Combinatorics: The Fine Art of Counting

Chinese Dice Group Activity

*The following table lists the probabilities of all the various types of throws. The notation (6 4,1,1) is a multi-nomial coefficient that indicates the number of distinct permutations of xxxxyz, which by the Mississippi rule is 6!/(4!*1!*1!). This is a generalization of the binomial coefficients where only one of the two numbers is listed, i.e. (6 3) = (6 3,3).*

Throw Type	***Count***	***Exact Probability***	***Approximation***
6	*6*	*1/7776*	*.00013*
5-1	*(6 5)*6*5 = 180*	*5/1296*	*.0039*
3-3	*(6 3)*(6 2) = 300*	*25/3888*	*.0064*
4-2	*(6 4)*6*5 = 450*	*75/7776*	*.0096*
1-1-1-1-1-1	*6! = 720*	*5/324*	*.015*
4-1-1	*(6 4,1,1)*6*(5 2) = 1800*	*25/648*	*.039*
2-2-2	*(6 2,2,2)*(6 3) = 1800*	*25/648*	*.039*
3-2-1	*(6 3,2,1)*6*5*4 = 7200*	*25/162*	*.15*
3-1-1-1	*(6 3,1,1,1)*6*(5 3) = 7200*	*25/162*	*.15*
2-1-1-1-1	*(6 2,1,1,1,1)*6*(5 4) = 10,800*	*25/108*	*.23*
2-2-1-1	*(6 2,2,1,1)*(6 2)*(4 2) = 16,200*	*25/72*	*.35*
All Types	6^6 *= 46,656*	*1*	*1*

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"Craps" Group Activity

Let W_1 be the event of winning on the initial roll, let L_1 be the event of losing on the first roll, and let P_k be the event of rolling the point value k on the first roll.

$P(W_1) = 6/36+2/36 = \mathbf{2/9}$ $\quad$ $P(L_1) = 1/36+2/36+1/36 = \mathbf{1/9}$

$P(P_4) = P(P_{10}) = 3/36 = \mathbf{1/12}$ $\quad$ $P(P_5) = P(P_9) = 4/36 = \mathbf{1/9}$

$P(P_6) = P(P_8) = \mathbf{5/36}$

Let W be the event of winning.

$$P(W) = P(W_1) + 2*P(P_4)*P(W|P_4) + 2*P(P_5)*P(W|P_5) + 2*P(P_6) *P(W|P_6)$$

Note that the probability rolling a given point value prior to rolling a 7 is the probability of ***not rolling either the point value or a 7*** *an arbitrary number of times (possibly zero) followed by rolling the point value.*

Probability of not rolling a 4 or a 7 = 1 – (1/12+1/6) = 3/4

$P(W|P_4) = 1/12 + (3/4)*(1/12) + (3/4)^2*(1/12) + (3/4)^3*(1/12) + \ldots$

$P(W|P_4) = 1/12 * [1/(1 - 3/4)] = \mathbf{1/3}$

Probability of not rolling a 5 or a 7 = 1 – (1/9+1/6) = 5/18

$P(W|P_5) = 1/9 + (13/18)*(1/9) + (13/18)^2*(1/9) + (13/18)^3*(1/9) + \ldots$

$P(W|P_5) = 1/9 * [1/(1 - 13/18)] = \mathbf{2/5}$

Probability of not rolling a 6 or a 7 = 1 – (5/36+1/6) = 25/36

$P(W|P_6) = 1/12 + (25/36)*(1/12) + (25/36)^2*(1/12) + (25/36)^3*(1/12) + \ldots$

$P(W|P_6) = 1/12 * [1/(1-(25/36))] = \mathbf{5/11}$

Putting this all together we obtain:

$\mathbf{P(W)} = 2/9 + 2*(1/12)*(1/3) + 2*(1/9)*(2/5) + 2(5/36)*(5/11) = 976/1980 = \mathbf{244/495}$

$\mathbf{P(W) \sim 0.4929}$

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"Set" Group Activity

Any two cards determine a set, i.e. there is one and only one third card that can be added to make a Set. If we count all pairs of cards, we will count each Set three times since there are three pairs we could choose from each Set. Thus there are ***(81 2)/3 = 27*40 = 1080*** *Sets. Any particular card is contained in 40 of these Sets, since 40*81/3 = 1080.*

3 properties in common:	*(4 3)*3^3	= ***108***	*probability* ***1/10***
2 properties in common:	*(4 2)*3^2*3!*	= ***324***	*probability* ***3/10***
1 property in common:	*(4 1)*3*$(3!)^2$	= ***432***	*probability* ***4/10***
No properties in common:	$(3!)^3$	= ***216***	*probability* ***2/10***

*The number of groups of 4 cards which contain a Set is 1080*78 so the probability that a group of 4 cards contain a Set is 1080*78 / (81 4) =* ***4/79 ~ .05***

Five cards can contain just one Set, or two overlapping Sets. We will count both cases separately:

Exactly one Set:	*1080*78*74/2*
Two overlapping Sets:	*1080*78*3/2*
Total:	*1080*77*39*

*The probability that five cards contain a Set is 1080*77*39 / (81 5) =* ***10/79 ~ .13***

Six cards can contain just one Set, two overlapping Sets, or two disjoint Sets:

Exactly one Set:	*1080*78*74*69/3!*
Two overlapping Sets:	*1080*78*3/2*72*
Two disjoint Sets:	*1080*(1079-78*3/2)/2*
Total:	*1080*13*17*641*

*The probability six cards contain a Set is 1080*13*5791 / (81 6) =* ***28955/115577 ~ .25*** *(note this is very close but not equal to 20/79).*