

Manipulating numbers numerically

Take these two numbers (they are called a conjugate pair):

$$z_1 = 1 + i\sqrt{3}$$

$$z_2 = 1 - i\sqrt{3}$$

- Firstly, what are the magnitudes and phase angles?

$$|z_1| = \sqrt{1^2 + (\sqrt{3})^2} = 2$$

$$|z_2| = \sqrt{1^2 + (\sqrt{3})^2} = 2$$

$$\angle z_1 = \tan^{-1} \sqrt{3} = \frac{\pi}{3} = 60 \text{ deg}$$

$$\angle z_2 = \tan^{-1} -\sqrt{3} = -\frac{\pi}{3} = -60 \text{ deg}$$

- How about their sum?

$$z_1 + z_2 = (1 + 1) + i(\sqrt{3} - \sqrt{3}) = 2$$

- And the product?

Use either the individual components, or the magnitude and phase:

$$|z_1 z_2| = |z_1| |z_2|, \angle(z_1 z_2) = \angle z_1 + \angle z_2$$

The product has magnitude 4 ($|z_1 z_2|$) and phase angle 0 ($\angle z_1 + \angle z_2$). So $z_1 z_2 = 4$.

Algebraic expressions

- If A is a constant, find:

$$1 - \frac{1}{1 + iA} = \frac{1 + iA}{1 + iA} - \frac{1}{1 + iA} = \frac{iA}{1 + iA}$$

- If ω, R, L and C are constants, and:

$$z_1 = R + i\omega L$$

and

$$z_2 = \frac{1}{i\omega C}$$

Find the following:

$$\frac{z_1 z_2}{z_1 + z_2} = \frac{\frac{R + i\omega L}{i\omega C}}{R + i\omega L + \frac{1}{i\omega C}}$$

$$\frac{z_1 z_2}{z_1 + z_2} = \frac{R + i\omega L}{1 - \omega^2 LC + i\omega RC}$$