Understanding Impedances

Why do we need to know this?

We're going to build speakers using two or more different drivers (for example, a woofer and a tweeter) to each cover a specific part of the audible range. For example, we might want to use a 7" cone woofer to play all sounds up to a frequency around 2 KHz, and a 1" dome tweeter to play the higher frequencies - so that together, they make good music. But speaker drivers don't naturally work that way; we need to add some kind of electrical circuit to each driver, to equalize its frequency-response just the way we want.

The power delivered into the a speaker driver (which is proportional to the amount of sound we get out) is the voltage across the terminals of the speaker times the current going through it. While the actual relationship between those is somewhat complicated, we can predict it very accurately by using Ohm's law ($V = IR$, or voltage equals current times resistance) and saying that the speaker has a resistance that changes with frequency. That frequency-dependent resistance is called impedance; it's measured in ohms like resistance, but it's represented by the letter Z. This is the real Ohm's law:

$$V = IZ$$

The trick is in figuring out how to design circuits to filter audio signals based on this principle. This is the basis for crossover design!

Mathematical background

Impedance is a property of any two-port passive network (that is, any circuit made from resistors, capacitors, and inductors). Such a network can be condensed into one component that might look like this in schematic form:
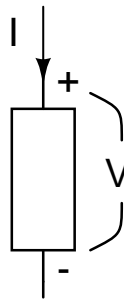
The voltage V is the potential difference (measured in Volts = Joules per Coulomb, or energy per unit charge) *across* the component. The current I (measured in Amps = Coulombs per second) is what flows through the component. The sign convention is that normally a *positive* current flows into the *positive* terminal. If you measure the voltage difference from the bottom to the top instead of top to bottom, your measurement of the current is flipped around too.

One important thing about impedance is that it's a function of frequency (i.e. $Z(\omega)$). That's because the scalar equation $V = IZ$ is actually only true for a sinusoidal wave (any one frequency). But if you decompose any voltage or current signals into their frequency spectra, $V(\omega) = I(\omega)Z(\omega)$ exactly. Engineers are lazy, so they assume you know that and just write $V = IZ$.

The angular frequency ω is measured in radians per second. Since it takes 2π radians to go around a circle, the absolute frequency (in Hz, or cycles per second) is $f = \frac{\omega}{2\pi}$. You will see f and $2\pi\omega$ used interchangeably.

Another important (and confusing) thing about impedance is that it needs to be described as a complex number with both real and imaginary ($i = \sqrt{-1}$) parts. To simplify things, instead we state impedance as a *magnitude* $|Z|$ (the overall amount of impedance) and *phase angle* $\angle Z$. Remember how the woofers in the open baffle made almost

no sound when you listened from the side? That's because the sound coming from the front and back of the speaker had the same strength (magnitude), but were actually opposing one another. Their phase angles were different by 180 degrees, so they cancelled each other out. Electric circuits affect the phase of audio signals just like speakers do; they can actually shift or delay the sound wave, because inductors and capacitors store and release energy over time. The main reason we think of it this way is because we don't want the tweeter and woofer to cancel each other out in that frequency range near the crossover point where they are both playing together. We're going to control the impedances in our crossover circuits so that the phase of the sound waves coming from the different drivers is aligned.

Passive components

Resistors, capacitors and inductors are 2-port passive components. Each of them has an impedance associated with it, and using simple rules for series and parallel components, you can compute the impedance of almost any combination of these components.

- **Resistors**: The impedance of a resistor is $Z_R = R$, or simply its resistance. There is no imaginary part; the magnitude of the impedance is R and the phase angle is zero at all frequencies. That makes sense, right? Adding a resistor to a system can't delay or shift a wave form, only decrease its magnitude.

