

The Strategy of Risk

Risk

Risk is a complex board game produced by Hasbro that involves both luck and skill. The goal is simple: take over the world. Despite this simple goal, the game is very complicated and dynamic. Players attempt to take over the world by eliminating all other players. Players are eliminated when they lose all of their troops on the game board. Players must be skilled in troop deployment and must be aware of the underlying probabilities present in the game. This paper will discuss the game board, rules, probabilities, and general strategies of the game.

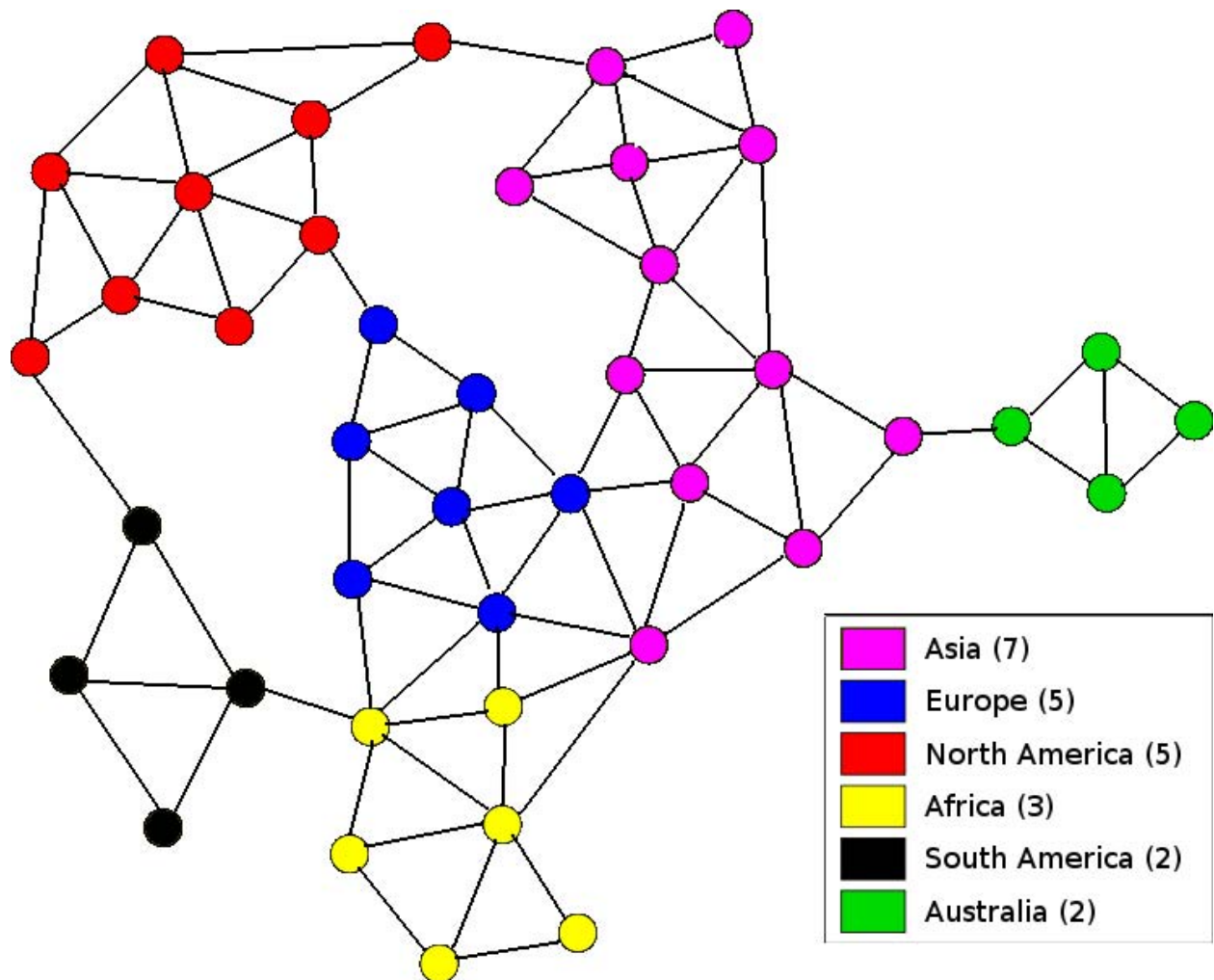
The Board

The game board is a map of the world divided into six continents comprised of 42 territories, shown below.



Image courtesy of caselee on flickr.

