Complexity of Games & Puzzles
[Demaine, Hearn & many others]

	0 players (simulation)	1 player (puzzle)	2 players (game)	team, imperfect info
unbounded	PSPACE	PSPACE	EXPTIME	Undecidable
bounded	P	NP	PSPACE	NEXPTIME

Courtesy of BigJ_Smack

Courtesy of Sam Cancilla. Used with permission.

Image courtesy of Herman Hiddema

Image courtesy of Nguyen Dai.

Rengo Kriegspiel?

Courtesy of Glenn Peters. Used with permission.

Image courtesy of Jason Whittaker.

Images by MIT OpenCourseWare.

Image courtesy of PartsnPieces.

Image is in the public domain.
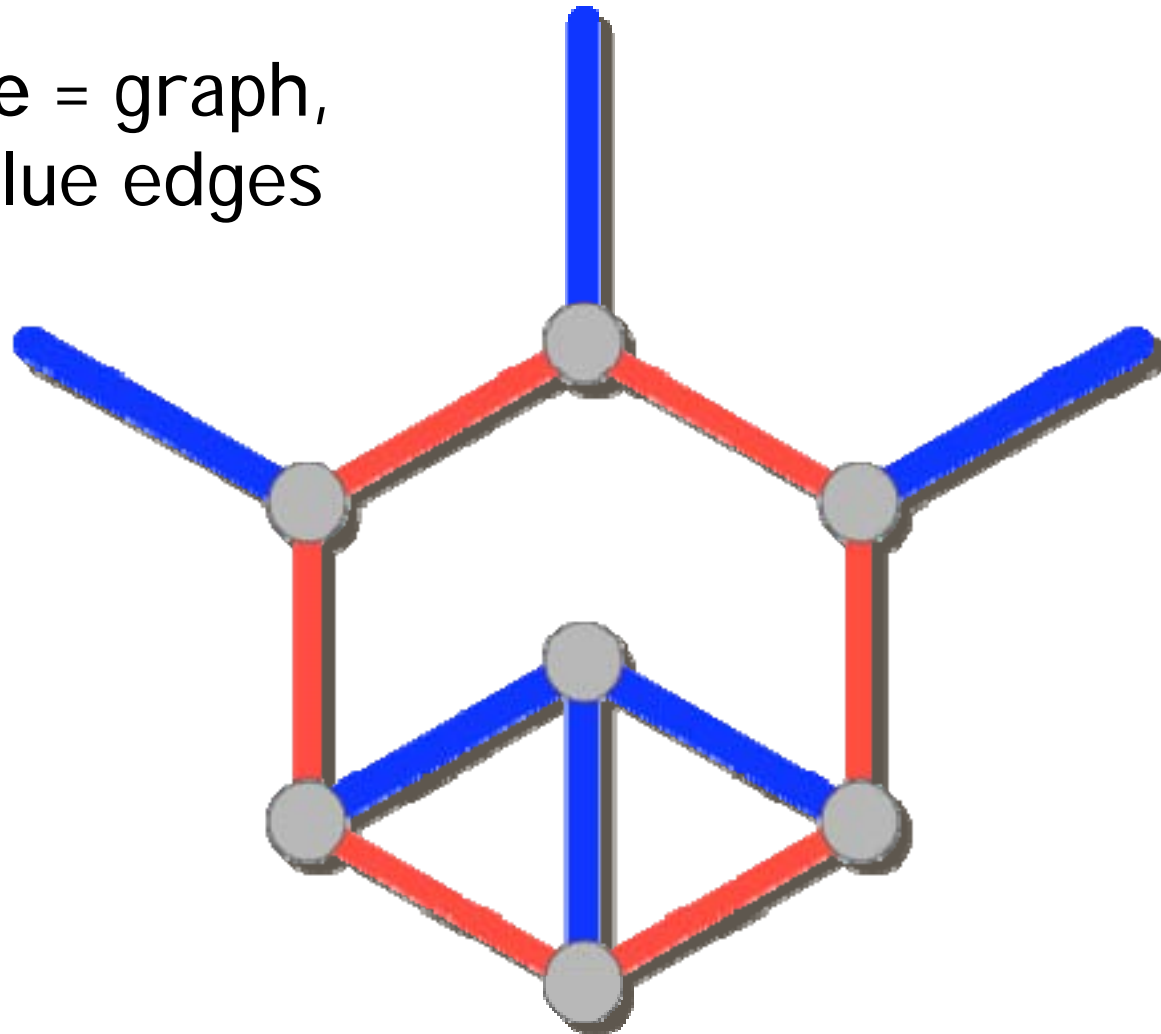
bridge?

Image courtesy of Marie-Lan Nguyen.

Constraint Logic
[Hearn & Demaine 2009]

Games, Puzzles, & Computation
Robert A. Hearn
Erik D. Demaine

	0 players (simulation)	1 player (puzzle)	2 players (game)	team, imperfect info
unbounded	PSPACE	PSPACE	EXPTIME	Undecidable
bounded	P	NP	PSPACE	NEXPTIME

false   true
variable
slow win
fast win
slower win

(a) AND   (b) FANOUT
(c) OR
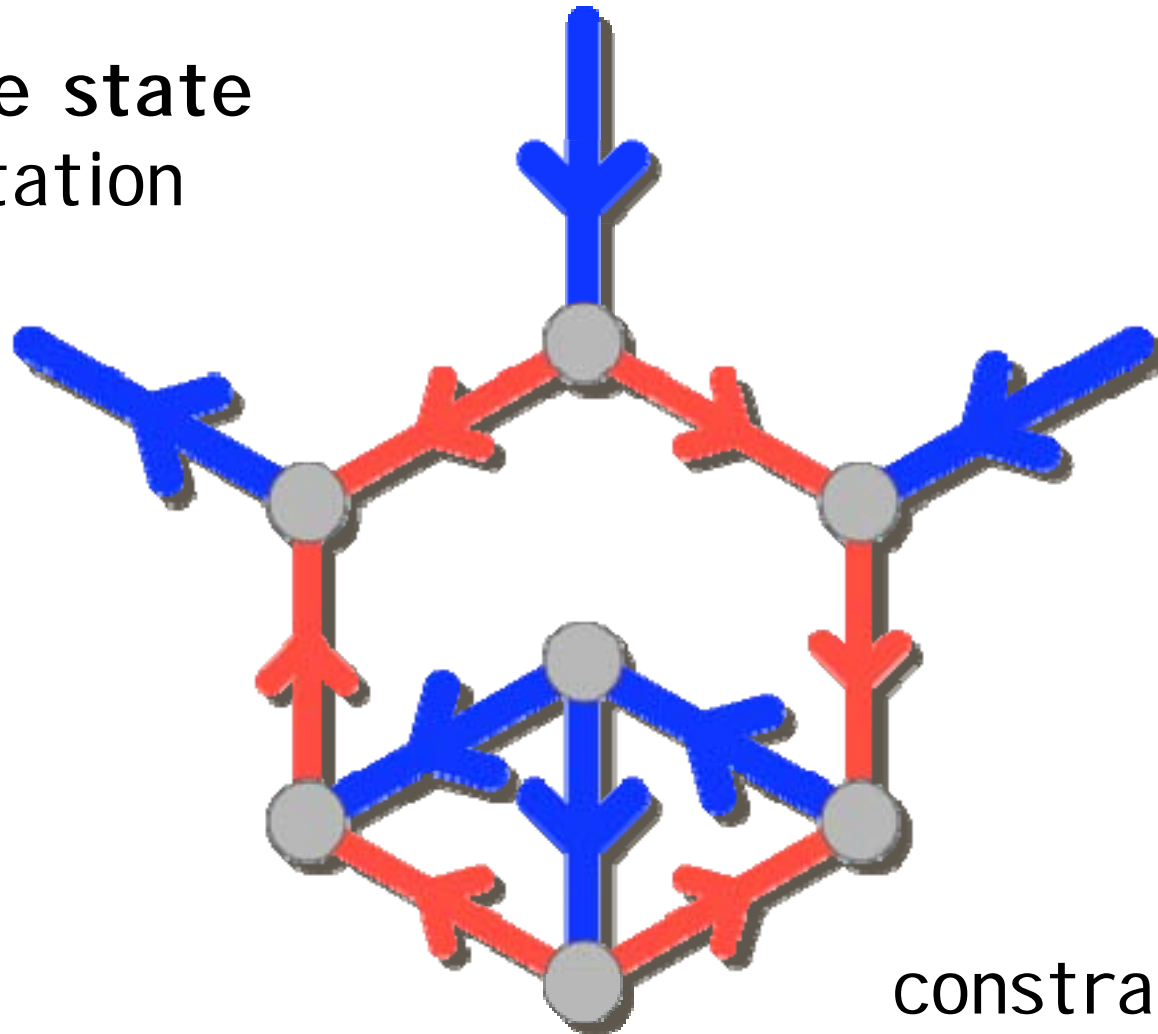
White win
Black win

Constraint Graphs

Machine = graph,
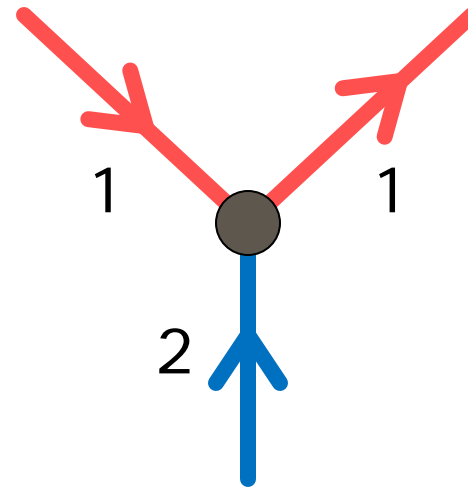red & blue edges

Constraint Graphs

Machine state
= orientation

constraint graph

Constraint Logic

= 1

= 2

1   1

2
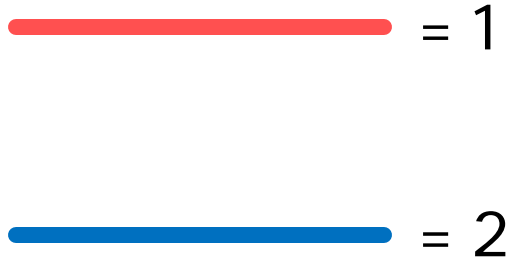
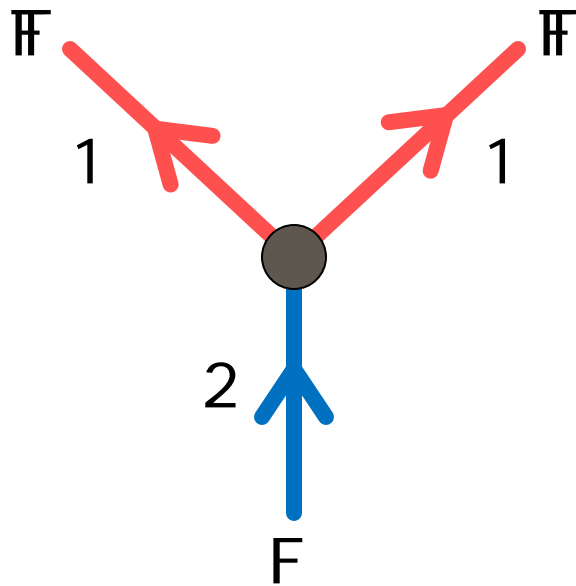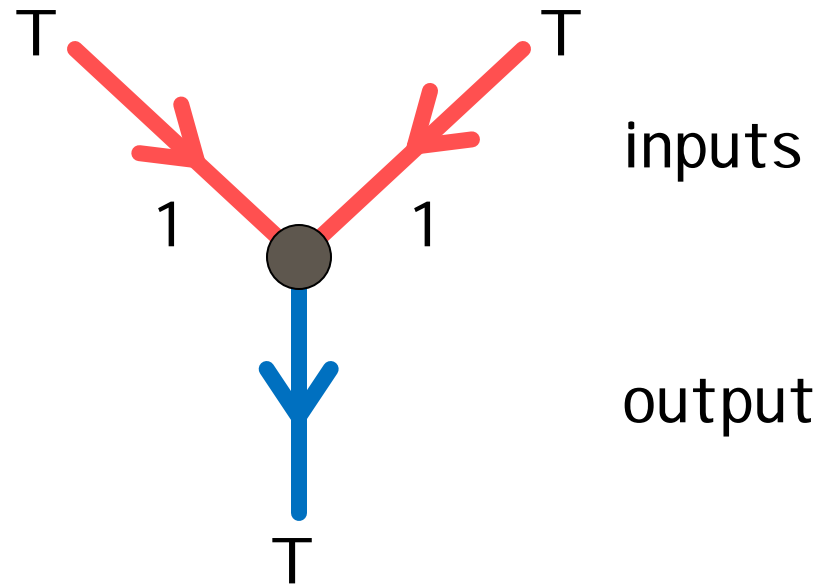**Rule**: at least 2 units
incoming at a vertex