  Resistances are measured in ohms (Ω), and can be made arbitrarily small or large; values between 1Ω and 50Ω are common in speakers.
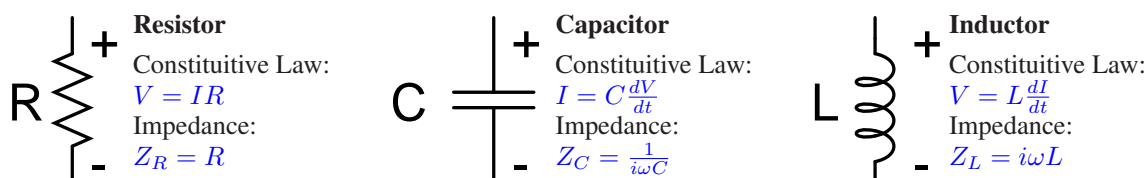
- **Capacitors**: The impedance of a capacitor is $Z_C = \frac{1}{i\omega C} = \frac{1}{i2\pi fC}$. The important thing to notice is that the frequency f times the capacitance C is in the denominator. For any given capacitance, the impedance decreases with increasing frequency. And the magnitude of the impedance goes the opposite way from the capacitance: larger capacitors have lower impedance. The i in there means that the impedance is imaginary. It has a phase angle of -90 degrees. What that means as far as we're concerned is that the current through a capacitor is actually shifted by $\frac{1}{4}$ of a cycle relative to the voltage. If you put in a pure tone where the voltage is alternating back and forth, the current will be highest when the voltage is zero! Don't you think it would be pretty tough for an amplifier to drive a capacitor?

  Capacitances are measured in farads (F), although feasible values of capacitors vary from around 1 pF (10^{-12} F) to 10 mF (0.01 F). In speakers you'll find capacitors between 1μF and 100μF.

- **Inductors:** The impedance of an inductor (coil) is $Z_L = i\omega L = i2\pi fL$. It's the opposite of a capacitor: the impedance is directly proportional to frequency, and larger inductors have higher impedance. Because the i isn't on the bottom of a fraction, the phase angle is +90 degrees. In other words, the current in an inductor is also shifted $\frac{1}{4}$ of a wave compared to the voltage, but in the opposite direction! What do you think would happen if you connected both an inductor and a capacitor in series with a speaker? Would they block all the current at the frequency where the impedance magnitudes matched?

  Inductances are measured in henries (H), and commonly show up between 1 μH (10^{-9} H) and 1 H. The inductors in speakers are typically 0.1 mH to 10 mH.

This picture summarizes the three types of passive components:

Resistor
Constituitive Law:
$V = IR$
Impedance:
$Z_R = R$

Capacitor
Constituitive Law:
$I = C\frac{dV}{dt}$
Impedance:
$Z_C = \frac{1}{i\omega C}$

Inductor
Constituitive Law:
$V = L\frac{dI}{dt}$
Impedance:
$Z_L = i\omega L$

Demonstrations and speakers

By hooking up these components in series with a speaker, we saw impedances in action. The capacitor added a lot of impedance at low frequencies, so when we connected it in series with a driver, it filtered out the bass - and the effect was stronger when we used smaller values of capacitance. The inductor added impedance at high frequencies and made the driver sound softer.

Our simple crossover network used an inductor on the woofer and a capacitor on the tweeter. We added a series resistor to the tweeter circuit because the tweeter had a higher sensitivity than the woofer we needed to decrease the signal strength going into the tweeter so the speaker would sound balanced.

Here's a picture to show what's going on when you connect an inductor in series with a woofer. There are three pictures:

- **On top:** This schematic shows that the inductor goes between the positive lead of the amplifier output and the positive connector on the driver. The little resistor is there because every real inductor has some resistance.
- **In the middle:** This is the electrical frequency response. It shows how the addition of the inductor alters the strength of each frequency component that the driver produces.
- **On the bottom:** Acoustic frequency response. This represents the sound coming from the driver. Its original frequency response has been multiplied by the electrical frequency response, which declines at higher frequencies.
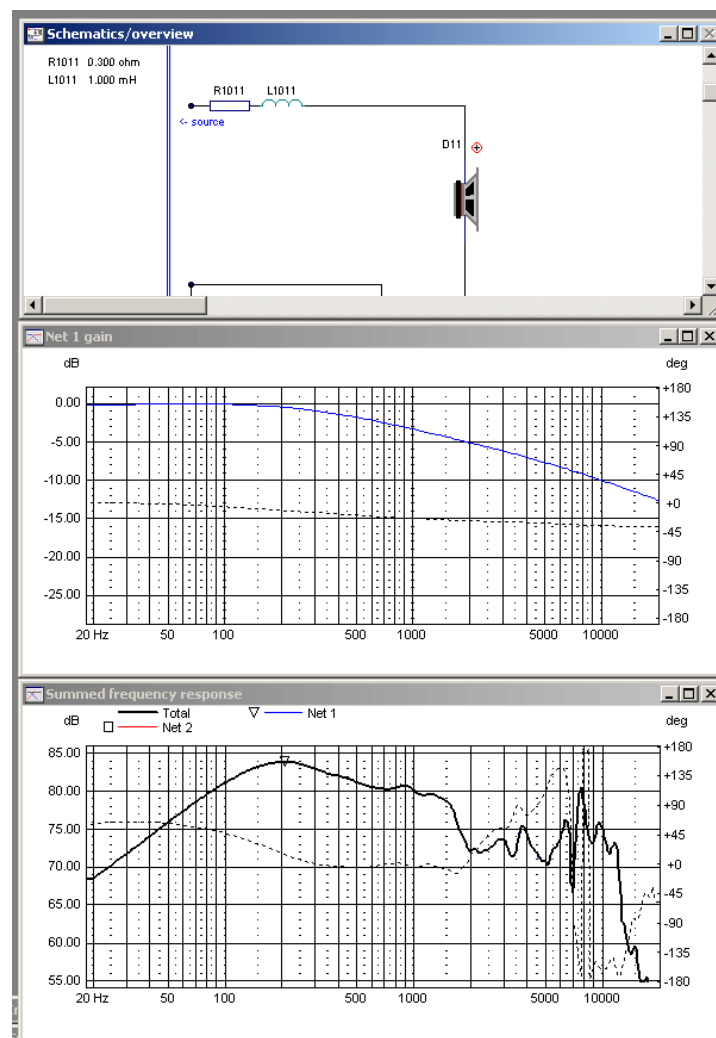
We also tried using a ribbon tweeter the funky-looking yellow rectangular one instead of the regular dome. With the resistor and capacitor connected like we had before, it sounded a lot better than the other tweeter. Why? There isn't a huge difference in quality between these two drivers.

Image of Dayton DC28F Soft dome tweeter due to copyright restrictions.

Image of Dayton PT2B Planar magnetic tweeter removed due to copyright restrictions

I went on the Internet to look up the impedance curves for those two drivers. Here is what I found:
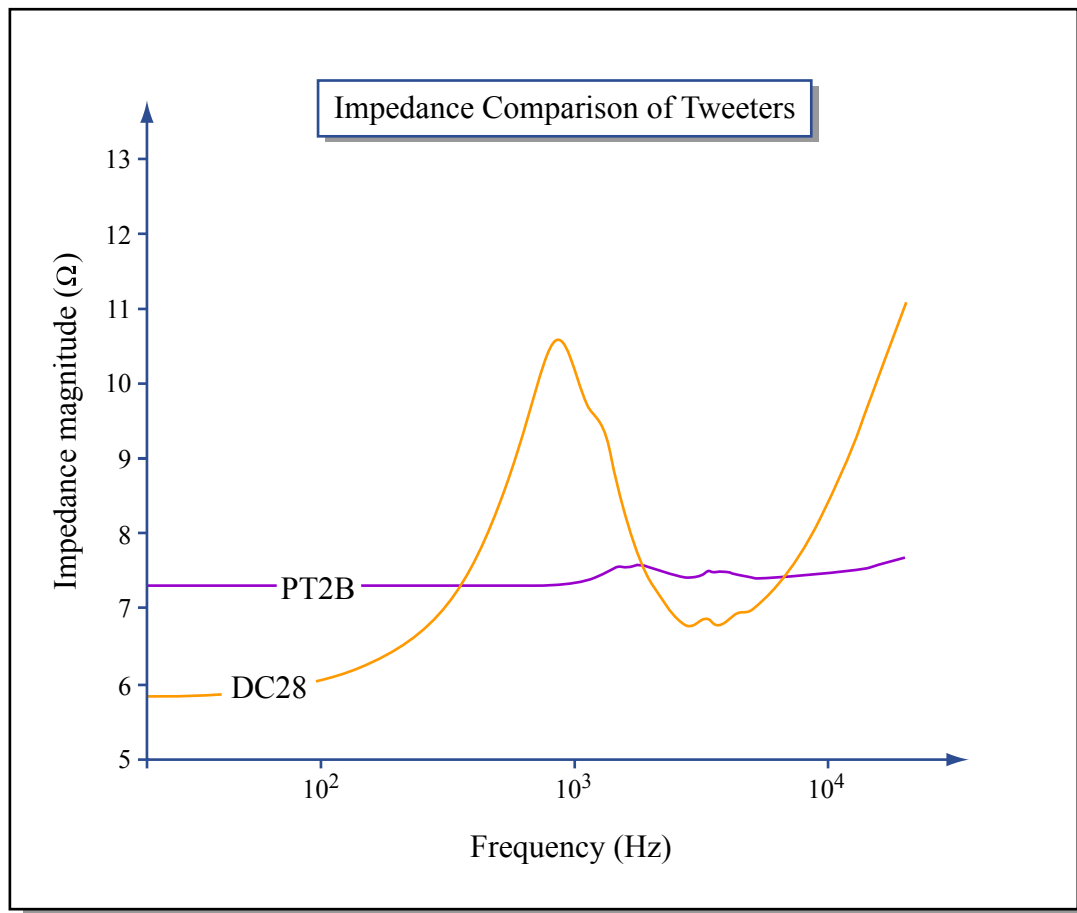