Players may only move between adjacent territories, with the exception of the territories connected by red lines over water. Additionally, Alaska connects to Kamchatka, which is the easternmost territory in Asia. The board can be simplified by turning it into a graph where the territories are the nodes and the lines between nodes are the potential paths that can be taken from territory to territory.



One key to victory is control over continents. Players that hold continents at the beginning of a turn get bonus reinforcements in an amount roughly proportional to the size of the continent (these bonuses will be detailed in the Rules section). Thus, the key positions on the board are the territories on the borders of continents. It is also important to know how to deal with the

game board, as path efficiency is a key to success. It makes no sense to leave troops in the center of an area controlled by a player, and choosing the quickest path from end to end of a continent is of the utmost importance.

Rules of the Game¹

Game Pieces

- 1 game board
- 6 colors of troops
- 42 territory cards + 2 wild cards
- 6 dice (3 white, 3 colored)

Game Set-Up

The game begins by distributing the territories among the players. There are two common ways to do this. The first method is to roll dice to determine the order of play. Once the order of play is determined, the first player chooses a territory to claim and places a troop on the claimed territory, followed by the player to his left and proceeding clockwise until all territories have been claimed. The second method is to take the 42 territory cards and divide them evenly among the players. Players then place one troop on each territory shown on the cards dealt to them. The second method is quicker than the first and is desirable for many players because it randomly distributes the territories and helps make each game more unique. Once all territories have been claimed, players proceed to place the remainder of their initial troop allotments one by one in a clockwise fashion. The initial troop allotment per player is

¹ Note: As set-up and play varies slightly for the 2 player game, these rules assume 3-6 players.

dependent on the number of players participating in the game and can be found in the game's rule book.

General Game Play

There are three phases of each player's turn: troop placement, attacking, and troop movement. At the beginning of each player's turn, he or she is given a certain number of troops to place on any territory controlled by that player. The number of troops given is equal to the number of territories controlled divided by 3 and rounded down to the next integer. Players are given a minimum of 3 troops per turn. Additional troop bonuses may be granted; these are detailed later in the Rules section.

The second phase of the turn is the attack phase. Players may choose to attack any territory adjacent to his or her own during the attack phase. The player must leave at least one troop back to occupy the territory. Thus, there must be more than one troop in the attacking country. Players attack by rolling up to three dice. Each dice represents one troop, so the attacker may not roll any more dice than the number of troops with which he or she is attacking. The defender rolls up to two dice, each representing one troop. Like the attacker, the defender may not roll more dice than troops they have to defend with. Once all dice have been cast, the highest roll of the attacker is matched up with the highest roll of the defender and the second highest roll of the attacker is matched up with the second highest roll of the defender (if applicable). The highest roll in each pairing wins, with ties going to the defender. The losing troop in each pairing is removed from the board and the attack phase continues. An attack ends in one of three ways: 1) The attacker decides to end the attack, 2) the attacker runs out of troops with which to attack, and 3) the defender loses all troops. In this third case, the

attacker takes over the territory and must move at least as many troops as dice rolled in the winning roll and at most the number of remaining troops in the attacking territory minus the one troop that must stay behind to occupy the territory. A player can attack as many territories as he or she wants during the attack phase of the turn. If the player captured at least one territory during the attack phase, he or she takes a territory card from the pile.

The third and final stage of the turn is the troop movement stage. Rules for this stage vary depending upon the established house rules, but the most common rule is that players may make a single move of as many troops as they want from one territory to an adjacent territory. Once the troop movement stage of the turn is complete, play passes to the left.

Bonuses

There are two kinds of bonuses in the game. The first bonus is for holding a continent. If at the beginning of a turn a player controls all territories on a continent, the player receives a bonus sum of troops equal to the number listed on the game board next to the name of that continent. The bonuses scale with continent size and are as follows: 2 troops for Australia or South America, 3 for Africa, 5 for North America or Europe, and 7 for Asia.

The second kind of bonus is derived from the territory cards received from conquering territories. Each territory card has a picture of a territory and one of three emblems, typically an infantryman, a cavalryman, and an artillery piece. There are also two bonus cards that have one of each emblem on the card. To get the bonus, players must have three matching emblems or one of each emblem at the beginning of the turn. There are two variations on troop dispensation. The first is to grant each set of cards an ascending number of troops. This is the most common method, as the ascending value sequence is printed on the board. The

second method is to assign a number of troops given for each emblem combination before the game. If a player holds 5 cards at the beginning of any turn, he or she is required to cash them in for the troop bonus.

