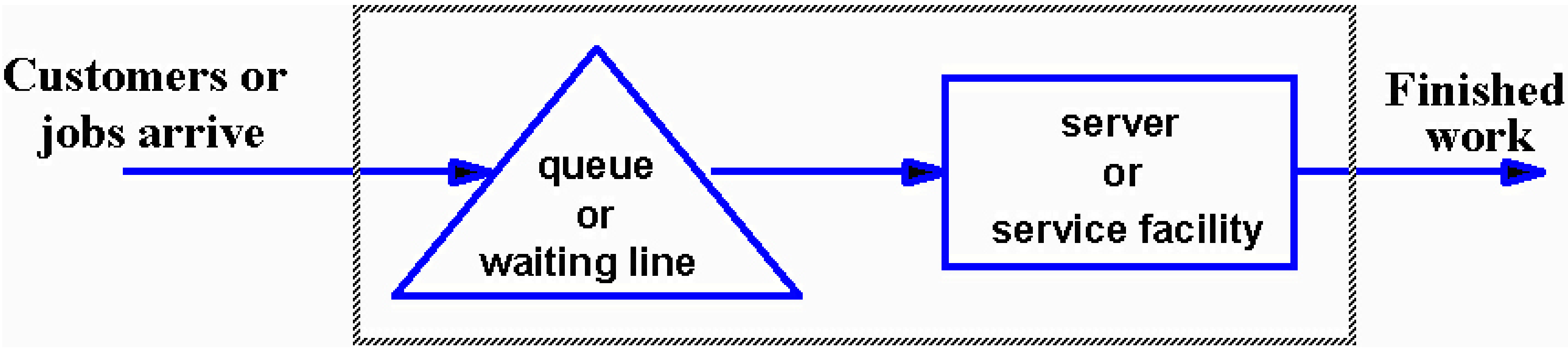
15.760 Class #8:
Basic Concepts in Queueing