Impedance Comparison of Tweeters

Image by MIT OCW.

The DC28 (a regular soft dome tweeter) had a varying impedance, but the PT2B tweeter's impedance was nearly constant over all frequencies (like a resistor). Even though the crossover network stayed the same, the impedance variation of the standard tweeter was causing problems in the frequency response! That doesn't mean that drivers with flat impedance are always better. In fact, we can add components to our circuit (called a "zobel") that make the impedance of any driver appear constant to the rest of the network. Then it's no problem at all.

Series and parallel

These are the rules for finding the impedances of combinations of components:

- **Series connection:** If you have two 2-port networks (with impedances Z_1 and Z_2) connected one after the other, the equivalent impedance of that series combination is $Z_{eq} = Z_1 + Z_2$. The impedance of a bunch of networks connected in one chain is the sum of all of the individual impedances.

- **Parallel connection:** If you have two 2-port networks (with impedances Z_1 and Z_2) connected together, the equivalent impedance of that parallel combination is $Z_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2}$.

If you put two equal impedances in series, the effective impedance is double what you started with. If you put them in parallel, you get half of what you started with.

Series connection

$Z_{eq} = Z_1 + Z_2$

Parallel connection

$Z_{eq} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2}} = \frac{Z_1 Z_2}{Z_1 + Z_2}$
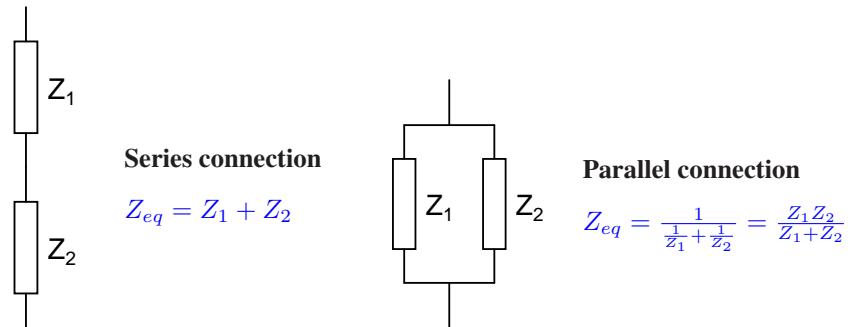
How about an example? Let's find the equivalent impedance of this network.
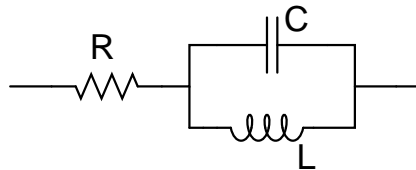

The circuit is a resistor R in series with the parallel combination of the inductor L and capacitor C. First, let's find the impedance of the parallel combination of L and C:

$$Z_1 = Z_L = i\omega L$$
$$Z_2 = Z_C = \frac{1}{i\omega C}$$
$$Z_{LC} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$
$$= \frac{\frac{i\omega L}{i\omega C}}{i\omega L + \frac{1}{i\omega C}}$$
$$Z_{LC} = \frac{i\omega L}{1 - \omega^2 LC}.$$