Player Elimination

A player is eliminated when he or she no longer controls any territories on the game board. The eliminated player must then surrender all territory cards to the player that took his or her last territory. The game ends when only one player remains.

Probability

One of the dominant principles in the game is probability. Understanding the principles of probability that are implicit in the game is a key building block of success. There are six different combinations of numbers of dice that can be thrown by the attacker and defender.

The probabilities of the outcomes of the rolls are shown below:

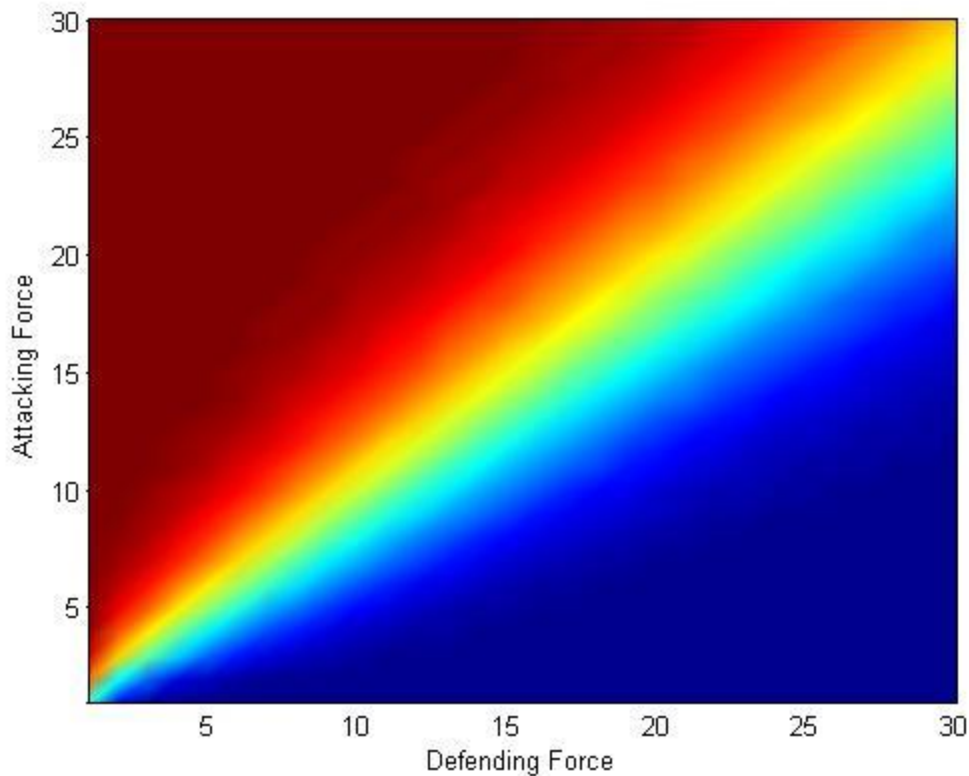
Defender Rolls One Die			
Number of Dice Rolled by Attacker:	1	2	3
Defender Loses One Troop	41.7%	57.9%	66.0%
Attacker Loses One Troop	58.3%	42.1%	34.0%

Defender Rolls Two Dice			
Number of Dice Rolled by Attacker:	1	2	3
Defender Loses Two Troops	25.5%*	22.8%	37.2%
Attacker Loses Two Troops	74.5%*	44.8%	29.2%
Both Lose One Troop	N/A	32.4%	33.6%

*- Only one troop lost

The probability problem is simple when the troop numbers are small, but the problem becomes much more difficult when the number of troops on each side becomes greater as the event tree expands and the number of possible outcomes and orders of outcomes increases exponentially. Thus, it is very difficult to explicitly predict the outcome of a scenario and it becomes necessary to implement numerical methods to quantify the probability.

One such method is the Monte Carlo simulation. A Monte Carlo simulation simulates each roll and calculates the outcome of the roll. In this case, the Monte Carlo method used factors the outcome of the roll into the troop strength and continues rolling until the troops of one side are completely depleted. Given enough simulation iterations for each starting troop strength combination, the percentage of victories for each side will approach the probabilistically expected outcome. The Monte Carlo simulation used here calculated the percentage of wins over a 100,000 battle sample set. One important thing to note is that in actual game play, the attacking player can choose to end an attack at any point during the attack, but the simulation did not break off the attack if the odds became unfavorable for the attacker. The code for the Monte Carlo simulation was implemented in MATLAB and is attached as an appendix to this paper. Once the simulation had been run for all attacking and defending troop strengths from one to thirty troops per side, the data was tabulated and turned into a "heat map" showing the probabilities graphically. This data is presented below, with dark red signifying a high probability of attacker victory and dark blue signifying a low probability of attacker victory. The green region of the graph is the area where the probability of victory is a virtual coin toss.