Move: reverse an edge, preserving Rule

AND vertex

F F   T   T
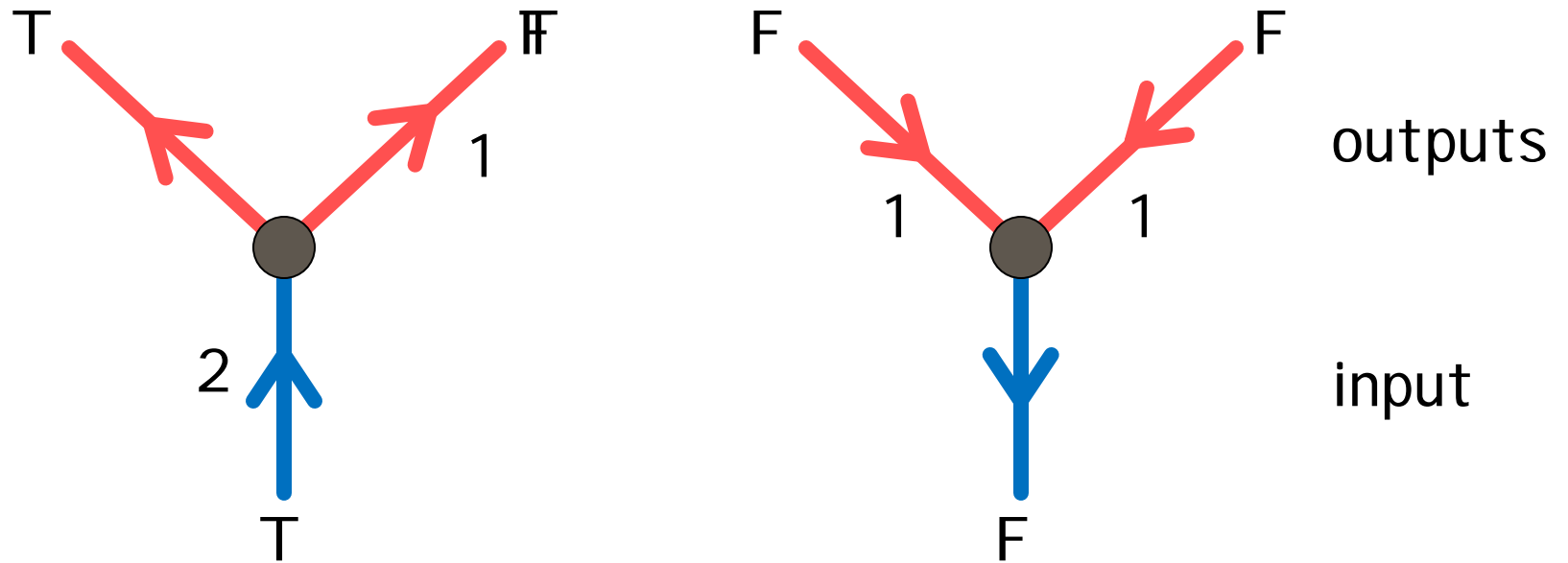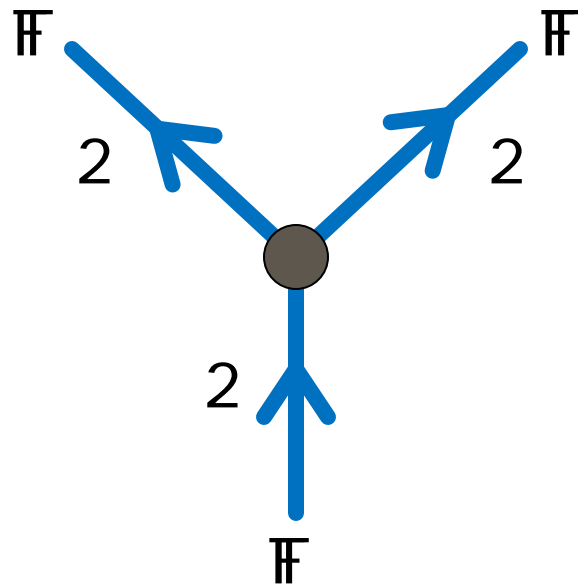
1   1   1   1
                                                              inputs

2   output

F   T

not your usual   **Rule**: at least 2 units
AND gate!   incoming at a vertex

SPLIT vertex

outputs

input

Rule: at least 2 units incoming at a vertex

OR vertex

not your usual
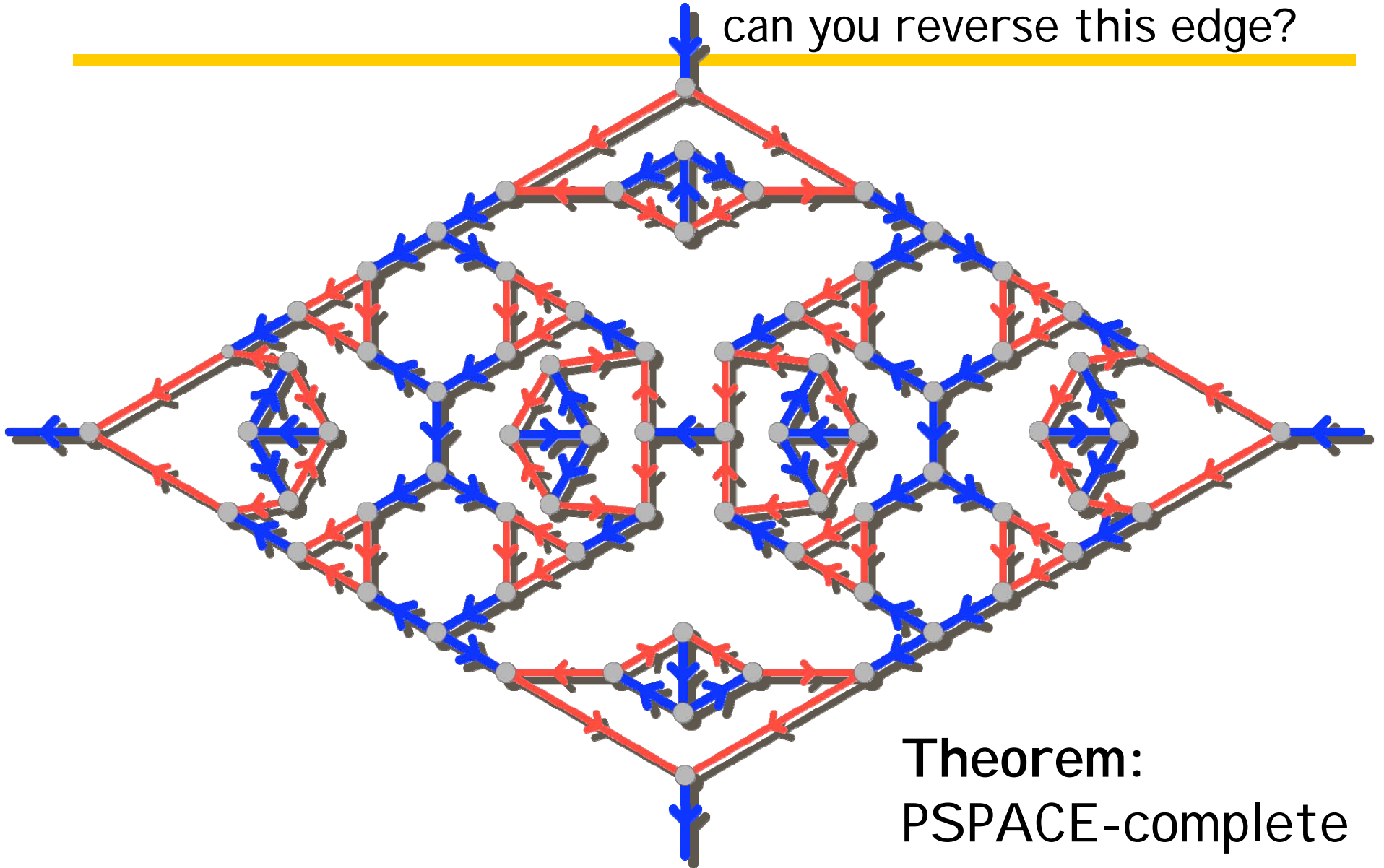OR gate!
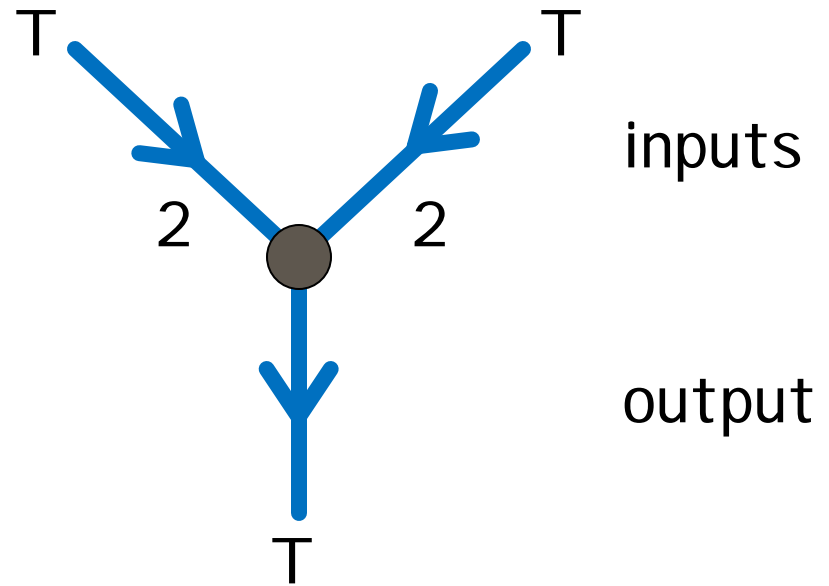
Rule: at least 2 units
incoming at a vertex
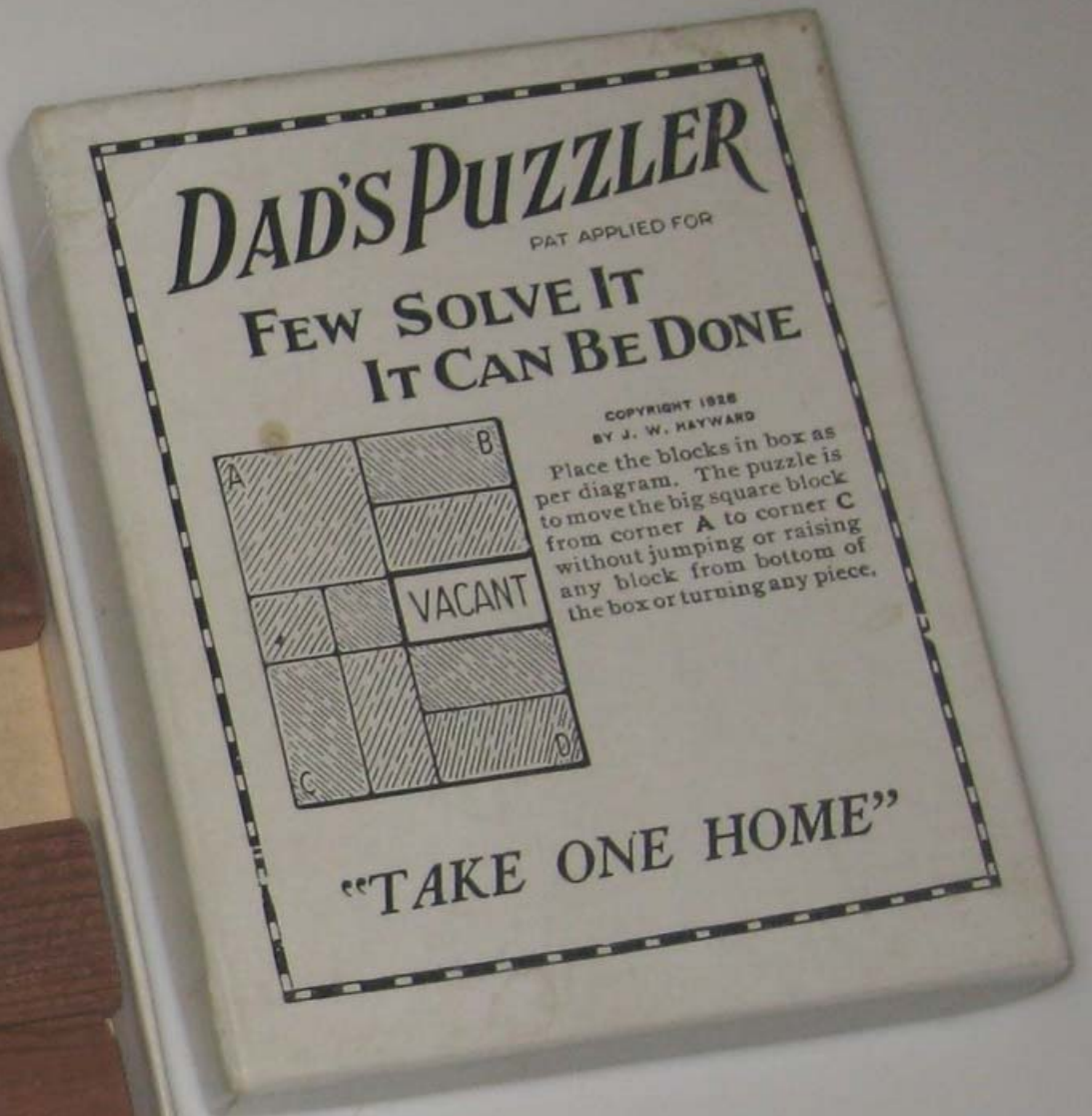
inputs

output

Decision Problem

can you reverse this edge?