Now we can add the resistor's impedance to that, because they are in series:

$$Z_{eq} = Z_R + Z_{LC}$$
$$= R + \frac{i\omega L}{1 - \omega^2 LC}$$
$$Z_{eq} = \frac{R(1 - \omega^2 LC) + i\omega L}{1 - \omega^2 LC}$$

That expression is a little complicated. Take a look at the graph of the impedance magnitude versus frequency, $Z_{eq}(\omega)$:
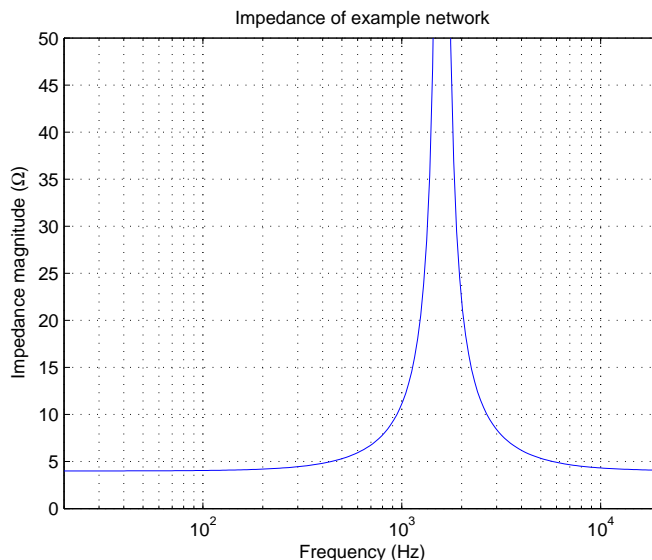
Isn't this interesting? The parallel combination of an inductor and a capacitor is a *resonant circuit* that has a relatively low impedance at most frequencies, but a very high impedance around the *resonant frequency*, $f_0 = \frac{1}{2\pi LC}$ (in this case it's around 1.6 KHz). The resistor gives this network a minimum impedance (in this case 4 Ω) at frequencies where the LC combination acts as a short circuit.

If you put this network in series with a speaker, frequencies around 1.6 KHz would get blocked out from the sound, and all of the other frequencies would get a little bit quieter.

Higher-order filters

In the next two weeks we're going to start using combinations of components in series and parallel, instead of everything in series with the driver. It turns out that when you put a component in parallel with a speaker, it tends to have the opposite effect of when you put it in series. Why?

Remember that a capacitor has impedance that decreases with increasing frequency. When you put it in series with the speaker, the high impedance at low frequencies blocks low-frequency current from flowing through the circuit. If you put it in parallel, connecting the capacitor across the terminals of the speaker and leaving it connected directly to the amplifier, it has no effect just by itself (besides giving your amplifier a hard time). But what if you add a series resistance or inductance? That will limit the overall current going into the circuit at high frequencies. Since the capacitor has low impedance at high frequencies, it "sucks" those currents from the source instead of allowing them to pass through the speaker. Just like a series inductor, with the right source impedance a parallel capacitor can be used to filter out high frequencies. And likewise, putting an inductor in parallel with a speaker will filter out low frequencies much like a series capacitor would.