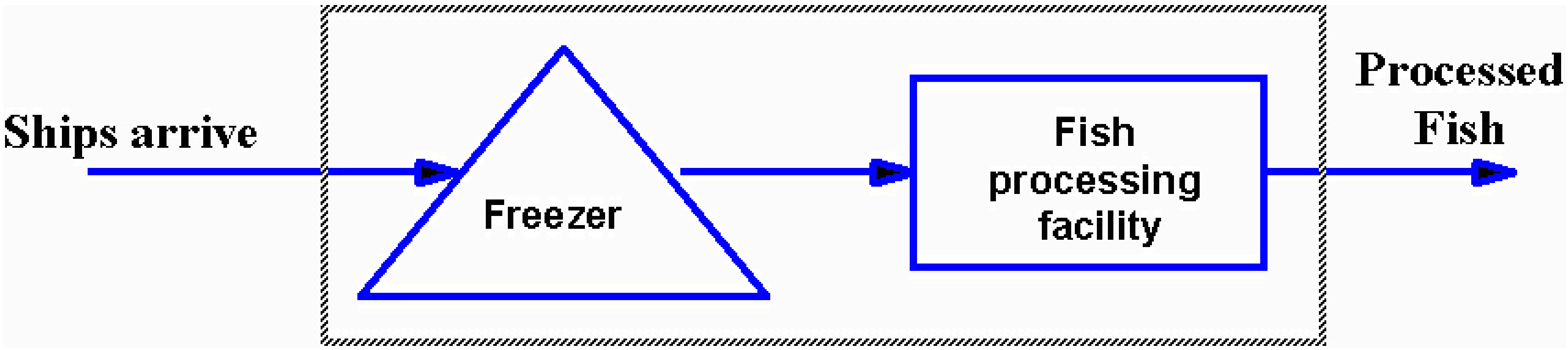


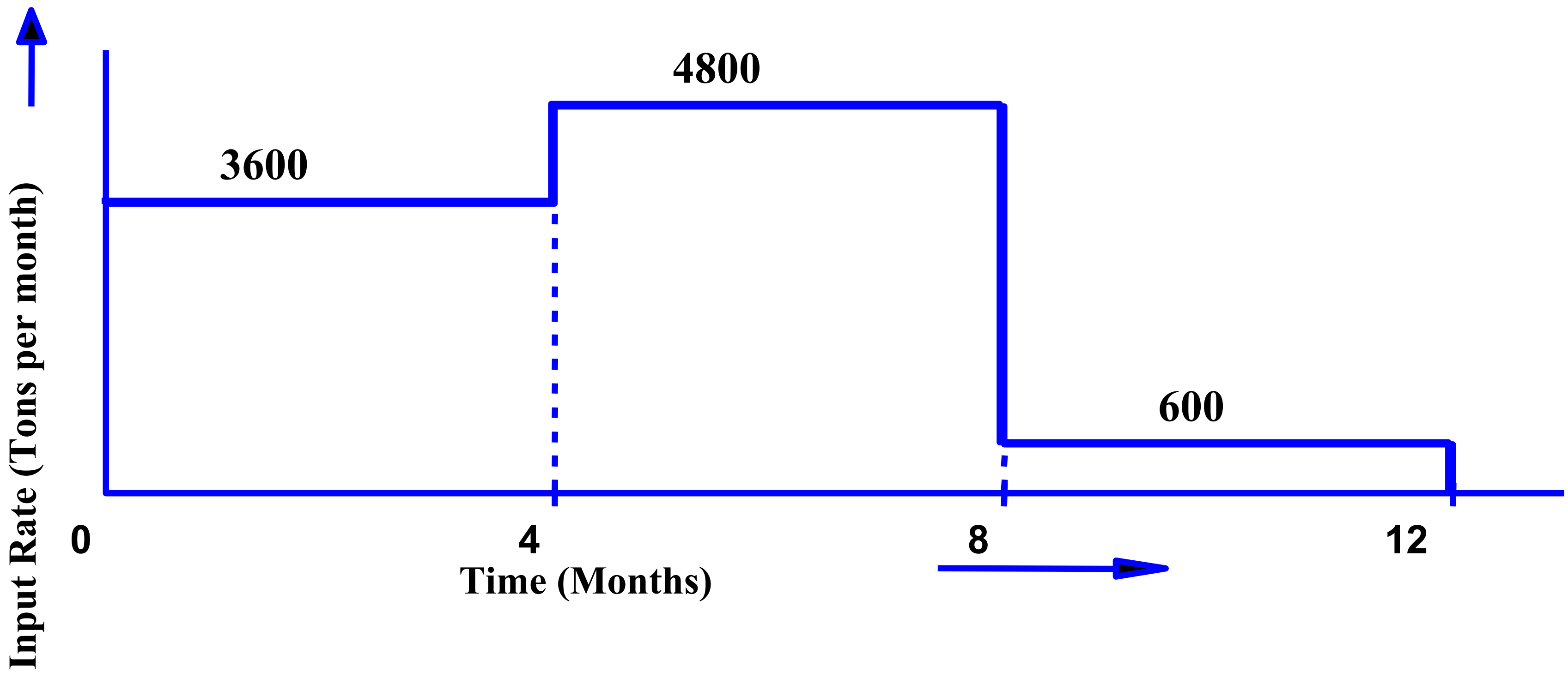
System Performance = f(System parameters)

Output/throughput rate	[λ]
Inventory Level/Queue Size/ Line length	[□]
Waiting Time/Cycle Time	[W]
Capacity or Server utilization	[ρ]
Probability that Queue is full	[P_{full}]

