Guided Study Program in System Dynamics
System Dynamics in Education Project
System Dynamics Group
MIT Sloan School of Management[1]

Solutions to Assignment #15
Monday, February 22, 1999

Reading Assignment:

Please refer to Road Maps 5: A Guide to Learning System Dynamics (D-4505-4) and read the following paper from Road Maps 5:

- *Dynamic Simulation Models: How Valid are They? by Ray Shreckengost (D-4463)*

Please refer to Road Maps 6: A Guide to Learning System Dynamics (D-4506-4) and read the following paper from Road Maps 6:

- *Exploring S-shaped Growth, by Leslie Martin (D-4476)*

Please refer to Road Maps 9: A Guide to Learning System Dynamics (D-4509) and read the following papers from Road Maps 9:

- *Mistakes and Misunderstandings: Table Functions, by Leslie A. Martin (D-4653)*
- *Graphical Integration Exercises Part Five: Qualitative Graphical Integration, by Manas Ratha (D-4675)*

Exercises:

1. Dynamic Simulation Models: How Valid are They?

Please read this paper carefully. This is a short paper, but very important in developing your modeling skills. You do not need to turn in anything for this paper but you will be asked to use the tests in this paper throughout the GSP.

2. Exploring S-shaped Growth

Please read this paper carefully. It covers fundamentals of the S-shaped growth structure that are vital to your understanding of system dynamics. Pay particular attention to the behavior of the net flow in a system generating S-shaped behavior.

A. How does a stock behave if its net flow is:

- *positive and increasing in magnitude*
  The stock increases faster and faster
- *positive and decreasing in magnitude*
  The stock increases slower and slower
- *negative and increasing in magnitude*
  The stock decreases faster and faster
- *negative and decreasing in magnitude*
  The stock decreases slower and slower
- *positive throughout, first increasing then decreasing in magnitude*
  S-shaped growth
- *negative throughout, first increasing then decreasing in magnitude*
  S-shaped decay

Often, "changing (increasing or decreasing) faster and faster" behavior may be described as "exponential-like" (for example, stock with flow that is positive and increasing in magnitude), and "changing slower and slower" behavior may be described as "asymptotic-like" (for example, stock with flow that is positive and decreasing in magnitude).

While such descriptions are acceptable, the terms "parabolic," "exponential," or "asymptotic" must not be used. Each of these has a particular mathematical meaning. Parabolic behavior is demonstrated ONLY by a stock that has a linearly changing flow, i.e. flow that is a straight line with non-zero slope. Exponential and asymptotic stock behaviors are generated only within feedback systems. Hence, be careful with the wordings used to describe stock and flow graph shapes.

B. Find a way to improve the healthy Immune Response model.[2]
Hint: the formulation for the "destruction" of "Foreign Cells" is flimsy. The time for white blood cells to destroy a foreign cell is not constant, but changes with the density of foreign cells. The correct formulation should not use an IF THEN ELSE statement.

There are a number of ways to improve the Immune Response model. The question suggests that you change the constant "time to destroy a foreign cell" to reflect a dependency on the density of foreign cells in the bloodstream. When the density of foreign cells in the blood stream is low, it takes a long time for a white blood cell to destroy foreign cells because it is difficult to isolate and attack them. Hence, the "foreign cell destruction fraction" is small. Past a certain density, however, it becomes very easy and quick for a white blood cell to destroy a foreign cell because foreign cells are everywhere. It takes little time to locate the foreign cells and then attack them, so the "foreign cell destruction fraction" is high. In order to reflect this nonlinear relationship

---

[2] The model in "Exploring S-shaped Growth" contains a unit error in the formulation of "strength of proliferation." "Antigens" has units of antigens, and both "strength of immune response" and "strength of proliferation" are dimensionless. The "strength of immune response" should have units of 1/antigens. We apologize for the confusion.

