Modeling the Evolution of Demand Forecasts with Application to Safety Stock Analysis in Production/Distribution Systems

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This summary presentation based on: Heath, D.C., and P.L. Jackson. "Modeling the Evolution of Demand Forecasts with Application to Safety Stock Analysis in Production/Distribution Systems." *IIE Transactions* 26, no. 3 (1994): 17-30.

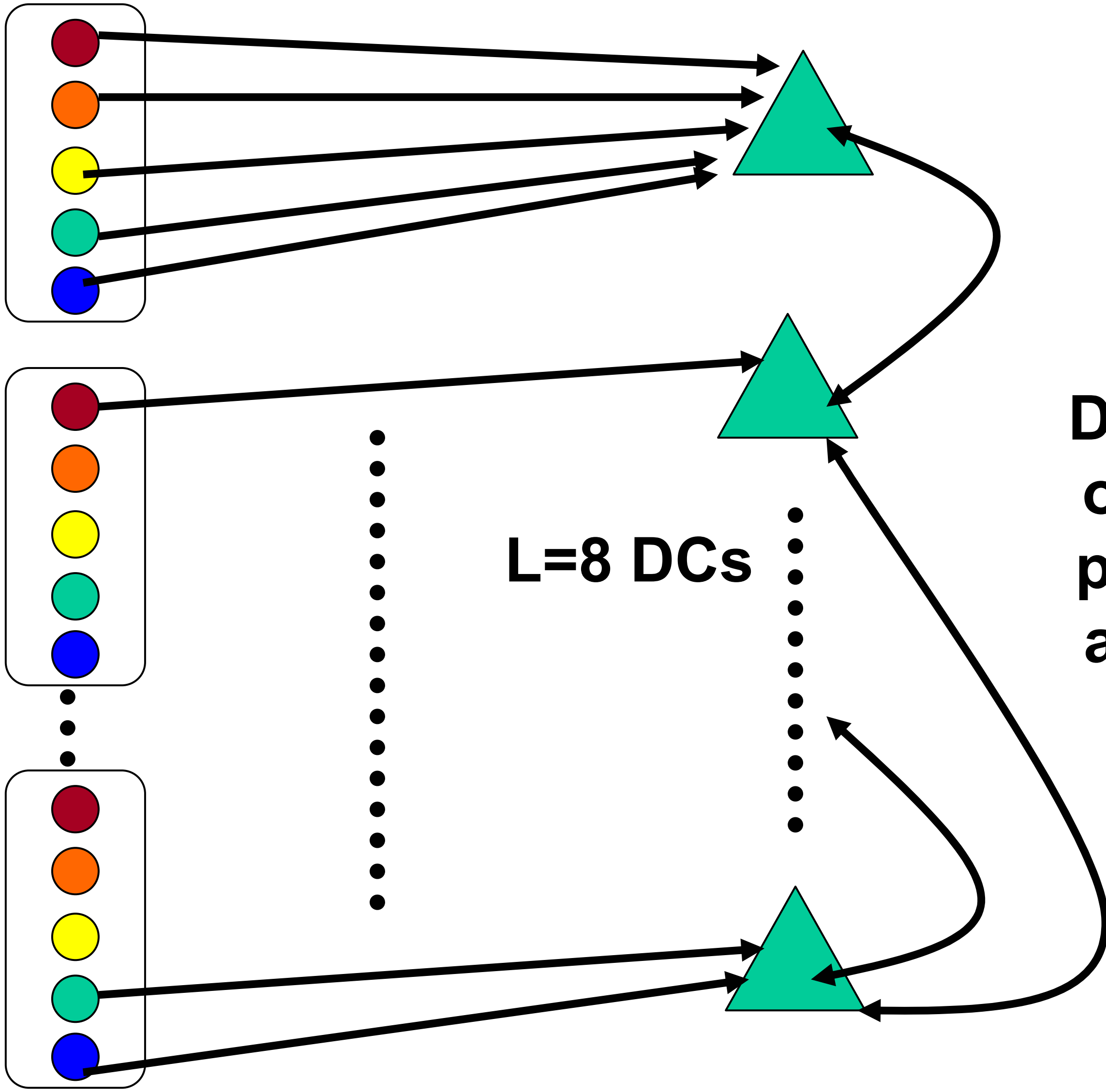
Overview

- Problem
- Methodology
- Key Results

Multi-product Multi-location Multi-period with Capacity and Trans-shipment

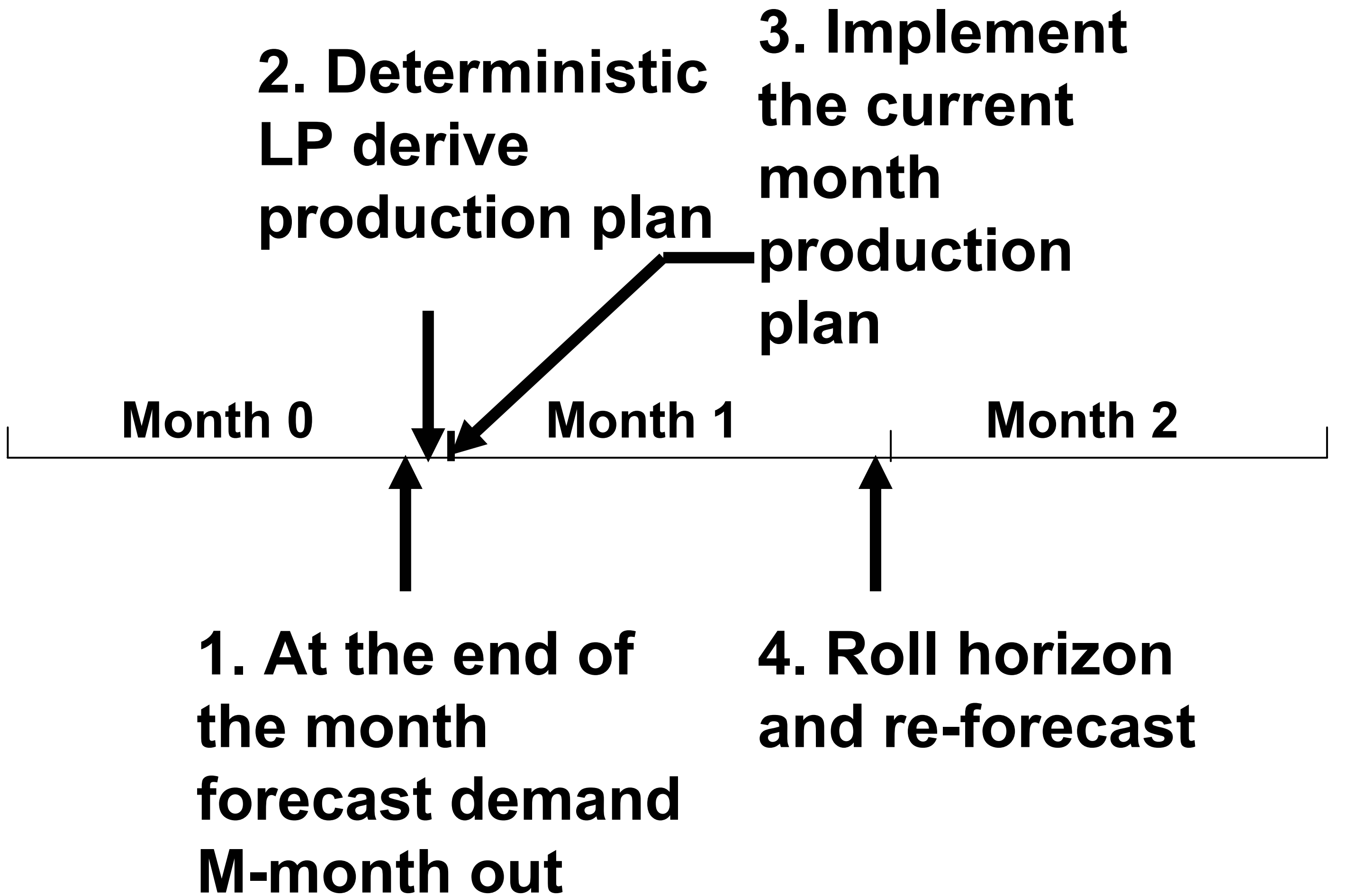
N=5 Products

Multi- Plants



Demand of each product at each DC

Planning Sequence of Events



Solving the Inventory Problem

- Find an economical safety stock factor for each product at each DC for each month
  - A DP approach difficult
  - A simulation approach
    - Use MMFE to simulate forecast