Constraint Logic
[Hearn & Demaine 2009]

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	0 players (simulation)	1 player (puzzle)	2 players (game)	team, imperfect info
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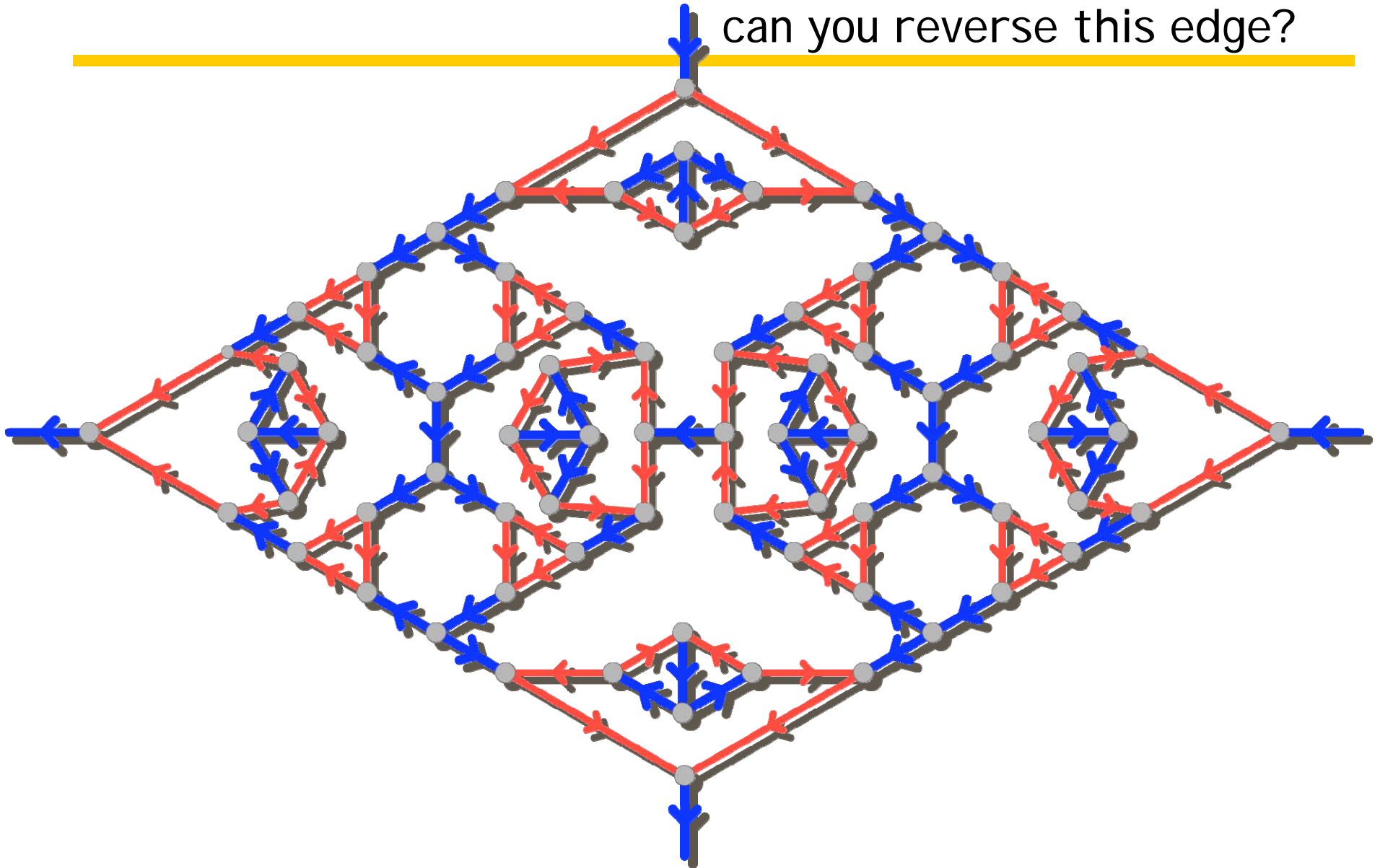
Decision Problem

can you reverse this edge?

Theorem:
PSPACE-complete

Sliding-Block Puzzles

Sliding-Block Puzzles
[Hearn & Demaine 2002]

Corollary:
PSPACE-complete
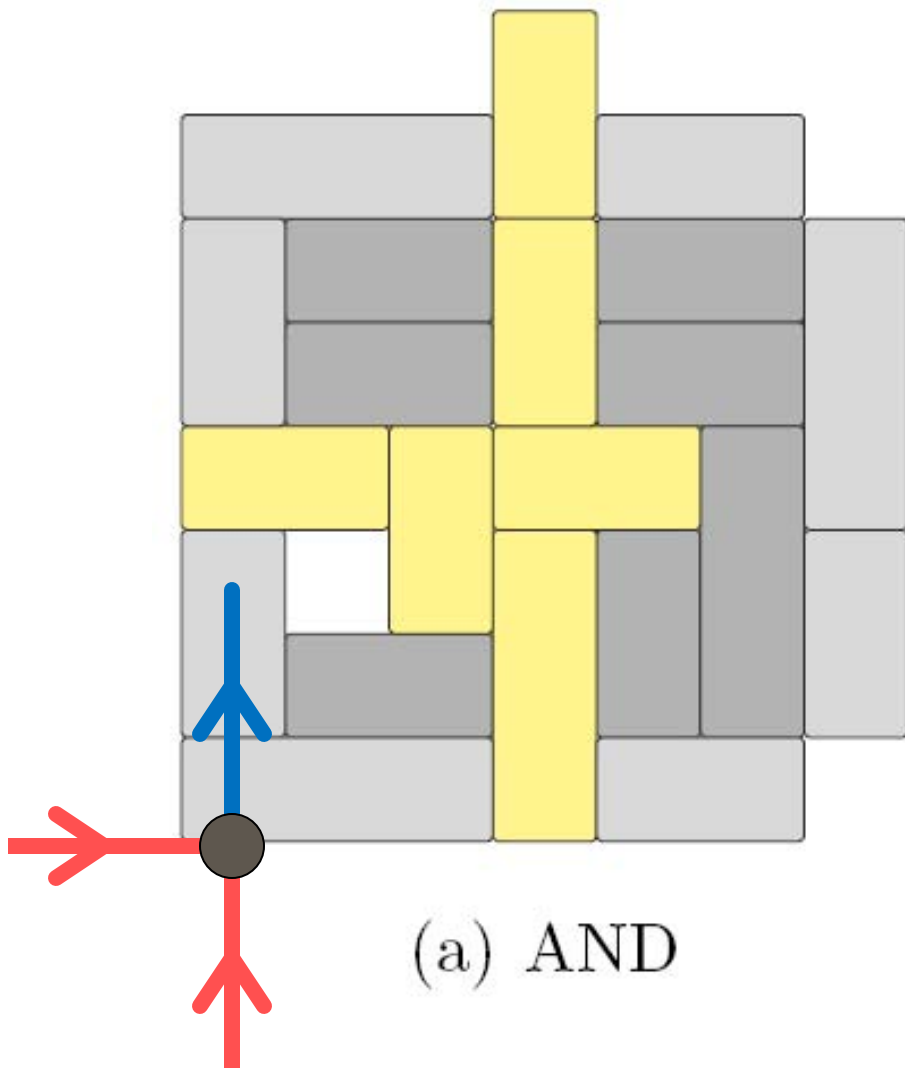
(a) AND
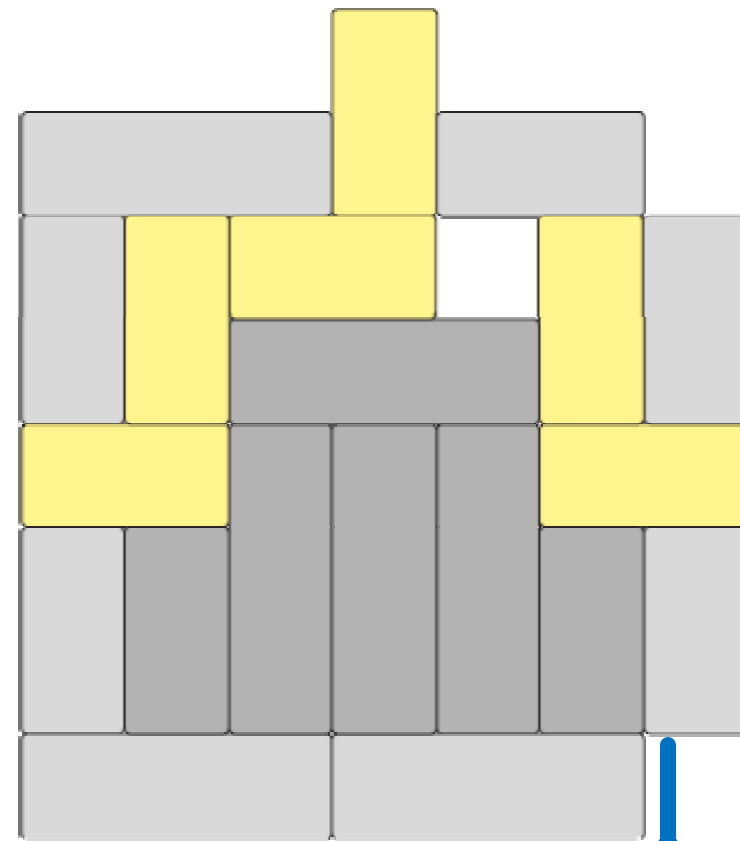
(b) OR

Sliding-Block Puzzles
[Hearn & Demaine 2002]