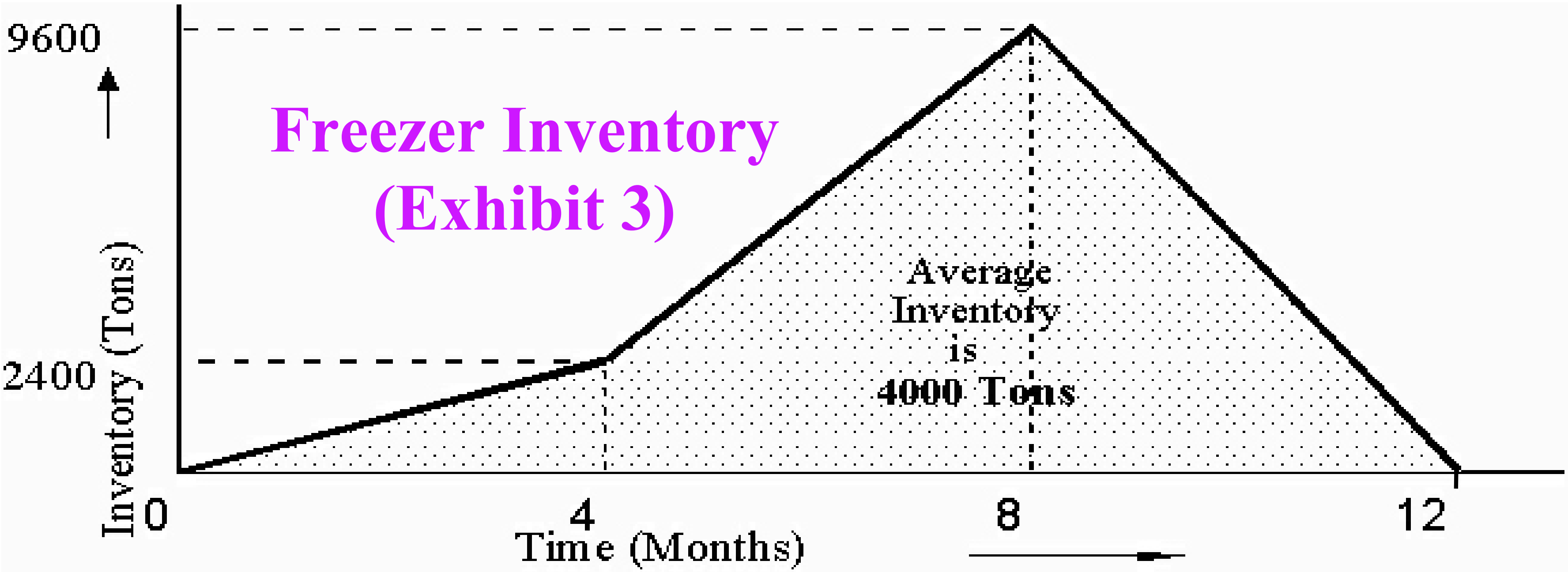
Arrival rate	[λ]
Service rate	[μ]
Service time	[M]
Number of servers	[S]
Queue/Buffer capacity	[R]
Capacity or Server utilization	[ρ]
Number of Service classes	[K]