between density of foreign cells in the blood and foreign cell destruction fraction, add a nonlinear lookup function to the model. We shall call the ratio of "Foreign Cells" to "normal foreign cell count" the "cell ratio." Remember to non-dimensionalize the input and output:

immune ————————————————

cell ratio on destruction fraction lookup

-X-
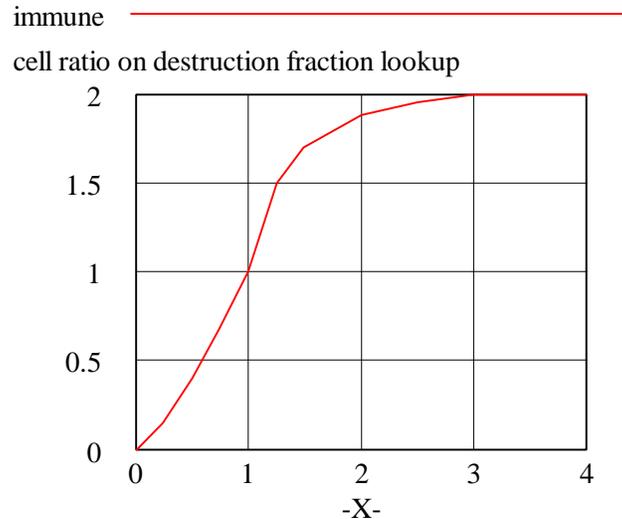
Incorporating the above lookup function into the original model, we define normal values for foreign cell count and foreign cell destruction fraction. The rate of "destruction" no longer depends directly on the number of foreign cells but is affected by the stock of "foreign cells" through the nonlinear function relating density and "foreign cell destruction fraction".

As a sidenote, some participants were confused as to why "White Blood Cells" has no outflow in this model. White blood cells do die, but not frequently enough that the deaths would be significant within the timescale of this model. To simulate the model for longer periods of time, the outflow could be added. A person who is battling a disease always ends up with more white blood cells than he or she started with. During the disease-battling time frame in the model simulation, therefore, it is not necessary to include an outflow. Here are the modified model and equations to reflect the non-constant "time to destroy a foreign cell":

Model diagram:
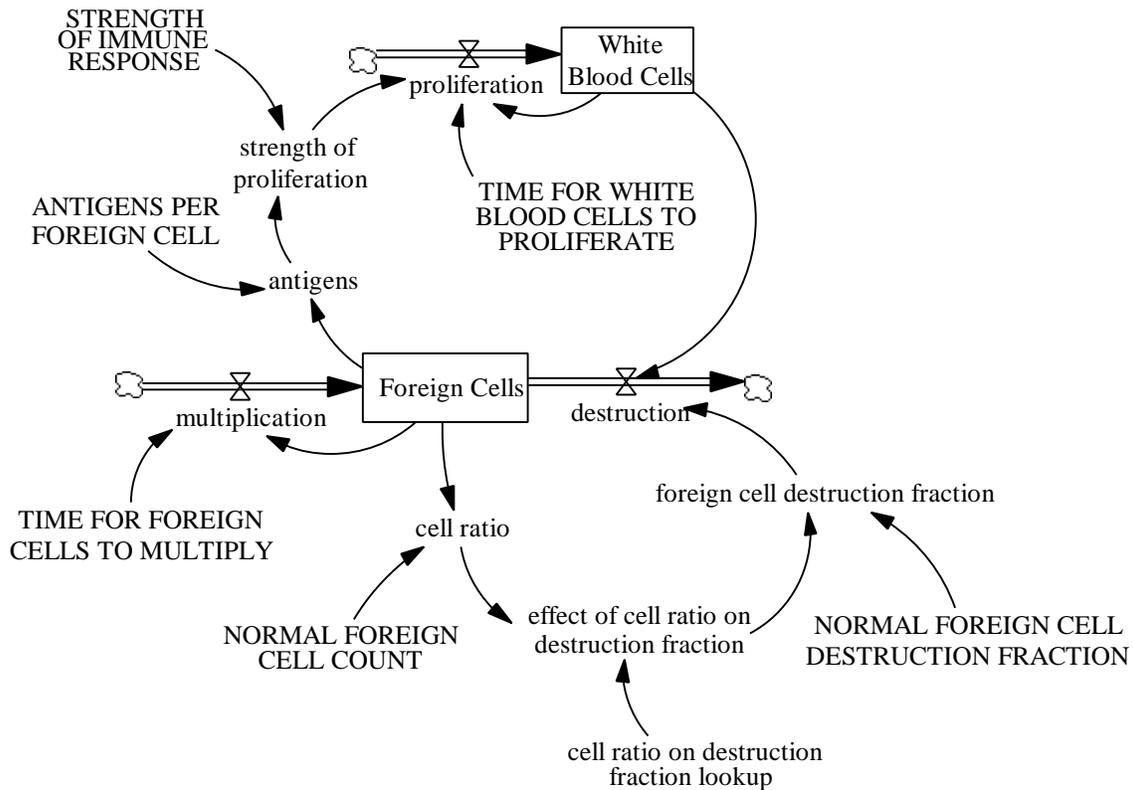
Model equations:

antigens = Foreign Cells * ANTIGENS PER FOREIGN CELL
        Units: antigens
        Antigens are macromolecule on foreign cells that, when identified by helper T cells, trigger an immune response.