Corollary:
PSPACE-complete

(a) AND

(b) Protected OR
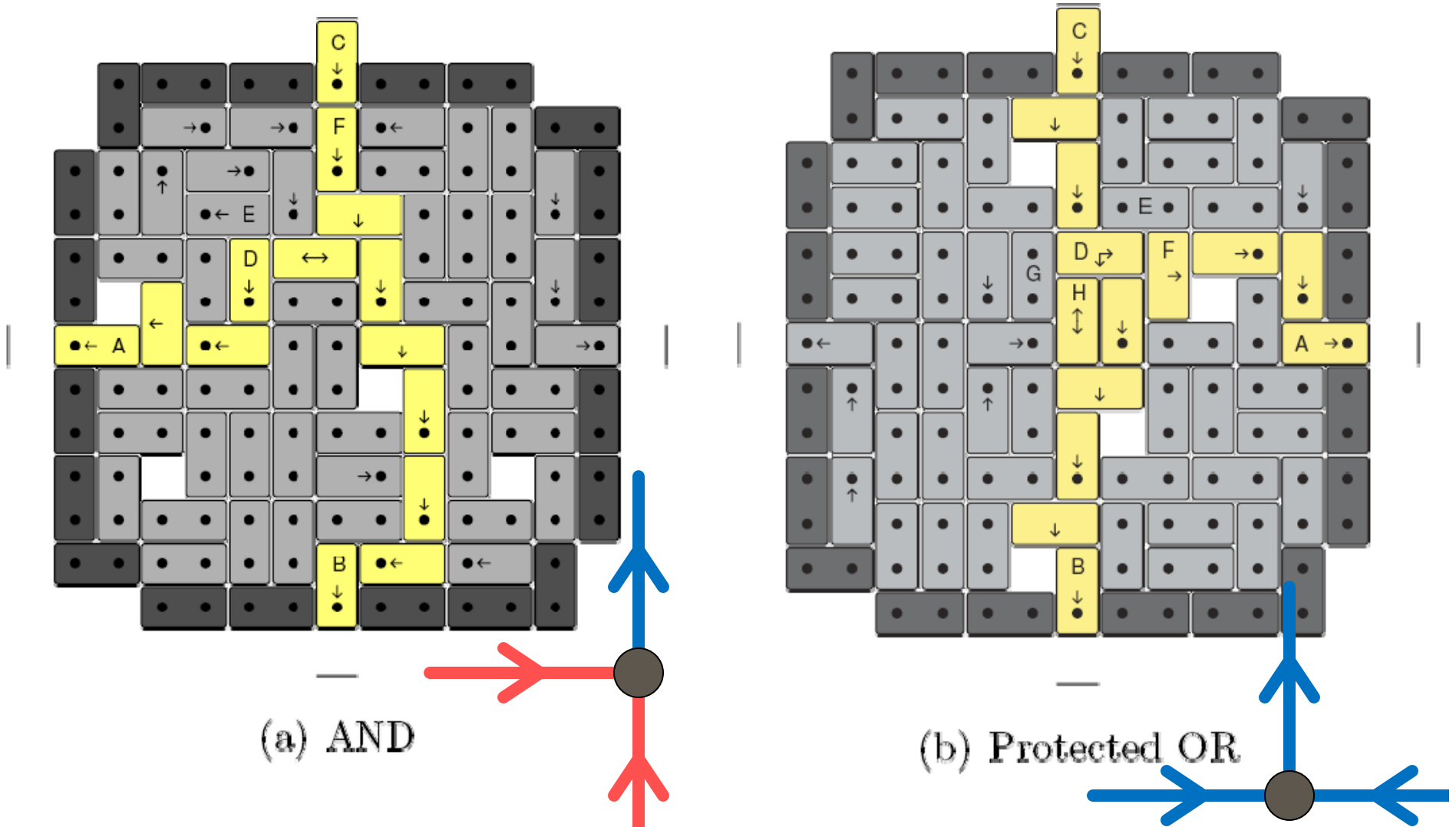
Wiring Vertices Together

AND

1

AND wants red
OR wants blue

2

OR
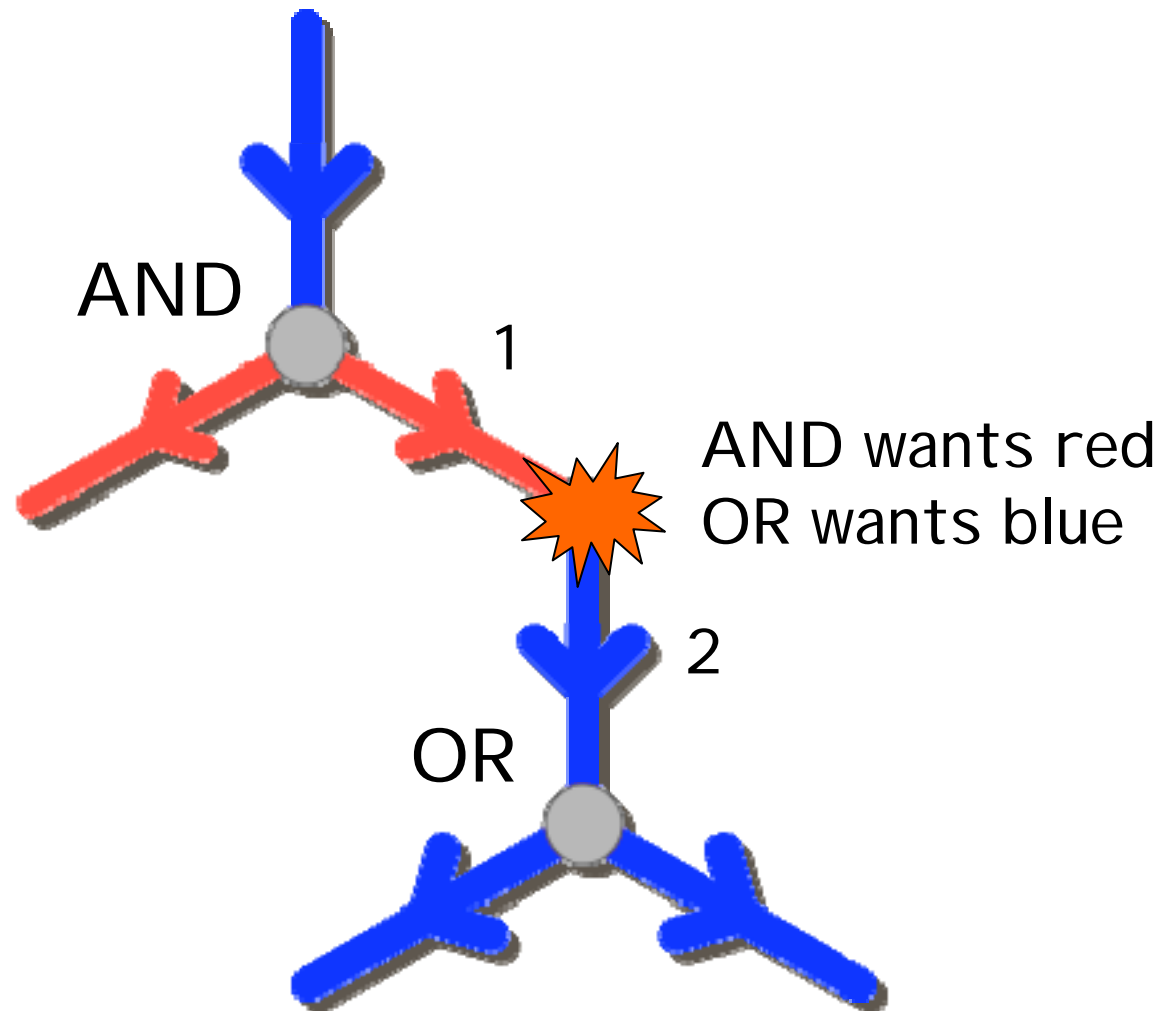
Red-Blue Conversion

assume an even number of conversions

Red-Blue Conversion
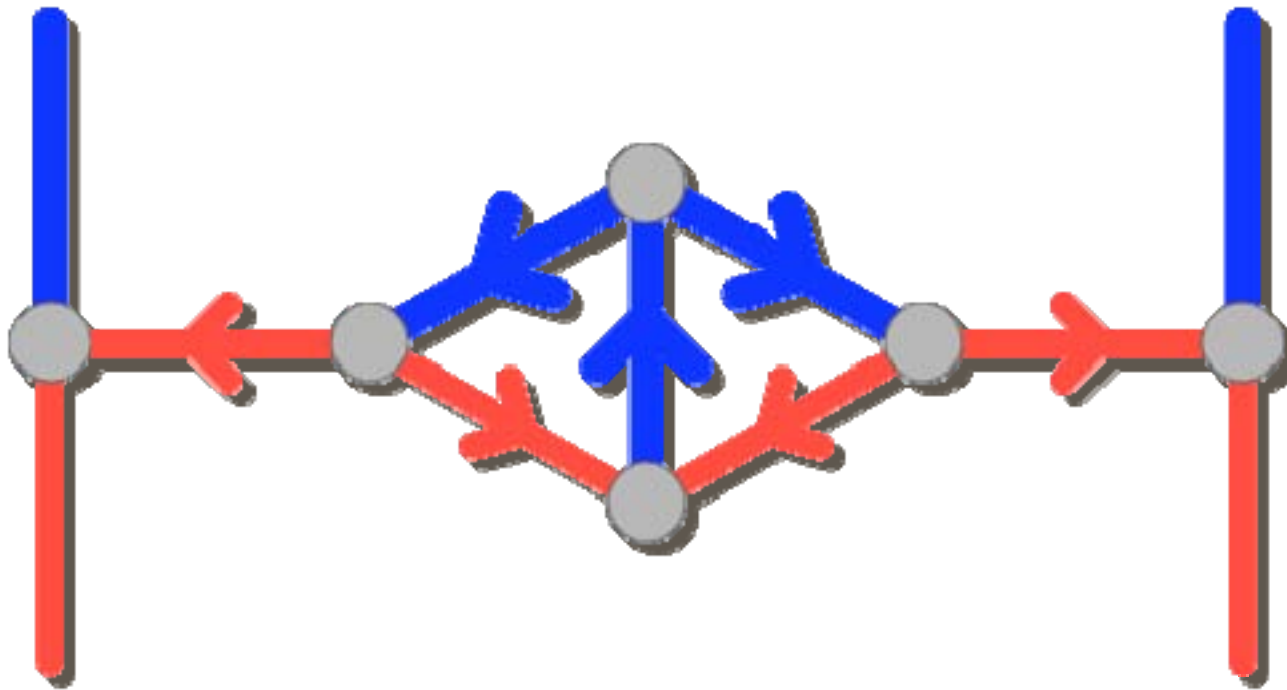
assume an even number of conversions

Boolean Formulas

$$(w \lor x \lor y) \land (w \lor \overline{x} \lor z) \land (x \lor \overline{y} \lor z)$$

$$(x \lor \overline{y} \lor z)$$

$$(w \lor x \lor y)$$

$$(w \lor \overline{x} \lor z)$$

w $\overline{w}$ x $\overline{x}$ y $\overline{y}$ z $\overline{z}$
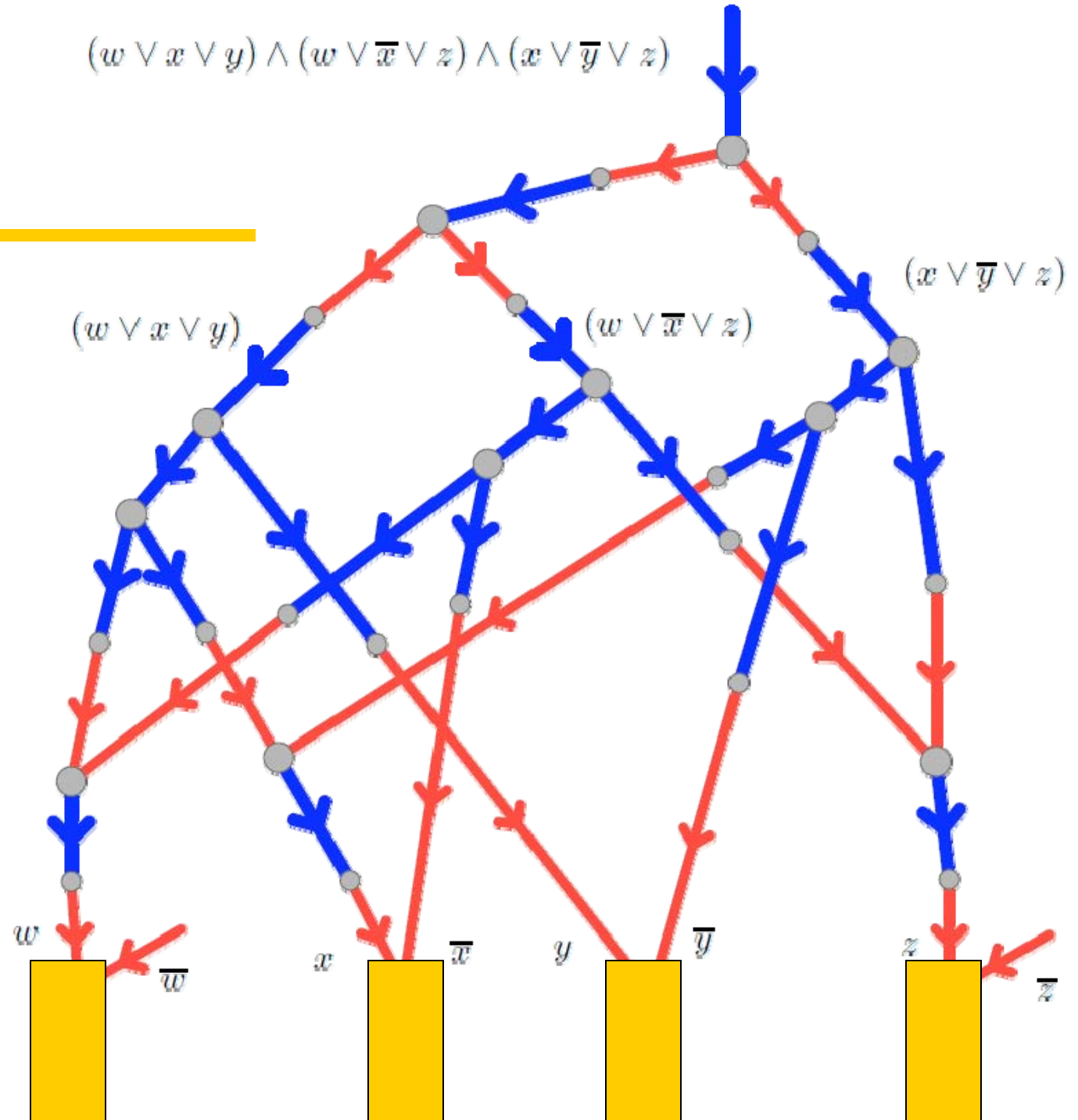
Quantified Boolean Formulas (QBF)