Fish Processing Example





Fish Processing Example (con't)
λ = 3600 then 4800 then 600
μ=3000, R= unlimited



Average Inventory = $\frac{1}{3} \times (2400/2)$

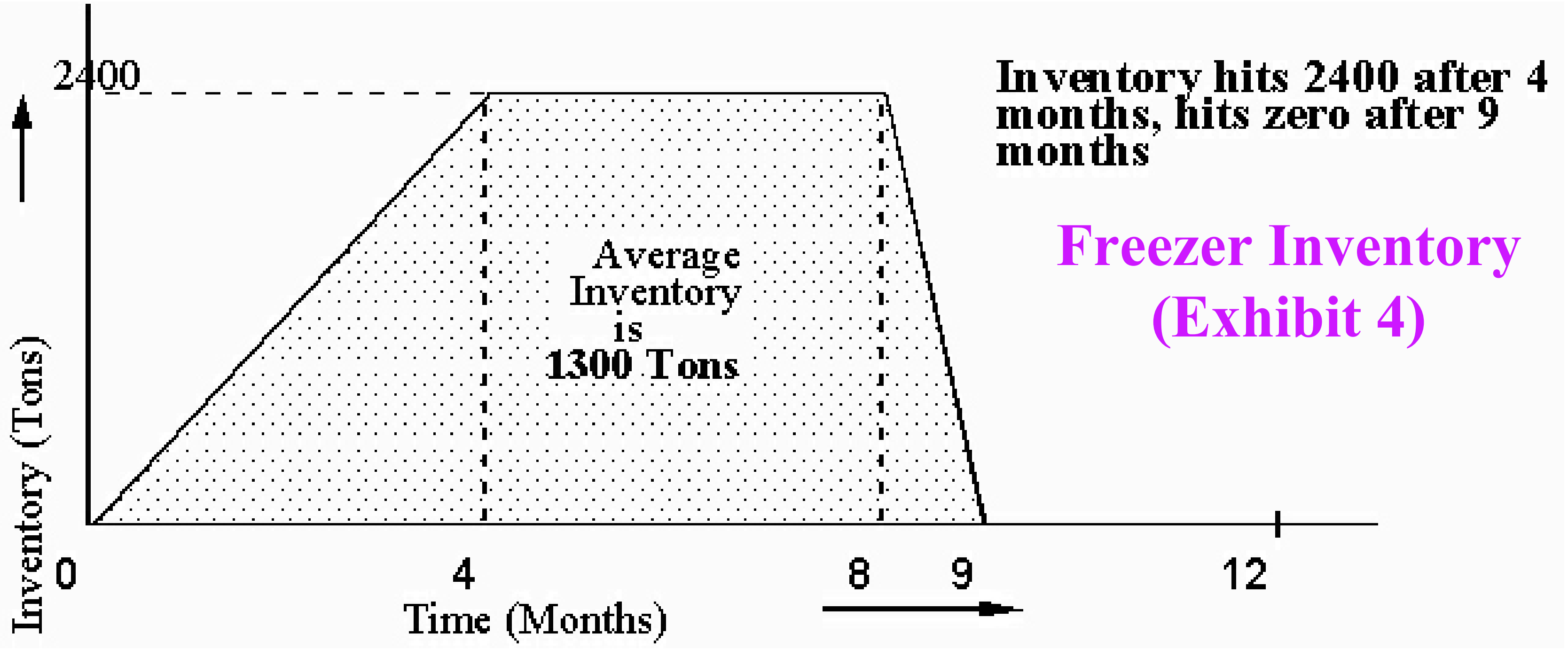
$+ \ \frac{1}{3} \times (9600 + 2400)/2$

$+ \ \frac{1}{3} \times (9600/2)$

$= \ (1200 + 6000 + 4800)/3$

$= \ 4000$ tons

Fish Processing Example (con't)
λ = 3600 then 4800 then 600
μ=3000, R= 2400



Average Inventory = 1/3 x (2400/2)

+ 1/3 x 2400

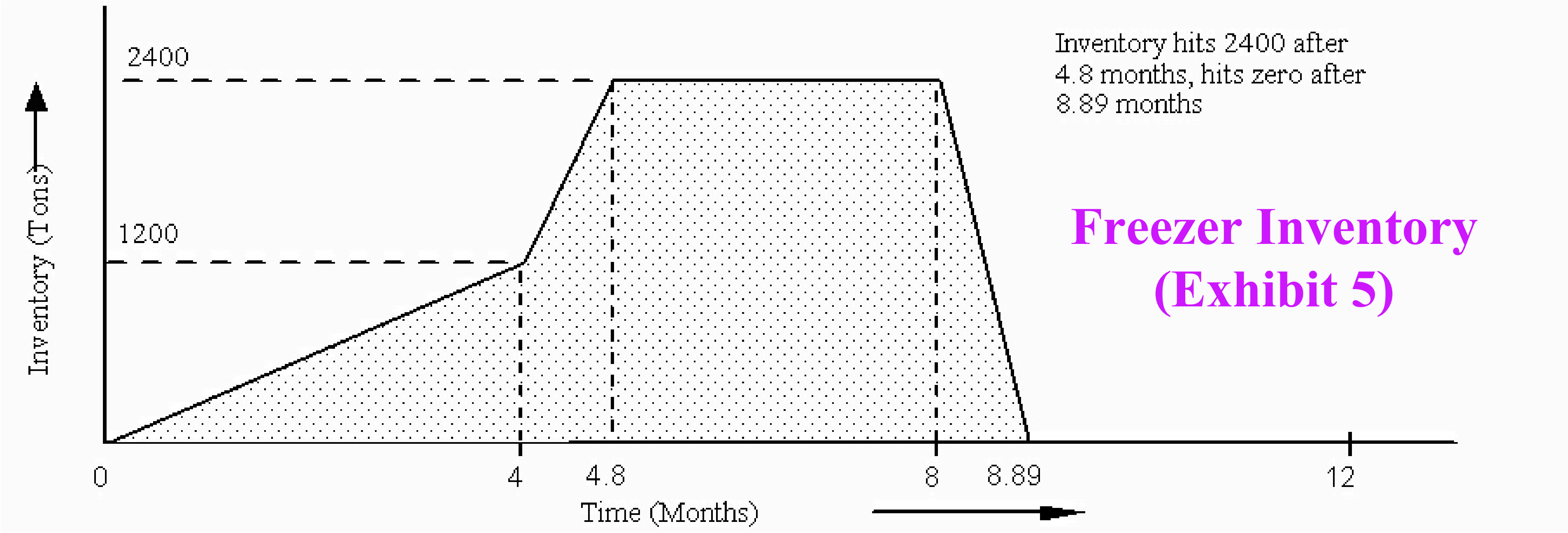
+ 1/12 x (2400/2)

