

MVT. MEAN-VALUE THEOREM

There are two forms in which the Mean-value Theorem can appear;¹ you should get familiar with both of them. Assuming for simplicity that $f(x)$ is differentiable on an interval whose endpoints are a and b , or a and x , the theorem says

$$\begin{aligned} (1) \quad & \frac{f(b) - f(a)}{b - a} = f'(c), \quad \text{for some } c \text{ between } a \text{ and } b; \\ (2) \quad & f(x) = f(a) + f'(c)(x - a), \quad \text{for some } c \text{ between } a \text{ and } x \end{aligned}$$

The first form (1) has an intuitive geometric interpretation in terms of the slope of a secant being equal to the slope of the graph at some point c . and in this form, it's easy to give an intuitive argument for the theorem.

The second form (2) looks less intuitive, but all that has been done is to multiply both sides of (1) by $b - a$, transpose a term, and change the name of b to x . Now it's not a theorem about slopes; instead, it says that the value of f at some point x can be estimated, provided you know the value of f at some fixed point a , and have information about the size of f' on the interval $[a, x]$. In other words, from information about f' , we can get information about f . (Such information can also be gotten by integration; one can think of the Mean-value Theorem as a poor-person's substitute for integration.)

The special case of (1) in which $f(a) = f(b) = 0$ is usually called **Rolle's theorem**; it says that if f is differentiable on $[a, b]$,

$$(3) \quad f(a) = f(b) = 0 \Rightarrow f'(c) = 0 \quad \text{for some } c, \text{ where } a < c < b.$$

Exercises: Section 2G

¹see Simmons, p. 76