$$\forall x \exists y \forall w \cdots \exists z \left[(x \vee y) \wedge \cdots \wedge (\bar{z} \vee x \vee \overline{w}) \right]$$
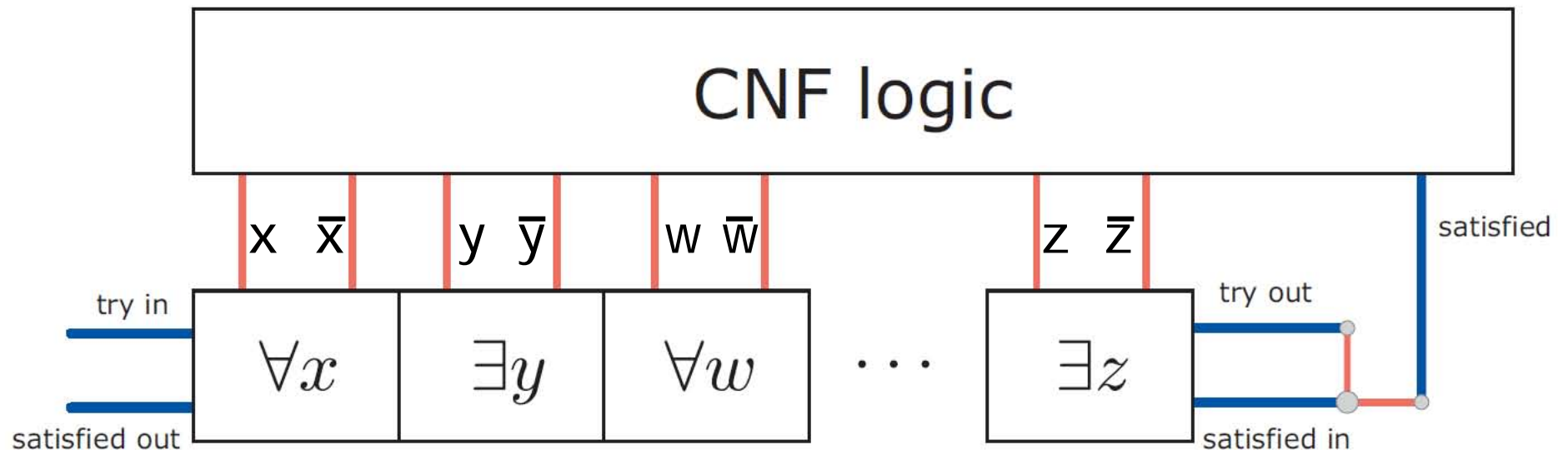
Existential Quantifier
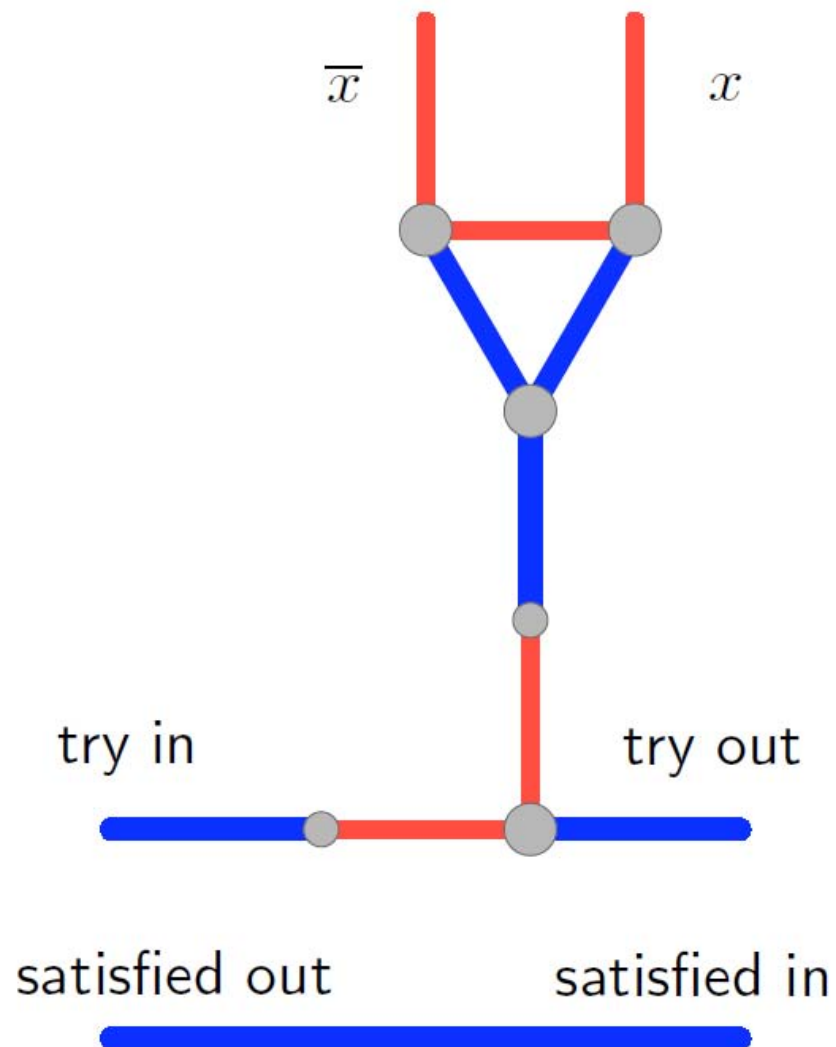
Universal Quantifier
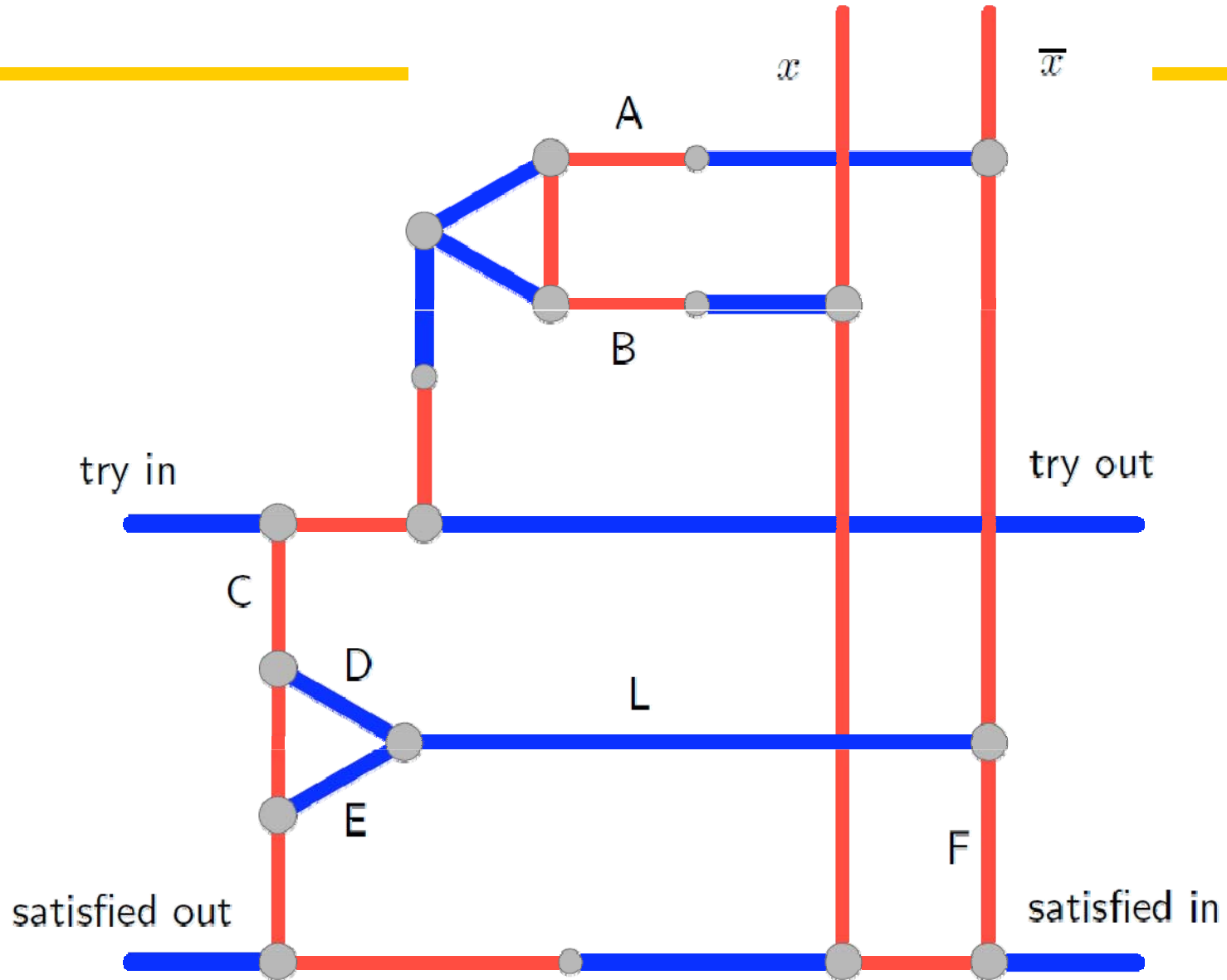
Latch
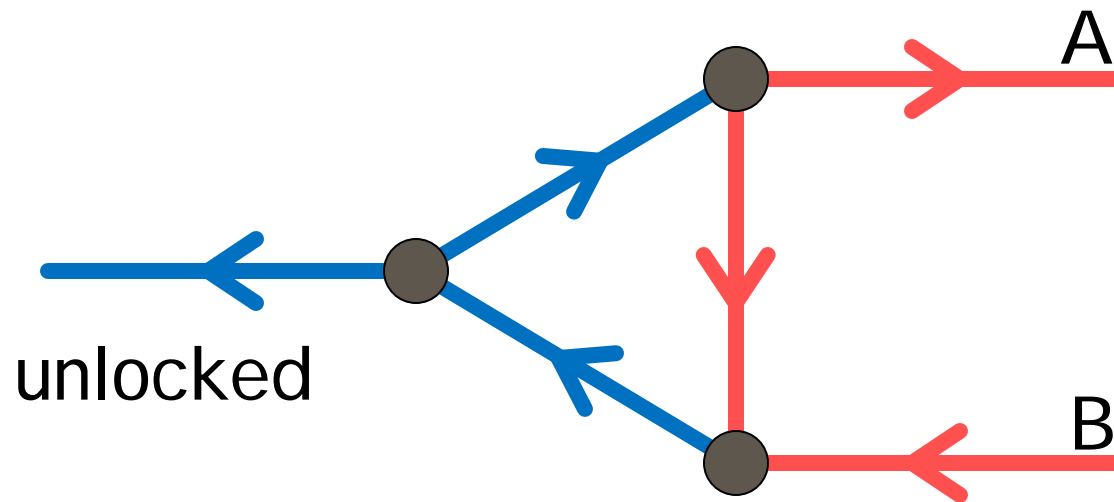
Universal Quantifier