+ 1/4 x 0

= (1200/3 + 2400/3 + 1200/12)

= 1300 tons

Fish Processing Example (con't)
λ = 3600 then 4800 then 600
μ=3300, R= 2400



Freezer Inventory (Exhibit 5)

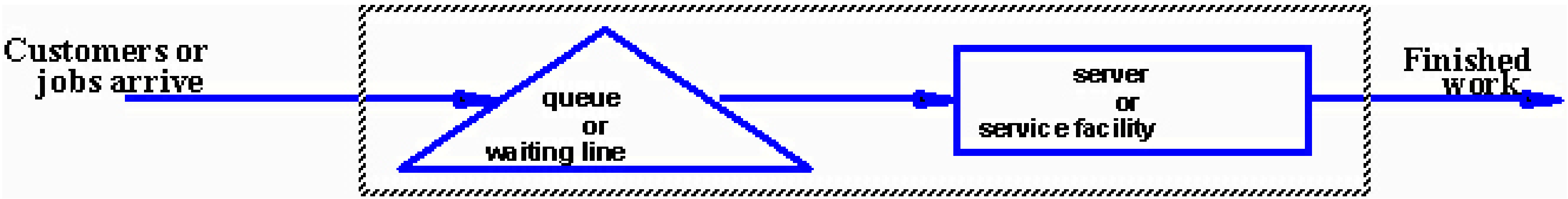
$$
\begin{aligned}
\text{Average Inventory} = \; & 1/3 \times (1200/2) \\
& + .8/12 \times 1800 \\
& + 3.2/12 \times (2400) \\
& + .89/12 \times (2400/2) \\
= \; & (200 + 120 + 640 + 89) \\
= \; & 1049 \text{ tons}
\end{aligned}
$$

Fish Processing Example (con't)
λ = 3600 then 4800 then 600

	Average Inventory	Thruput per month	Capacity Utilization
μ=3000, R=∞	4000 tons	3000 (.63)	100%
μ=3000, R= 2400	1300 tons	2400 (.63)	80%
μ=3300, R= 2400	1049 tons	2600 (.63)	79%