ANTIGENS PER FOREIGN CELL = 2
        Units: antigens/foreign cells
        The approximately number of antigens attached to each foreign cell.

cell ratio = Foreign Cells / NORMAL FOREIGN CELL COUNT
        Units: dimensionless
        The proportion of current foreign cells to normal foreign cell count.

cell ratio on destruction fraction lookup ([(0,0) - (4,2)], (0,0), (0.25,0.15), (0.5,0.4), (0.75,0.67), (1,1), (1.25,1.5), (1.5,1.7), (2,1.88), (2.5,1.95), (3,2), (4,2))
        Units: dmnl
        The lookup function relating cell ratio to the foreign cell destruction fraction.

destruction = White Blood Cells * foreign cell destruction fraction
        Units: foreign cells/hour

Destruction is the average rate at which white blood cells destroy foreign cells.

effect of cell ratio on destruction fraction = cell ratio on destruction fraction lookup(cell
        ratio)
        Units: dimensionless
        The foreign cell destruction fraction is a function of the cell ratio.

foreign cell destruction fraction = effect of cell ratio on destruction fraction * NORMAL
        FOREIGN CELL DESTRUCTION FRACTION
        Units: 1 / ((white blood cells / foreign cells) * hour)
        The actual fraction of foreign cells that a white blood cell destroys per hour.

Foreign Cells = INTEG (multiplication - destruction, 100)
        Units: foreign cells
        An approximate number of foreign cells in one microliter of blood.

multiplication = Foreign Cells / TIME FOR FOREIGN CELLS TO MULTIPLY
        Units: foreign cells/hour
        The rate at which foreign cells multiply.

NORMAL FOREIGN CELL COUNT = 100
        Units: foreign cells
        The normal number of foreign cells in the blood.

NORMAL FOREIGN CELL DESTRUCTION FRACTION = 1
        Units: 1 / ((white blood cells/foreign cells) * hour)
        The normal fraction of foreign cells destroyed by a white blood cell per hour.

proliferation = strength of proliferation * White Blood Cells / TIME FOR WHITE
        BLOOD CELLS TO PROLIFERATE
        Units: white blood cells/hour
        Proliferation is determined by multiplying a normal proliferation rate (white blood
        cells / time for white blood cells to proliferate) by the variable strength of
        proliferation.

STRENGTH OF IMMUNE RESPONSE =0.001
        Units: 1/antigens
        The strength of the immune response is a fraction representing the ability of
        helper T cells to recognize antigens and activate the proliferation of white blood
        cells. When someone is infected with HIV, the human immune system is crippled
        and the strength of the immune response is severely weakened.

strength of proliferation = STRENGTH OF IMMUNE RESPONSE * antigens
        Units: dimensionless
        Strength of proliferation is the multiplier that increases the proliferation rate as the
        number of recognized antigens increases.

TIME FOR FOREIGN CELLS TO MULTIPLY = 5
      Units: hour
      The time it takes foreign cells to multiply.

TIME FOR WHITE BLOOD CELLS TO PROLIFERATE =5
      Units: hour
      The time it takes white blood cells to proliferate.

White Blood Cells = INTEG (proliferation, 10)
      Units: white blood cells
      An approximate number of antigen-specific white blood cells that are present in
      one microliter of blood before an antigen invasion is recognized. White blood
      cells is the name given to cells that constitute the human immune response: helper
      T cells, killer T cells, B cells, memory cells...

Simulating the model for 60 hours, we obtain the following behavior:

White Blood Cells

White Blood Cells : immune ————————————— white blood cells

Foreign Cells

Foreign Cells : immune ——————————————— foreign cells
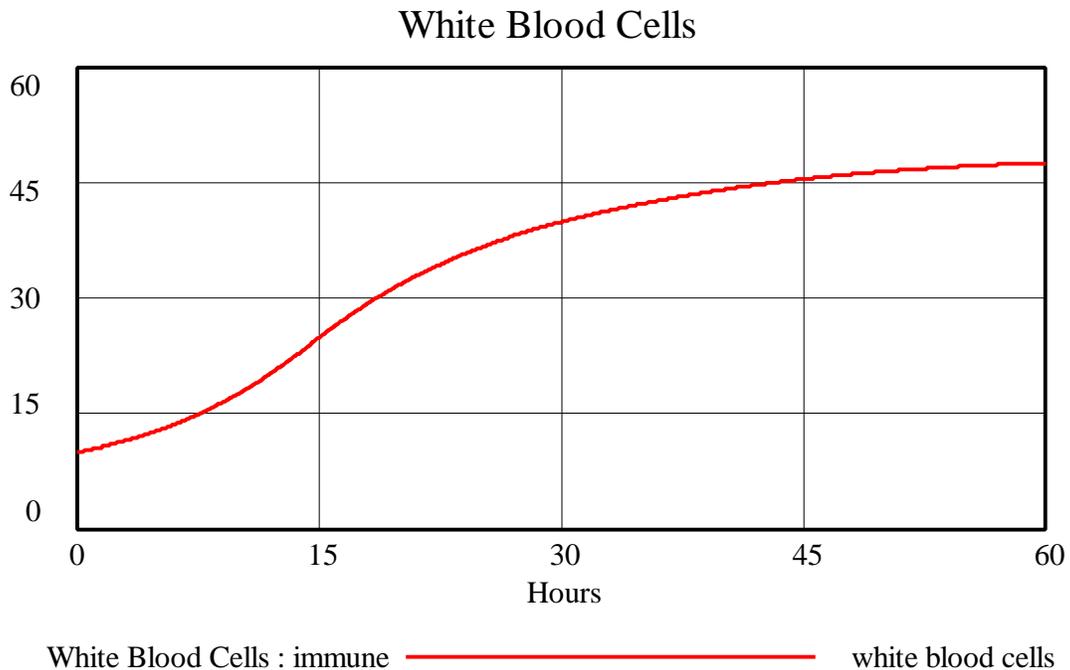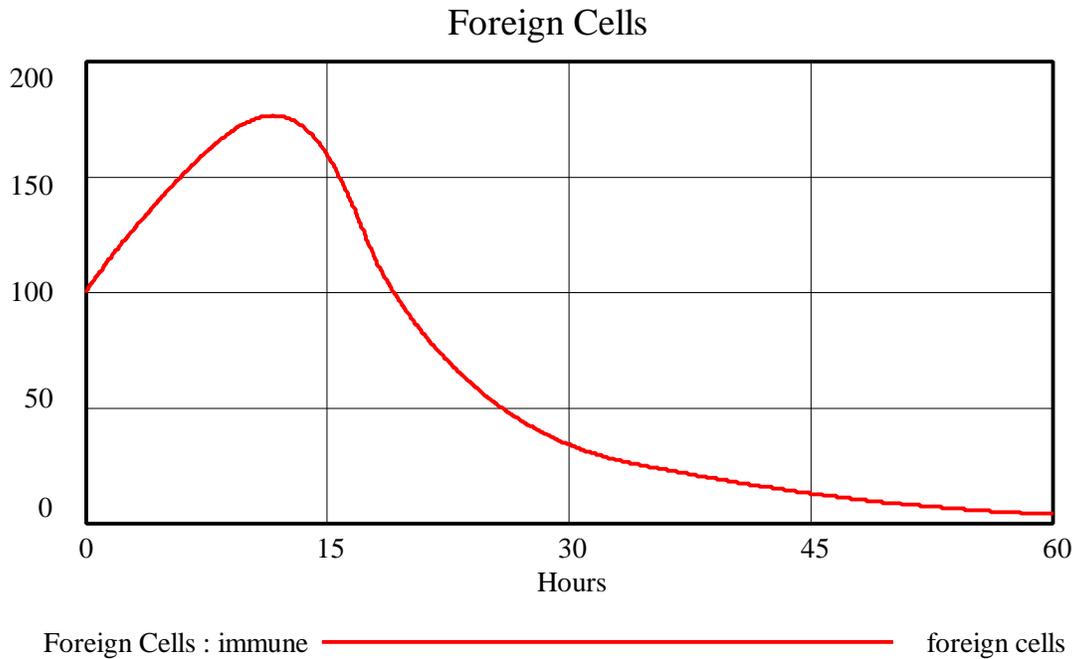
3. Mistakes and Misunderstandings: Table Functions

Please read this paper carefully. You do not have to answer any questions for this paper, but please let us know if you have any questions or comments about it.

4. Graphical Integration Exercises Part Five: Qualitative Graphical Integration

Please read this paper and do all the included exercises. Based on the skills that you acquired from this paper, integrate the following flows.[3] Assume that the initial values of the stocks are zero.