Crossover Gadget
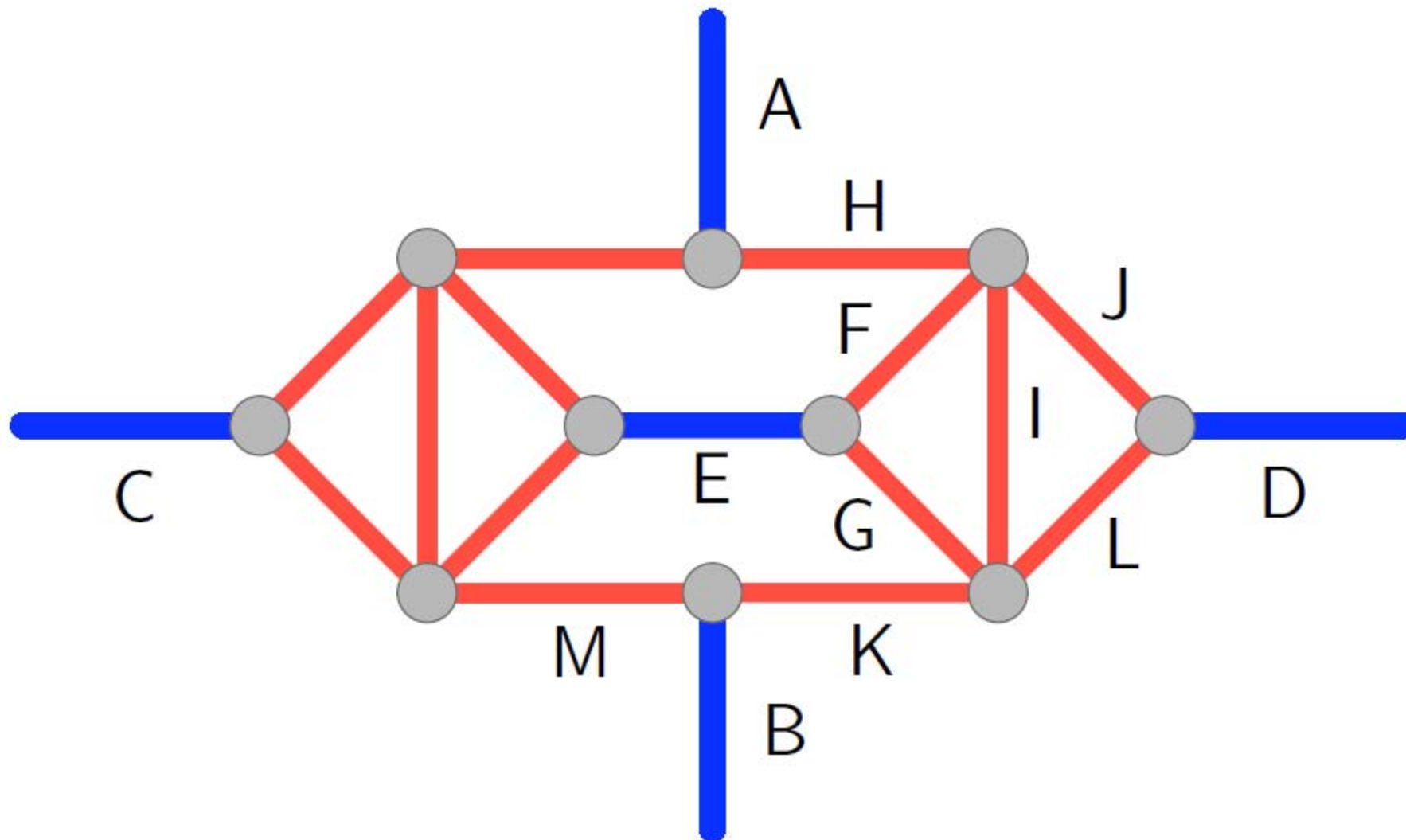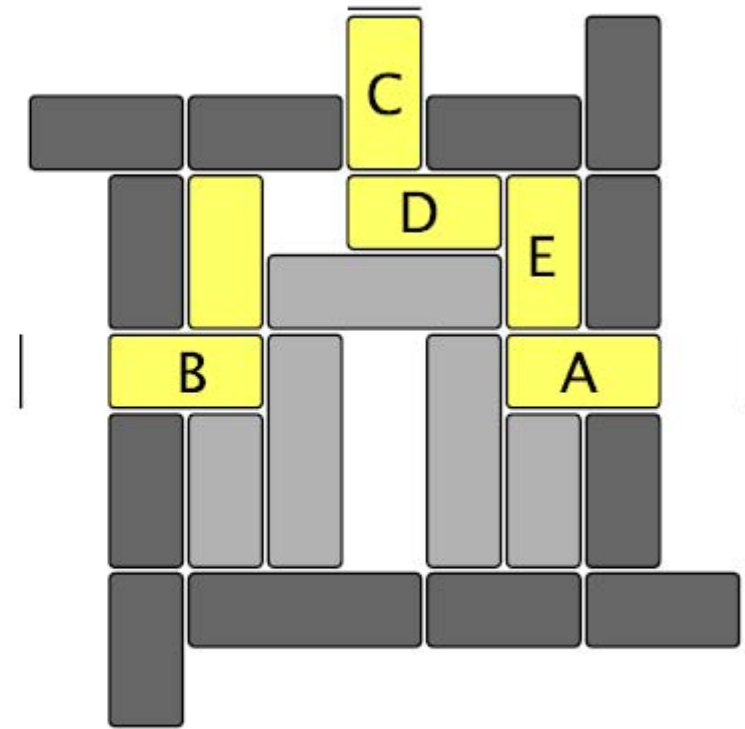
OR from Protector OR
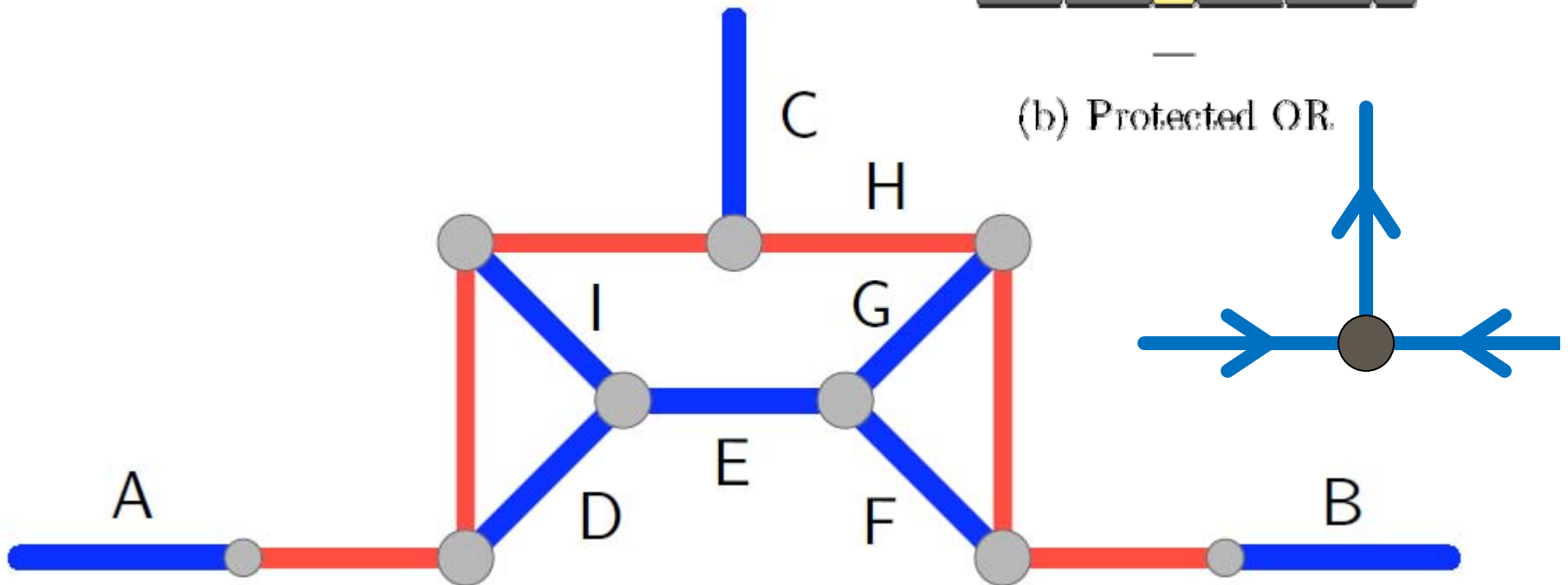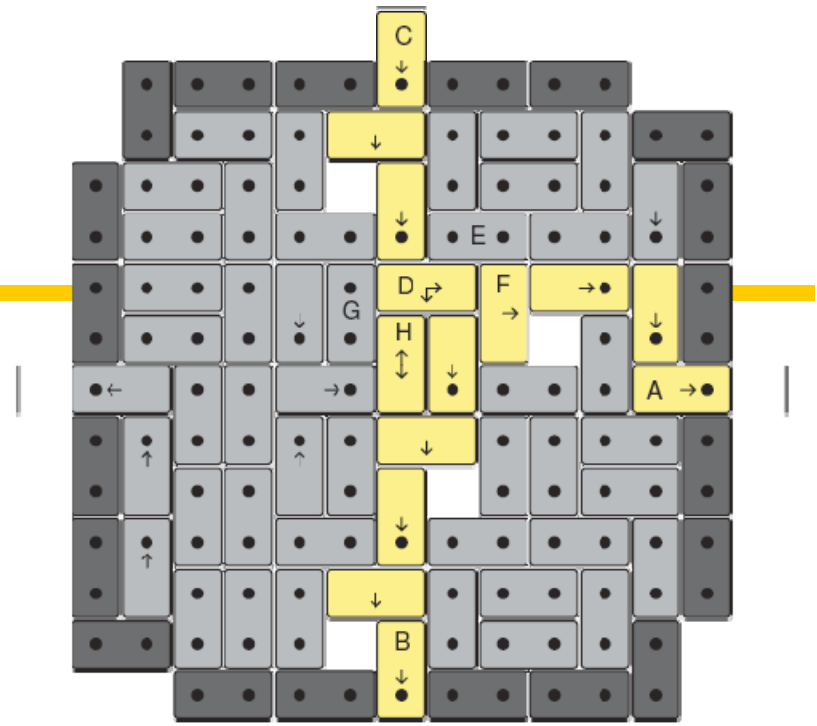
(b) Protected OR
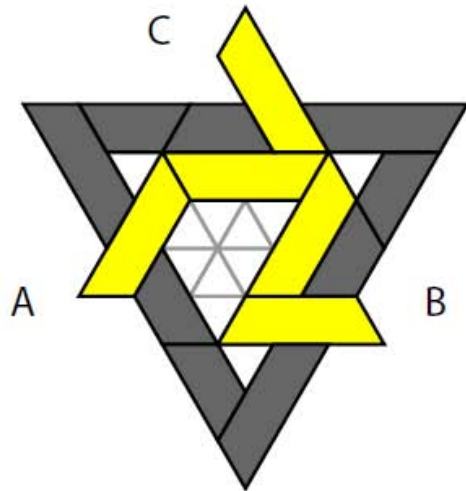
Rush Hour
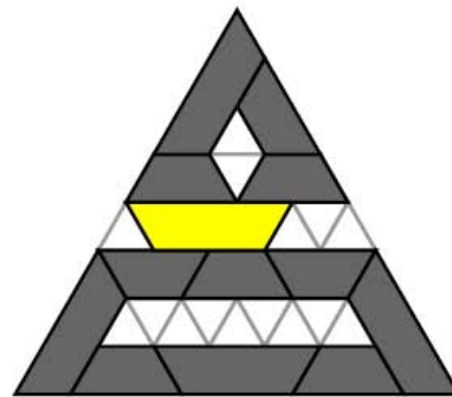## [Hearn & Demaine 2002]
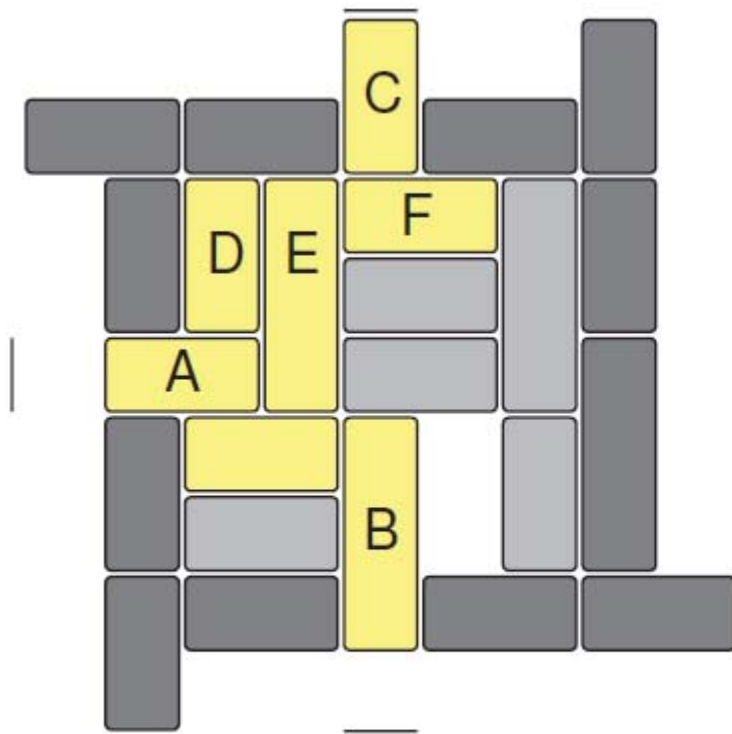
(b) AND

