Combinatorics: The Fine Art of Counting

Week Six Problems

In an effort to trim the fat, this week's menu has been pared to the bone. Please take a look at all four problems. The last one is especially fun.

Assume that all dice, decks, coins, etc… are standard (i.e. six-sided dice, 52 card decks, coins with heads and tails, etc...) and fair. Balls are drawn from urns without replacement unless otherwise stated.

Standard Fare

1. Which is more likely, rolling a sum of 9 with two dice, or rolling a sum of 9 with three dice? (Compute exact probabilities for both cases).

   As a follow-up, what are the most likely sums when rolling two or three dice and how likely are they?

   If you are feeling ambitious, construct a table of all possible sums for two dice with the probabilities of each sum listed and then do the same thing for three dice (three dice are much more interesting). Keep your probabilities as unreduced fractions for easy comparison, and be sure to check that your tables add up to 1.

2. Consider a randomly dealt hand of five cards from a standard deck. Let A be the event that the hand contains an ace. Let B be the event that the hand contains one pair (2 cards of the same rank and 3 cards of different ranks). Let C be the event that the hand contains one pair of aces (2 aces and 3 cards of different ranks). Note that the hand (A♠,4♣,4♦,K♠,J♥) is contained in both A and B but not C. Compute the following probabilities:

   P(A) = ? P(B) = ? P(AUB) = ? P(A∪B) = ? P(C) = ?

   Now suppose you know that one of the cards is an ace. Compute the following conditional probabilities:

   P(B|A) = ? P(C|A) = ?

3. An urn contains 3 red balls and *n* blue balls. The probability of drawing two blue balls is the same as the probability of drawing two balls with different colors. Determine *n*.

   As a follow-up, replace 3 with *m* in the question above and find *n* in terms of *m*.

4. Perhaps the most famous probabilistic puzzler of all-time is what is known as the "Monty Hall" problem. This problem has baffled a surprising number of people, including more than a few mathematicians, but you should be able to solve it easily using what you have know about conditional probability.

   You are a contestant on a game show and are shown three closed doors and told that behind one of them is a brand new car while the other two have goats behind them. You are asked to pick the door you think hides the car. After you have made your choice, the game show host (Monty Hall) opens a door you did not pick and there is a goat behind it. He then asks you whether you want to change your mind and switch doors, or stick with the door you originally picked.

   Should you switch or stick? What is the probability that you will get the car?

   If you want to try playing this game, visit the following web-site:

   http://math.ucsd.edu/~crypto/Monty/monty.html