---

[3] Use a graphics application to create a graph of the stock behavior and then paste the graph into your assignment solutions document. You can also draw the stock behavior by hand and fax it to us at (617) 258-9405, or you can provide a detailed verbal description of the stock behavior.

A.

1: flow
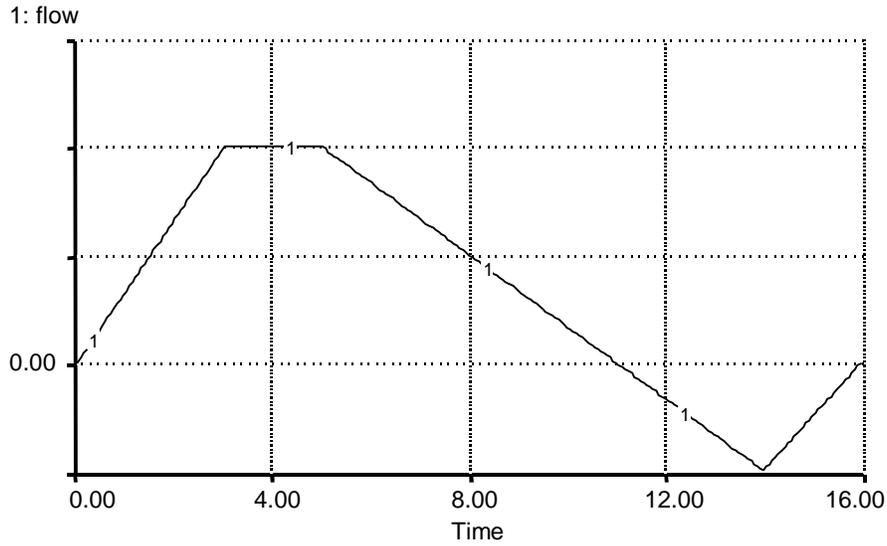
From time = 0 to time =3, the flow is increasing linearly, so the stock is increasing parabolically. From time = 3 to time = 5, the flow is constant, so the stock increases linearly. From time = 5 to time = 10, the flow is still positive but decreases linearly. Hence, the stock continues increasing but at a slower and slower rate. At time = 10, the flow becomes negative and linearly grows in magnitude until time = 14. Therefore, the stock is decreasing at an increasing rate. From time = 14 to time = 16, the flow remains negative but starts decreasing in magnitude, until it reaches zero at time = 16. Hence, the stock continues decreasing, but at a slower and slower rate. At time = 16, the stock is not changing.
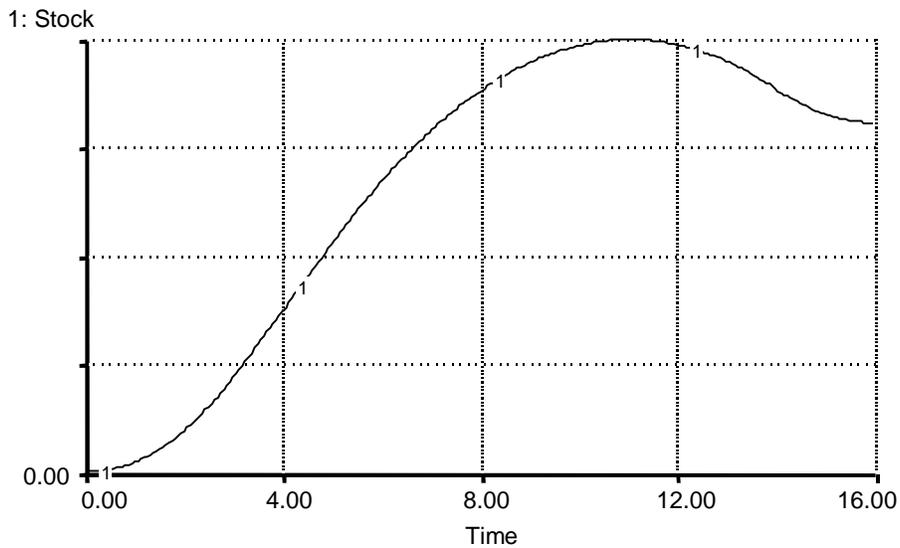
1: Stock

