

18.01 Exam 4

Problem 1. (15 points) Evaluate $\int \frac{dx}{x(x+1)^2}$

Problem 2. (15 points) Evaluate $\int (\ln x)x^2 dx$

Problem 3. (20 points) Use a trigonometric substitution to evaluate $\int_0^1 \frac{dx}{(4+x^3)^{3/3}}$
(Be careful evaluating the limits)

Problem 4. a. (10 points) Find an integral formula for the arc length of the curve $y = 2\sqrt{x+1}$ for $0 \leq x \leq 1$. Do not evaluate.

b. (10 points) Find an integral formula for the surface area of the curve in part (a) rotated around the x -axis. Simplify the integrand and evaluate the integral.

Problem 5. a. (7 points) Sketch the spiral $r = \theta^2$, $0 \leq \theta \leq 3\pi$. Say how many times the curve meets the x -axis counting $\theta = 0$ as the first times, and mark those points with X-s. (Your sketch need not be accurate to scale.)

b. (8 points) On your picture, shade in the region $0 \leq r \leq \theta^2$, $0 \leq \theta \leq 2\pi$, and find its area.

Problem 6. a. (10 points) Find the equation in polar coordinates for the line $y = x - 1$ in the form $r = f(\theta)$

b. (5 points) Find the range of θ for the portion of line $y = x - 1$ in the range $0 \leq x \leq \infty$. (It helps to draw a picture.)