(c) Protected OR

PSPACE-completeness known [Flake & Baum 2002]
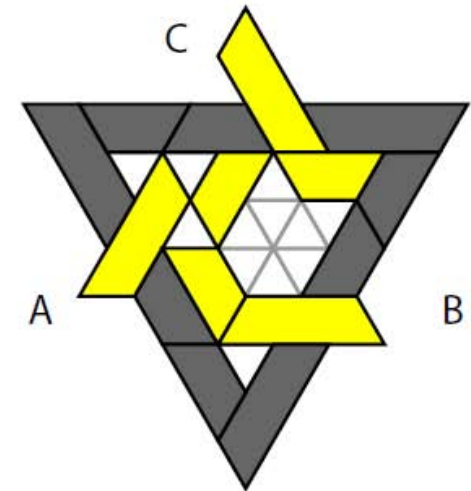
Triangular Rush Hour

[Hearn & Demaine 2009]
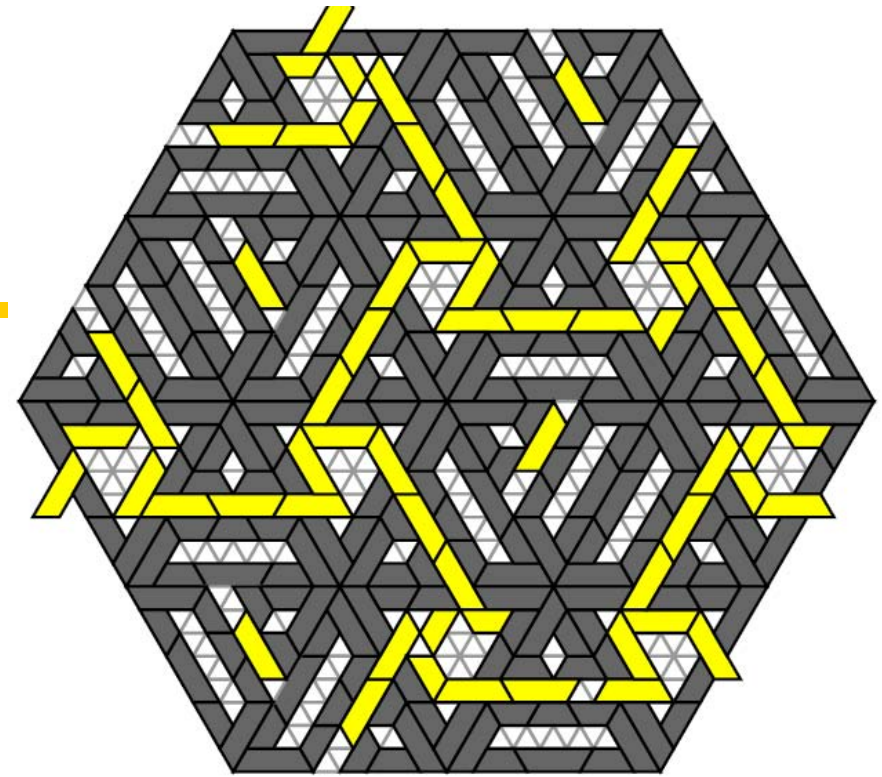
(a) AND vertex   (b) Connector   (c) OR vertex

Open: 1×1 Rush Hour
[Tromp & Cilibrasi 2008]
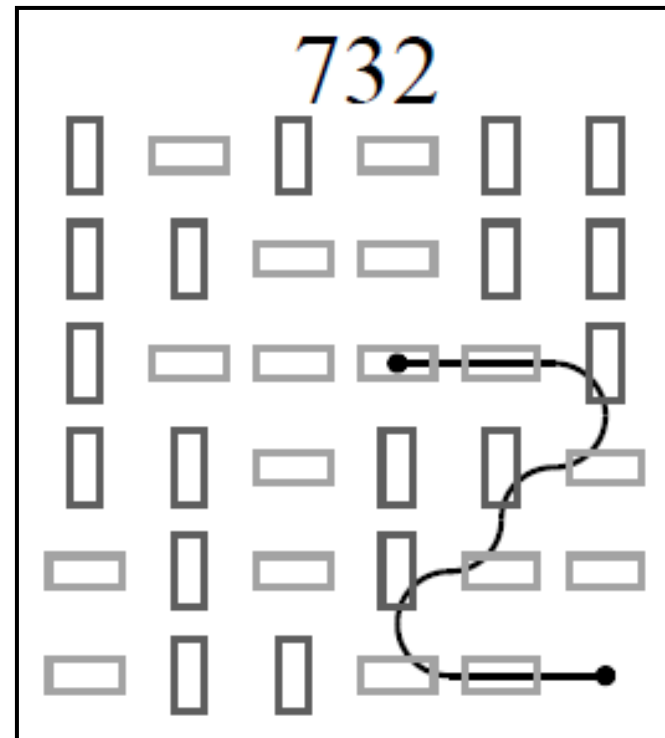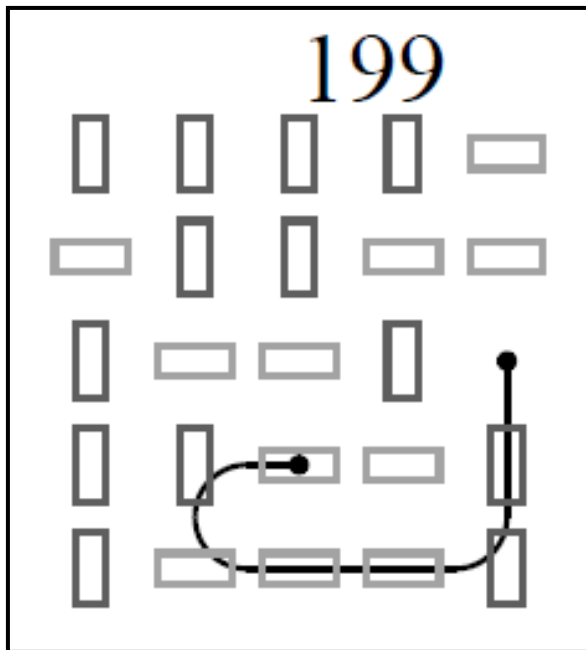
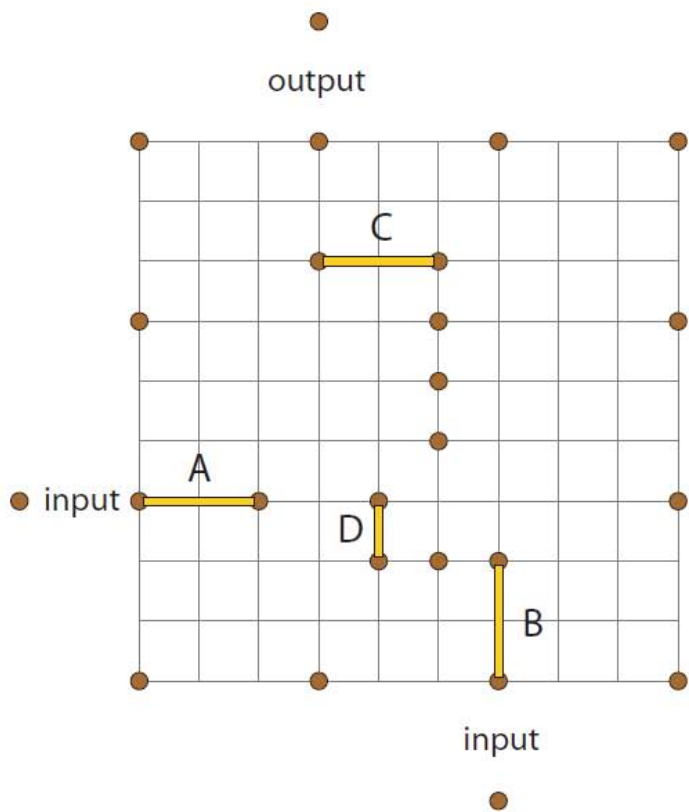- P or PSPACE-complete or …?
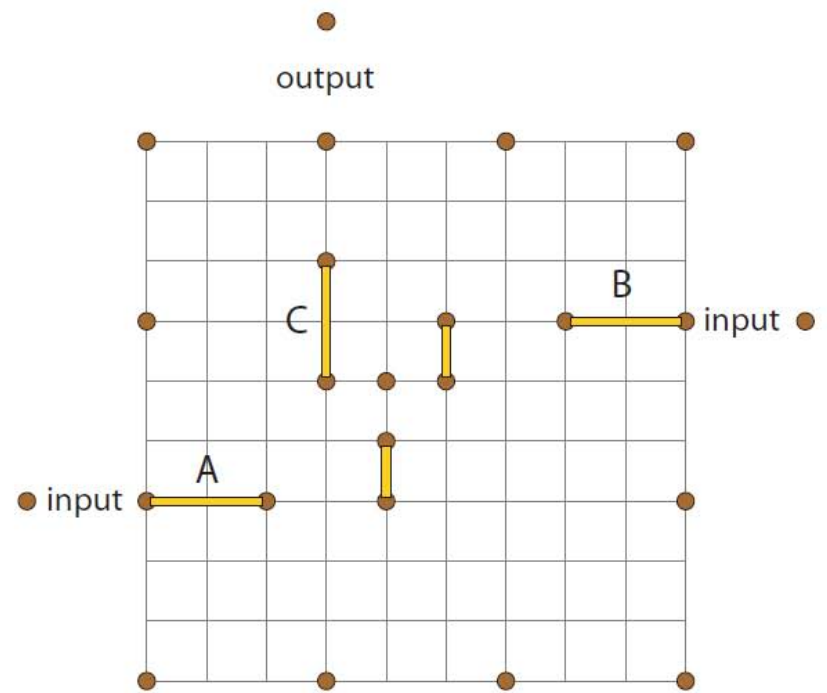

Image courtesy of John Tromp. Used with permission.
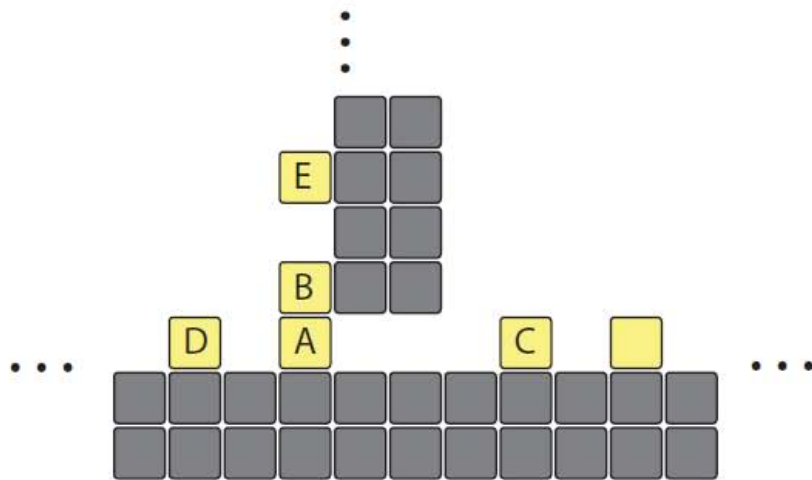
Plank Puzzles   [Hearn 2004]