The picture below is a comparison of the effect of a single series inductor, versus that of a series inductor followed by a capacitor in parallel with the speaker. The filter with the capacitor (called an LC filter) is a *2nd-order* filter, and it causes the frequency response to roll off twice as fast (look at the dB scale on the bottom graph).
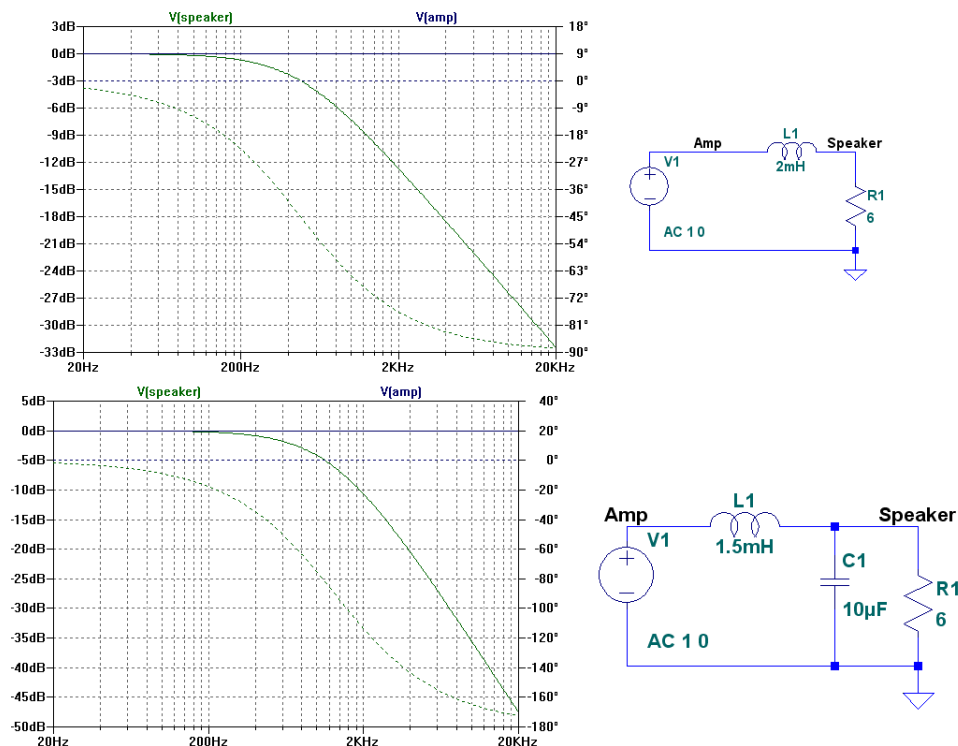
Let me throw out some numbers for example. The impedance of an inductor, for example, increases proportional to frequency. Let's say you have a speaker that's compensated to have a constant impedance of 6 ohms, and the inductor's value is $L = 1$ mH (0.001 H). At low frequencies that impedance is small compared to the impedance of the speaker, so it doesn't do anything. For example, at 100 Hz the inductor's impedance magnitude is $2\pi \times 100 \times 0.001$, which is about 0.6 Ω... not much effect. Then you get to the frequency where the two impedances are equal, and the power delivered to the speaker is reduced by half (-3 dB). That's around 1 KHz, because $2\pi \times 1000 \times 0.001$ is about 6 ohms, and the speaker is 6 Ω as well. Then the inductor starts playing a significant part; the frequency response curve tilts downward. That point is called called the "corner frequency" (F_c) of the low-pass filter:

$$F_c = \frac{R}{2\pi L}$$

Above that frequency, the impedance of the whole circuit is dominated by the inductor and rises proportional to frequency; then the output of the speaker decreases with increasing frequency. For a doubling of frequency, or one *octave*, the output falls by half, or 6 dB. So the 1 mH inductor with a 6 Ω speaker forms a 6 dB/octave low-pass filter with a corner frequency of about 1 KHz.

You can show in the same way that a single capacitor in series with a resistive speaker forms a 6 dB/octave high-pass filter: below a certain frequency given by $F_c = \frac{1}{2\pi RC}$, the output is just about proportional to the frequency. These simple crossovers are called *1st-order*, meaning the output in the undesired "stop band" is some linear, or 1st-order, function of frequency.

> Whoops, there's some more engineering jargon:
>
> - The *pass band* is the range of frequencies in which we want a driver to play.
> - The *stop band* is the range of frequencies we want to prevent it from playing.

If your drivers are perfect, this will work great: provided the F_c of the high and low sections are the same, the output from the two drivers will add together to a perfectly flat frequency response. The trouble is when the drivers are less than perfect; especially when they have problems like resonances or low power-handling outside the desired range. The 1st-order crossover has a wide overlap between drivers, placing noticeable stress on them at the edges of the pass band. If you're using a woofer with a 10 dB high breakup peak at 2 KHz, then the nasty breakup sound will still be noticeable even with a low 1KHz crossover frequency, because by 2 KHz the crossover will only reduce

the woofer's output by about 6 dB. The unwanted sound from the woofer will be 4 dB louder than the desired sound coming from the tweeter!

