however, if it's possible to predict with certainty how the wavefunction, and therefore the position probability distribution, of a particle changes in time. The answer is that it *is* possible, and an equation called the **Schrödinger equation** tells precisely how it changes. In this sense, quantum mechanics *is* a deterministic theory. One can then ask which represents the ultimate reality: the wavefunction, or the results of measurement? If it's the case of the former, then reality is deterministic; in the latter case, it isn't.]

Quantum Tunneling

One of the reasons I love quantum mechanics is that it gives me hope. It gives me hope, for example, that I can walk through a wall, fly to the top of the Empire State Building, and survive a fall off a cliff. If this sounds extremely silly to you, that's a *good* thing – it means you're in for a wonderful surprise! It turns out that all of these scenarios can actually happen, according to quantum mechanics, through a nifty little effect called **quantum tunneling**.

Suppose you're riding your bicycle in some hilly area. As you know, going uphill is harder than going downhill, and that's because to go uphill you need to apply more energy. In fact, there's a *minimum* amount of energy you need to put in, or else you simply wouldn't make it up the hill. This is obviously true, right?

Wrong. According to quantum mechanics, there's a *slight, slight, slight* probability that you can make it up the hill with any arbitrary speed. This is where the wave-particle duality of objects comes in. Ordinarily, the particle-like aspect of everyday objects is exceedingly more apparent than the wave-like aspect. For this reason, we don't generally see weird events like dogs and cats interfering (in the physics sense!) with one another. However, the wave-like aspect of objects is what's responsible for the probabilistic nature of quantum mechanics. It's because of the wave-like nature of electrons, for instance, that we're unable to predict with certainty the result of any one measurement of an electron's position.

In exactly the same manner, according to quantum mechanics, we're unable to predict with certainty where we'll find a human. Now, I know what you're thinking: "What?! I can predict with great certainty where I'll find a human! I see people move all the time, and it's very easy (disregarding psychological considerations) to predict where a person will move from point in time to the next!" It's true that you can predict with "great" certainty where a person will be found, but quantum mechanics even says this: you'll find a human where you expect to find a human with 99.99999999999999% chance. (Actually, there are probably many more 9's than that.) The reason is that, in general, quantum effects disappear – for

all practical purposes – for large objects. They’re most readily observed – and have been most readily confirmed – for extremely small objects, like electrons and atoms. However, *in principle*, quantum effects apply to *all* objects, and *in principle*, they apply even to humans. The result of a measurement of a human is uncertain, and quantum mechanics predicts that it’s possible (although with an extraordinarily small probability) that at one moment you’ll find yourself at the bottom of a hill on your bike, and at the next moment at the top of the hill, or even on the other side of it. For a better visual understanding, see the figures below.

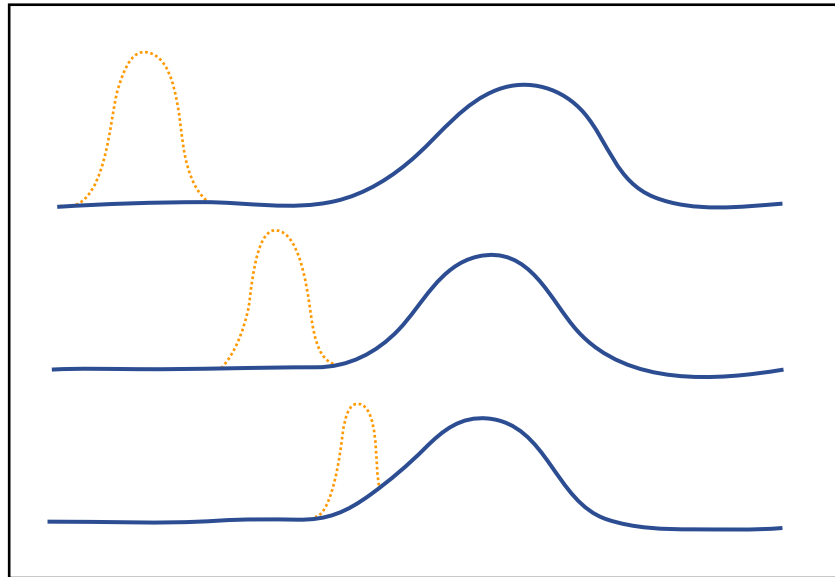


Figure by MIT OCW.

Of course, all of this reasoning needs to be made much more precise, but ultimately that would require some mathematics. The reader is therefore wholeheartedly encouraged to study calculus and differential equations, as well as classical mechanics and electromagnetism, and then pick up an actual textbook on quantum mechanics and learn precisely how quantum tunneling works!

We can have more fun with quantum tunneling and use our imagination to think of various peculiar things that the tunneling effect implies. [Add more in later... “magical” worlds, etc.]