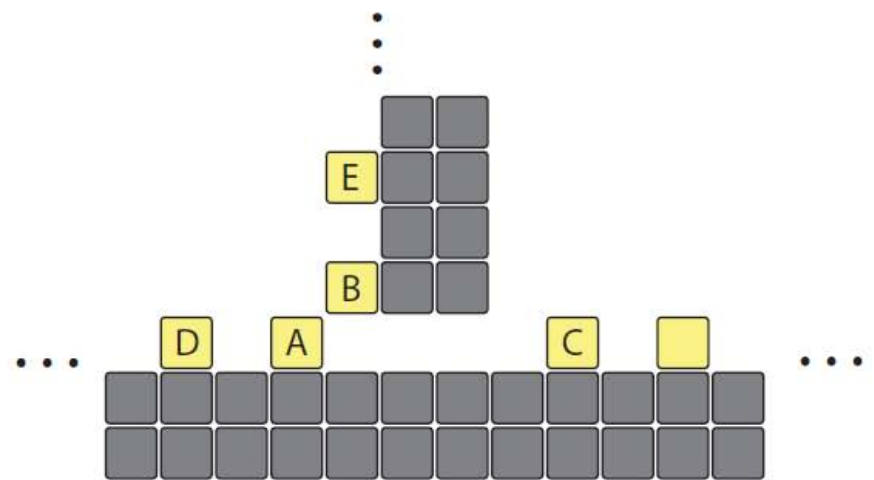(a) AND   (b) OR
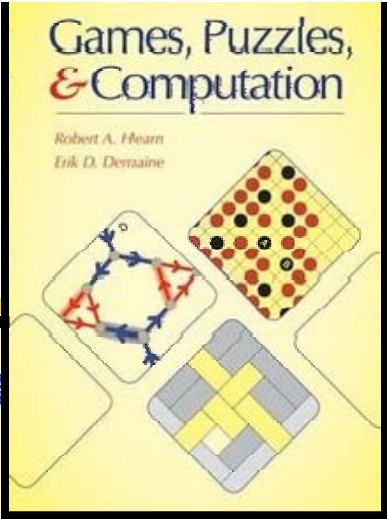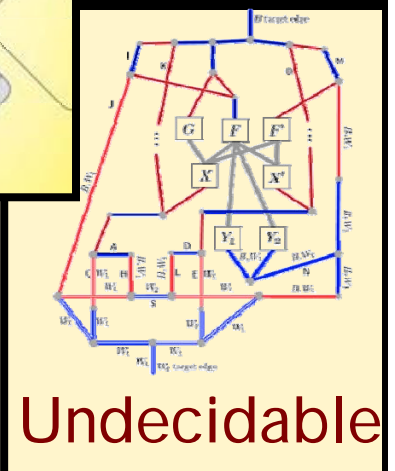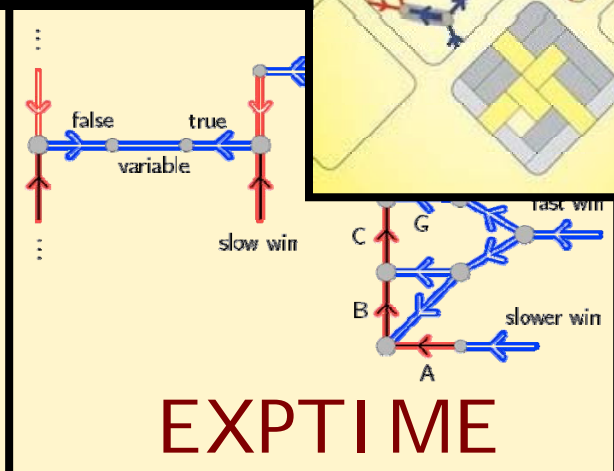
Sokoban
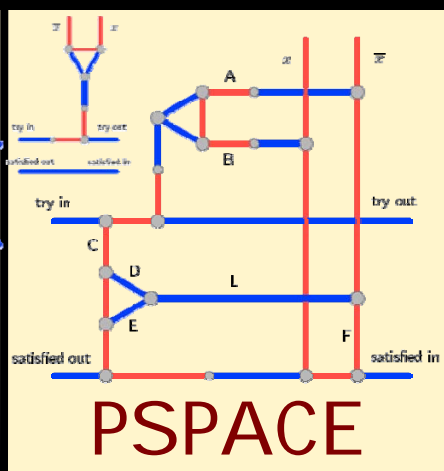## [Hearn & Demaine 2002]

(a) AND

(b) OR

PSPACE-completeness known [Culberson 1998]
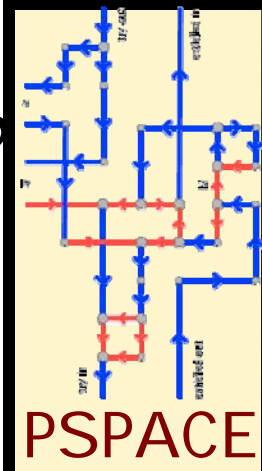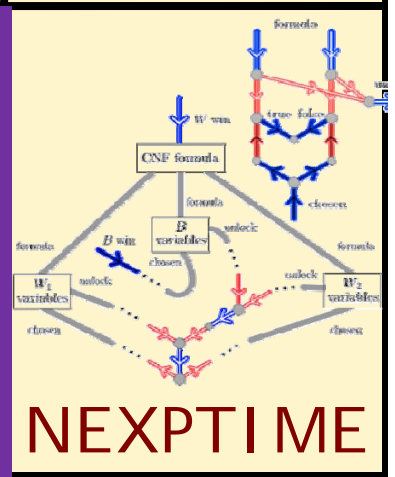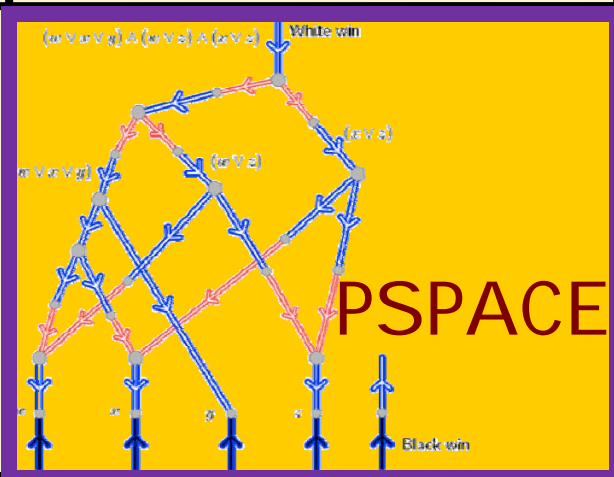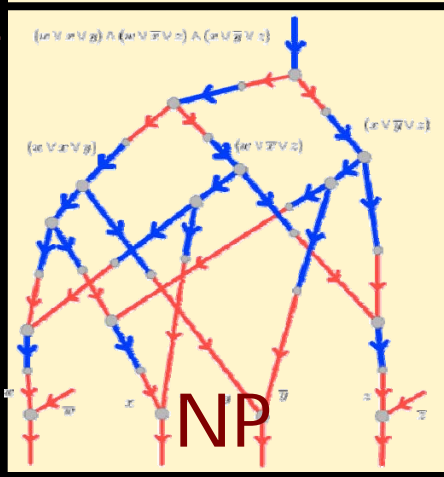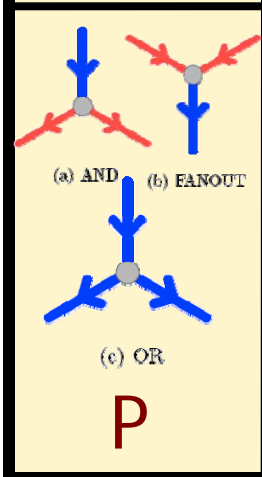
Push-2F
[Demaine, Hearn, Hoffmann 2002]

(b) AND

(c) OR

Constraint Logic
[Hearn & Demaine 2009]

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bounded	P	NP	PSPACE	NEXPTIME

Constraint Logic
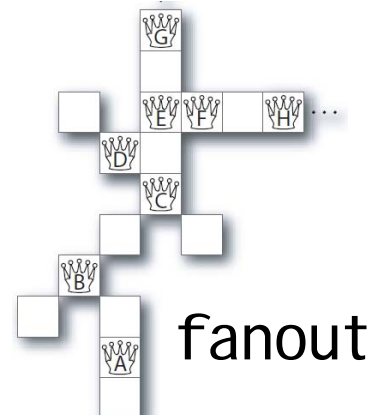## [Hearn & Demaine 2009]

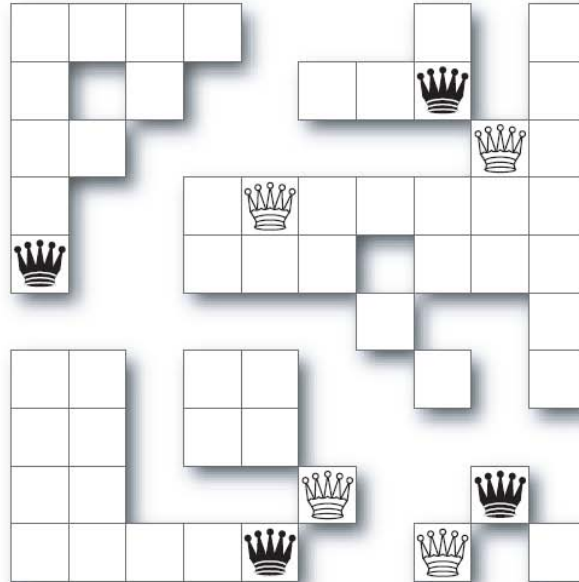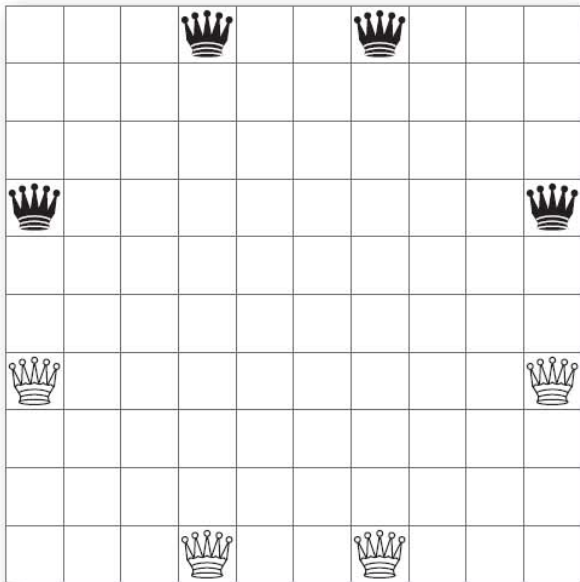|	0 players (simulation)	1 player (puzzle)	2 players (game)	team, imperfect info
unbounded	PSPACE	PSPACE	EXPTIME	Undecidable
bounded	P	NP	PSPACE	NEXPTIME

Amazons
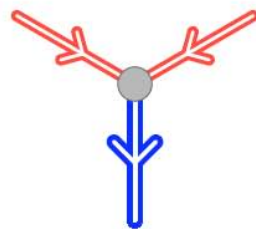## [Hearn 2005]

formula

false   true
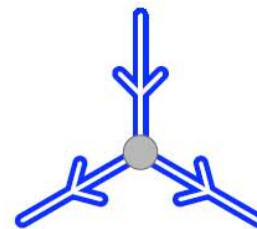
variable

slow win

fast win

slower win

D

F

C

G

B

A
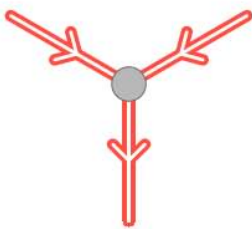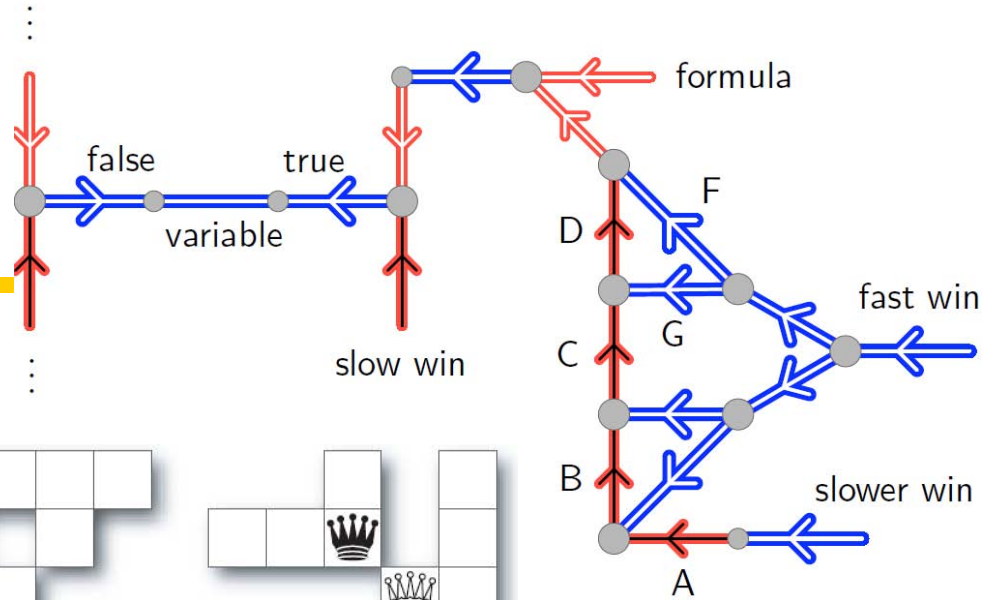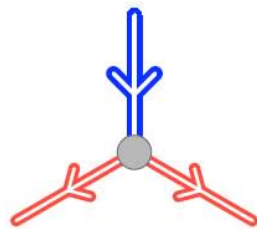
fanout

(a) CHOICE   (b) AND   (c) FANOUT   (d) OR   (e) VARIABLE

SP.268 / ESG.SP268 The Mathematics in Toys and Games

Spring 2010