B.
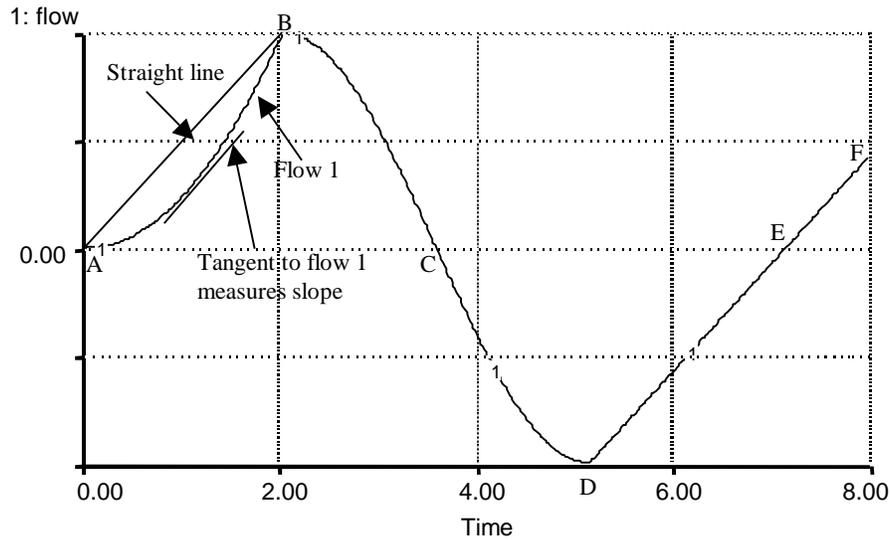
From time = 0 to time = 2, the flow is positive and increasing with an increasing slope. Even though the flow is not linear, the general characteristics of the flow (its sign and increasing magnitude) let us conclude that the stock is increasing at a faster and faster rate. For those who want a more detailed explanation, read the paragraphs in italics below.

In this example, the flow is not growing linearly. This flow can be approximated by drawing many linear flows, each over short periods of time (exactly how elementary calculus is taught: by drawing short, straight lines to approximate a curve). Thus, the stock for this curve will be the composite of the results of graphically integrating each short linear flow. Now each of these flows produces a parabolically growing stock; but each successive flow has a larger slope; hence, the stock behavior produced by it will have a sharper upward-sloping curve. Thus, the stock will grow faster and faster as time passes.

Looking at it another way:
Draw a straight line from A to B and let this line be called flow AB. Between time 0 and about one, the slope of the nonlinear flow is increasing, but is still smaller than the slope of flow AB. Therefore, the stock corresponding to the nonlinear flow will grow more slowly than the stock corresponding to flow AB. After time = 1, however, the slope of the nonlinear flow is greater than the slope of line AB, and the stock corresponding to the nonlinear flow grows more rapidly than the stock corresponding to flow AB. Now you can draw a stock corresponding to flow AB first, and then drawing another stock for the above flow by making the adjustments noted above.

From time = 2 to approximately time = 3.5, the flow stays positive but decreases, again in a nonlinear way. Hence, the stock continues increasing, but at a decreasing rate. Because the slope of the flow becomes increasingly negative, in this time segment the

stock will initially be growing faster than a stock filled by a positive decreasing linear flow that joins B and C. As soon as the nonlinear flow becomes steeper than the linear flow BC, the stock will start growing at a slower rate than a stock filled by the linear flow.

From approximately time = 3.5 to approximately time = 5, the flow is negative and nonlinearly increasing in magnitude, with a negative slope whose magnitude gradually decreases to zero. Hence, the stock starts decreasing at a faster and faster rate. Compared to a stock drained by a linear negative flow (that joins C and D) with increasing magnitude, the stock in this example will initially be decreasing faster, until the slope of the nonlinear flow equals the slope of the linear flow CD. As soon as the magnitude of the slope of the nonlinear flow becomes smaller than the magnitude of the slope of the linear flow CD, the stock will start decreasing more slowly than a stock drained by the linear flow CD.

Notice that the flow from time =3.5 to time = 5 is the reverse of the flow from time = 2 to time = 3.5. Therefore, whatever was added to the stock between time = 2 and time = 3.5 is then removed from the stock between time = 3.5 and time = 5, and the stock behavior from time = 3.5 to time = 5 is the mirror image of the stock behavior from time = 2 to time =3.5.

From approximately time = 5 to time = 7, the flow remains negative but linearly decreases in magnitude towards 0. Hence, the stock continues decreasing but at a slower and slower rate. Finally from time = 7 to time = 8, the flow is positive and linearly increasing, so the stock increases at an increasing rate.