    - Use a similar LP to simulate decisions
    - Tally costs and service level for different safety stock factor

The Martingale Model of Forecast Evolution (Additive)

Future Month t

Current Month s

	Month 0	Month 1	Month 2	Month 3
Month 0	$D_{0,0}$	$D_{0,1}$	$D_{0,2}$	$\underline{D}$
Month 1		$D_{1,1}$	$D_{1,2}$	$D_{1,3}$
		$\varepsilon_{1,1}= D_{1,1}-D_{0,1}$	$\varepsilon_{1,2}= D_{1,2}-D_{0,2}$	$\varepsilon_{1,3}= D_{1,3}-\underline{D}$
Month 2			$D_{2,2}$	$D_{2,3}$
			$\varepsilon_{2,2}= D_{2,2}-D_{1,2}$	$\varepsilon_{2,3}= D_{2,3}-D_{1,3}$

i.i.d. multivariate normal with mean 0

Justifying ε are i.i.d. multivariate normal with mean 0

$$\varepsilon_{s,t} = D_{s,t} - D_{s-1,t}$$

$$\varepsilon_s = (\varepsilon_{s,t})_{t=s}^{+\infty}$$

- Information set F_s grows with time *s*
- ε_s is uncorrelated with all ε_u for $u \leq s-1$ and $E[\varepsilon_s]=0$
- ε_s is a stationary process
- ε_s is normal

Why is it called a Martingale Model?

