

Review the rules for complex numbers:

$$\text{Definition: } i = \sqrt{-1} \quad (1)$$

$$i^2 = -1 \quad (2)$$

$$\text{Addition: } (a + bi) + (c + di) = (a + c) + (b + d)i \quad (3)$$

$$\text{Multiplication: } (a + bi)(c + di) = (ac - bd) + (bc + ad)i \quad (4)$$

$$\text{Division: } \frac{a+bi}{c+di} = \frac{a+bi}{c+di} \frac{c-di}{c-di} \quad (5)$$

$$= \frac{(ac + bd) + (bc - ad)i}{c^2 + d^2} \quad (6)$$

$$\text{Magnitude } (z = a + bi) : |z|^2 = a^2 + b^2 \quad (7)$$

$$\text{Phase: } \angle z = \tan^{-1} \frac{b}{a} \quad (8)$$

Manipulating numbers numerically

Take these two numbers (they are called a conjugate pair):

$$z_1 = 1 + i\sqrt{3}$$

$$z_2 = 1 - i\sqrt{3}$$

- Firstly, what are the magnitudes and phase angles?

$$|z_1| = \quad |z_2| =$$

$$\angle z_1 = \quad \angle z_2 =$$

- How about their sum?

$$z_1 + z_2 =$$

- And the product?

Use either the individual components, or the magnitude and phase:

$$|z_1 z_2| = |z_1| |z_2|, \angle(z_1 z_2) = \angle z_1 + \angle z_2$$

$$z_1 z_2 =$$

Algebraic expressions

- If A is a constant, find:

$$1 - \frac{1}{1 + iA} =$$

- If ω, R, L and C are constants, and:

$$z_1 = R + i\omega L \quad \text{and} \quad z_2 = \frac{1}{i\omega C}$$

Find the following:

$$\frac{z_1 z_2}{z_1 + z_2} =$$