As is expected, a large number of troops attacking a much smaller defending force results in victory almost all the time, and in many cases has a statistically insignificant probability of losing. The converse is also true in that a small attacking force fighting a large defending force has a statistically insignificant probability of success. The most interesting region of the graph is the area where the armies are of relatively equal strength, as it is these battles that will swing the momentum and perhaps even the outcome of the game.

Strategy

There are several points of offensive and defensive strategy that arise from the dice probabilities and from the game board:

Take Small Continents First

Australia and South America are the two smallest continents, each with four territories. Both are relatively easy to defend, as Australia only has one entry point and South America has two. South America and Australia are of critical importance at the beginning of the game because the territories are spread out evenly and the two-troop bonus for holding either of these continents is a big boost to troop strength at the beginning of the game. Additionally, both of these continents are valuable in the later stages of the game as they have entry points into North America, Africa, and Asia which can be used to attack other players and keep them from holding those continents.

Attack Conservatively

While it is clearly advantageous to control lots of territories, taking over too many territories too quickly can leave a player's forces very spread out and vulnerable to attack. Many players fall into the trap of trying to take over a large continent like Asia or North America in just one or two turns because the interiors of the continents are typically sparsely defended. What these players fail to remember is that holding a continent does no good unless it can be held through the beginning of his or her next turn. Even with a large starting force, the attacker is bound to lose troops along the way and is forced to leave at least one troop in each territory, so the force left to defend the borders of the continent is much smaller and more vulnerable than it was at the beginning of the attack phase.

Play Defense

Playing sound defense, while not exciting, is the key to winning the game. By the middle of the game, troops tend to end up concentrated on the borders of continents. Players must

act accordingly if they wish to hold continents and maintain an advantage in the game. It is important to build up forces that can withstand attacks from several smaller armies from bordering countries and that can act as a deterrent for large attacking forces. This is especially critical in many of the countries in the middle of the board because they have many entry points from outside their continents. One example is the Middle East, which can be attacked from Ukraine, Southern Europe, Egypt, and East Africa. Without a strong presence in the Middle East, it is nearly impossible to hold Asia.

Final Words

Perhaps the best strategy for the game is to play a few times and develop a unique strategy. The game is different each time you play and each person you play will have different tendencies. Being able to account for the dynamic nature of the game and to adjust playing style to fit each game are the biggest keys to success in Risk.

Appendix – Monte Carlo Code

```
function [AttackWinPCT]=SP268a(A,D)
close all

simnum=100000;

n=1;
Wins=[0,0];
while n<=simnum
AttackTroop = zeros(30,1);
DefendTroop = zeros(30,1);
AttackTroop(1) = A;
DefendTroop(1) = D;
i=2;

while AttackTroop(i-1)>0 && DefendTroop(i-1)>0

    if AttackTroop(i-1)<=3
        xa=AttackTroop(i-1);
    else
        xa=3;
    end

    if DefendTroop(i-1)<=2
        xd=DefendTroop(i-1);
    else
        xd=2;
    end

    Dice = min(xa,xd);
    a = 6*rand(1,xa);
    AttackDice=sort(ceil(a),'descend');

    d = 6*rand(1,xd);
    DefendDice=sort(ceil(d),'descend');

    ADice(1)=AttackDice(1);
    DDice(1)=DefendDice(1);

    if ADice(1)>DDice(1)
        DefendT=DefendTroop(i-1)-1;
        AttackT=AttackTroop(i-1);
    else
        AttackT=AttackTroop(i-1)-1;
        DefendT=DefendTroop(i-1);
    end

    if Dice==2
        ADice(2)=AttackDice(2);
        DDice(2)=DefendDice(2);

        if ADice(2)>DDice(2)
            DefendT=DefendT-1;
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        else
            AttackT=AttackT-1;
        end
    end
end
AttackTroop(i)=AttackT;
DefendTroop(i)=DefendT;

i=i+1;
end

if DefendT==0
    Wins(1)=Wins(1)+1;
else
    Wins(2)=Wins(2)+1;

end
n=n+1;
end

AttackWinPCT=round(1000*Wins(1)/simnum)/10;
```

Sources

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<http://graffletopia.com/stencils/455>

[http://en.wikipedia.org/wiki/Risk_\(game\)](http://en.wikipedia.org/wiki/Risk_(game))

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