As you might guess, if we gave the name "1st-order" to that kind of crossover, then there must also be circuits that are 2nd-order, 3rd-order, 4th-order, and so on! Those correspond to rolloff rates of 12, 18, and 24 dB/octave, which are helpful for minimizing the problems with normal speaker drivers. Going back to that nasty woofer example, switching to a 4th-order or 24 dB/oct circuit would reduce the output at 2 KHz by 24 dB, so the bad sound from the woofer would be 14 dB quieter than the tweeter at that point (much less noticeable). These steeper rolloff rates are achieved by alternating between series inductors and parallel capacitors for a low-pass, or series capacitors and parallel inductors for a high-pass.

A 3rd-order lowpass uses two series inductors with a parallel capacitor between them, for example. You can use these combinations of series and parallel components to achieve steeper rolloffs and a more accurate crossover. In my own personal speakers, I'm using a crossover that causes rolloffs of about 36 dB/oct... 6th-order. The number in dB/oct is literally the "slope" of the graph of frequency response, once you're out of the pass band of the driver. Adding more capacitors and inductors increases that rolloff slope, making the crossover "steeper."

Here is a simulation of a 4th-order filter applied to that same woofer used in the graphs above. Notice how the frequency response is much flatter, and the breakup peaks in the treble have significantly diminished? With an aluminum cone woofer, it sounds better this way!
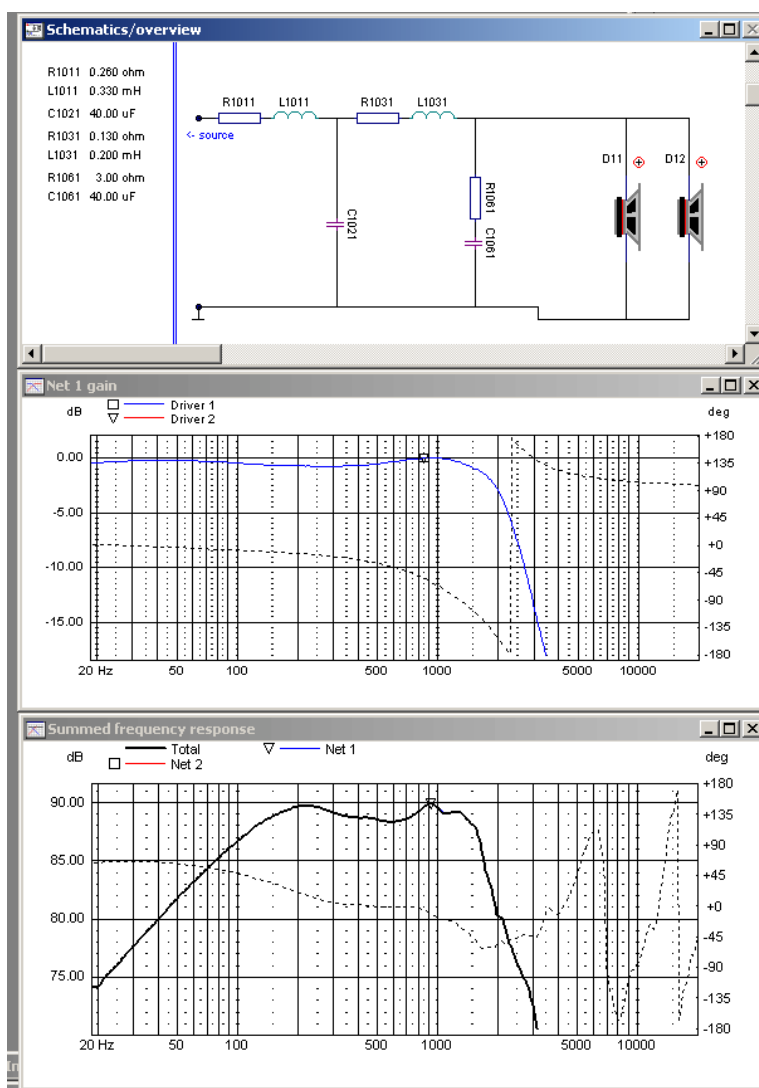
One more thing...

Why not just stack up large numbers of components to make ridiculously steep crossovers with no overlap between the drivers? Well, firstly, all of these components cost money; the crossover components for a speaker with steep filters can exceed the cost of the drivers themselves. Secondly, each additional component in the signal path can contribute a small amount of distortion. And finally, remember how each capacitor or inductor introduces its own phase angle? Complicated circuits can have rapidly shifting phase versus frequency, or worse, phase which is very sensitive to the exact value of each component. That makes it very hard to align the phase angle of the different drivers so they don't cancel each other out. Simpler crossovers are easier to design and more likely to work the way you want.

