Combinatorics: The Fine Art of Counting

Week Two Menu

This week's menu consists of many small tasty dishes, some light, some hearty. The Spanish refer to this type of meal as *Tapas*. Please select several. All of these problems have a straight-forward solution, but some might be considered difficult to the uninitiated. Contest problems are identified. I have made the contest problems easier/harder by leaving out the answer choices offered on the test.

Tapas Menu

1. How many different ways can you rearrange the letters in BOSTON? How about MASSACHUSETTS?
2. 20 students show up to HSSP looking for open classes. Only 3 classes are still open, one has 3 spots, one has 11 spots, and one has 6 spots. How many different ways can the students be arranged in the 3 classes?
3. How many license plates with 3 decimal digits followed by 3 letters do not contain both the number 0 and the letter O?
4. How many ways can you paint the faces of a regular tetrahedron with four colors if each face is painted a different color? (Assume that two paintings that can be oriented to look the same are considered indistinguishable)
5. A circular table has 60 chairs around it. There are N people seated at this table so that the next person seated must sit next to someone. Find the smallest possible value of N. (AHSME 1991 #15)
6. How many different sequences of the numbers {0,1,2} of length 10 do not contain any of the subsequences 12, 23, or 31? 3222132111 is such a sequence. (AIME 2003B #3)
7. A decimal number is called "increasing" if each digit is greater than the previous one (e.g. 24589 is one). How many 5 digit increasing numbers are there? (AIME 1992 #2)
8. Let S = {1, 2, …, 10 }. Find the number of unordered pairs A, B where A and B are disjoint non-empty subsets of S. (counting unordered pairs simply means we don't distinguish the pair A,B and B,A) (AIME 2002B #9)
9. A 7 digit phone number $d_1d_2d_3d_4d_5d_6d_7$ is called memorable if $d_1d_2d_3$ is exactly the same sequence as $d_4d_5d_6$ or $d_5d_6d_7$ (possibly both). (e.g. 4357435 is memorable). Assuming each d_i can be any decimal digit (so d_1 could be 0), how many memorable telephone numbers are there? (AHMSE 1998 #24)
10. How many triangles can be formed with vertices on a 4x4 grid of points? (AHSME 1993 #28)