If forecast is conditional expectation based on current information set F_s

$$D_{s,t} = E[D_{t,t} \mid F_s]$$

Then

$$\begin{aligned} & E[D_{s,t} \mid F_0,\ldots,F_{s-1}] \\ &= E[E[D_{t,t} \mid F_0,\ldots,F_s] \mid F_0,\ldots,F_{s-1}] \\ &= E[D_{t,t} \mid F_0,\ldots,F_{s-1}] \\ &= D_{s-1,t} \end{aligned}$$

Thus, $D_{s,t}$ is a martingale and

$\varepsilon_{s,t} = E[D_{t,t} \mid F_s] - E[D_{t,t} \mid F_{s-1}]$ **is uncorrelated with F_{s-1}**

$$E[\varepsilon_{s,t}] = 0$$

Conditional Expectation as Best Mean-Square Predictor of $D_{t,t}$

Using the best Mean-square predictor definition of conditional probability

$$E[(D_{t,t} - D_{u,t})w] = E[(D_{t,t} - D_{s,t})w] = 0$$

$$\forall w \in r.v.\ observed\ at\ u$$

Then

$$E[(D_{t,t} - D_{u,t})w] = E[(D_{s,t} - D_{u,t})w] = 0$$

Thus, true also under linear predictor

The Multiplicative Model

$$\nu_{s,t} = \log(D_{s,t}) - \log(D_{s-1,t})$$

$$R_{s,t} = \exp(\nu_{s,t}) = D_{s,t} / D_{s-1,t}$$

$$\nu_s = (\nu_{s,t})_{t=s}^{+\infty}$$

i.i.d. multivariate normal with mean of each coordinate being the negative of one half of its variance

Characterize the Simulation System

- **The variance-covariance matrix Σ for ε_s (*MN* X *MN*) – estimated using past demand and forecast**
- **The initial state of the system ($D_{0,0}$, $D_{0,1}$, …, $D_{0,M}$, $\underline{D}$)**

Why not Simulate the Time Series of Forecast Directly

- "*Simulate the complicated forecast process based on past demand, competitors' prices, weather forecast etc. is no more credible than assuming the forecast process is MMFE and estimating the variance-covariance matrix*"

Simulating the Forecast Evolution Using the Variance-covariance Matrix Σ

- **Properties of Σ**
  - *Σ* **is symmetric and PSD**
  - $\Sigma = CC' = (UD^{1/2})(UD^{1/2})'$ ***where D is the diagonal matrix with eigenvalues sorted in decreasing order***

- **The standard multivariate normal representation:** $\varepsilon_s = CZ$ ***where Z is a standard normal random vector***

Forecast Variability Resolving Over Time

- **The 1st column of *C* captures the 1st order magnitude of ε_s**
- **The signs and values of entries in the 1st column of *C* reveals how forecast variability resolves over time and how they are correlated**
  - ***e.g.* $C_{0,1} = -.5511$, $C_{0,41} = -.4343$ → *61.7%* variability resolved in month of sale, *38.3%* variability resolved 1 month out**
  - **e.g. $C_{0,1} = .1616$, $C_{0,41} = .3143$, $C_{0,81} = -.0880$, $C_{0,121} = -.2636$ → 12%, 49%, 4%, 34%**

The Experiment

- **Traditional Forecast Method (2 month out)**
  - **80 X 80 Σ from four years of past forecast and actual demand data**
- **Statistical Forecast Method (4 month out)**
  - **(160 X 160) Σ from two years actual demand and simulated forecast**
- **Initial state – forecast for the year of 1990-1991 fiscal year**
- **Cost and service metrics averaged over 10 – 20 simulated years**

The Results

- **Safety stock factor can be reduced without sacrificing much fill rate, if using the Statistical Forecast Method → resulting in significant cost savings**
- **Reducing safety stock factor using the Traditional Forecast Method does not show much benefit**
- **More important to increase forecast accuracy than to increase capacity**

Recap

- MMFE to model forecast evolution
- Simulate the system using MMFE
- Evaluate the performance of the two systems with two different forecast methods