When you're designing a speaker, you need to look at the drivers you want to use, and ask yourself these questions:

- What should the crossover frequency (or frequencies) be?
- How much do I need to compensate for differences in driver sensitivity? (How much resistance do I need to put in series with the tweeter to get my preffered tone?)
- Just how steep do you need the crossover slopes to be? (In general, the stiffer metallic drivers are less forgiving, and need steeper crossovers.)

The filter slopes can even be different between the lowpass and highpass networks. If you understand impedances, you can make crossover circuits do almost anything to the sound of your speakers. Let's move on to rolling our own!
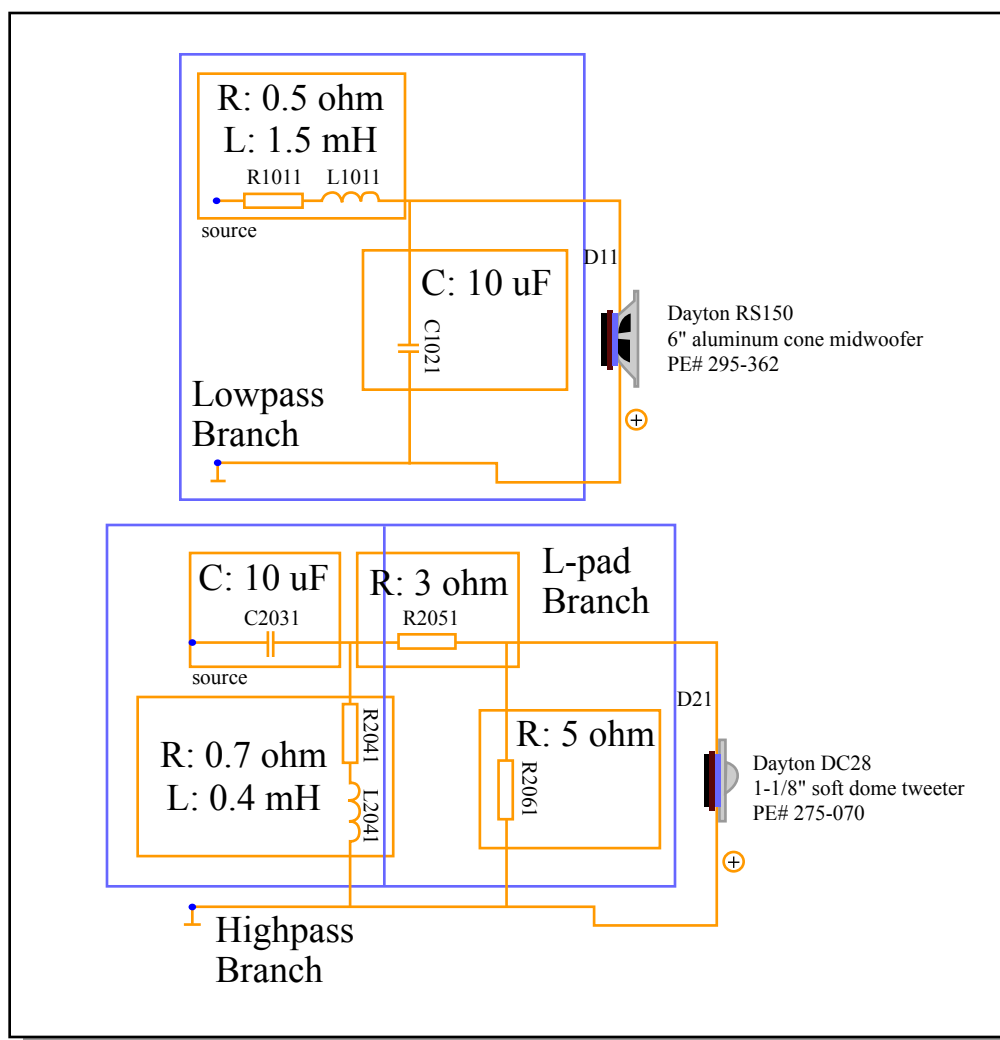

Image by MIT OCW.