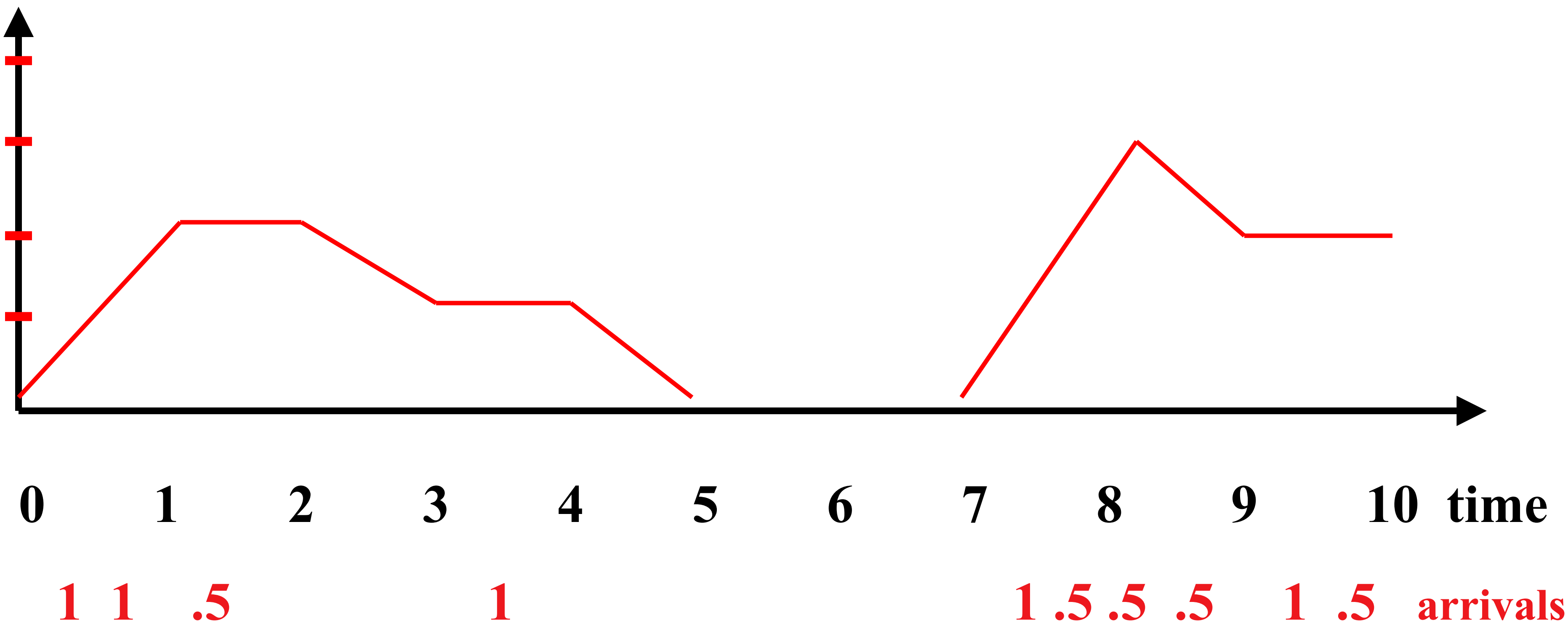
Tradeoffs: Cost of processing capacity vs.
Cost of Storage Capacity vs.
Value of output (net of holding costs)

Basic Concepts in Queueing: Capacity Utilization in Stochastic Systems

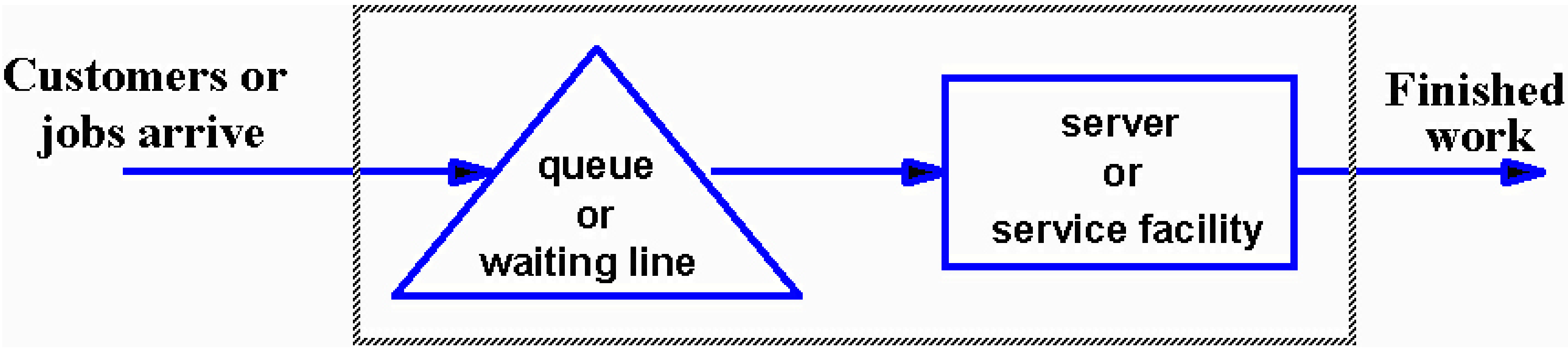


Suppose avg arrival rate = 1/minute

Avg service rate = 1.33/minute (or avg service time = 45 seconds)



Basic Concepts in Queueing: Capacity Utilization in Stochastic Systems



System Performance = f(System parameters)

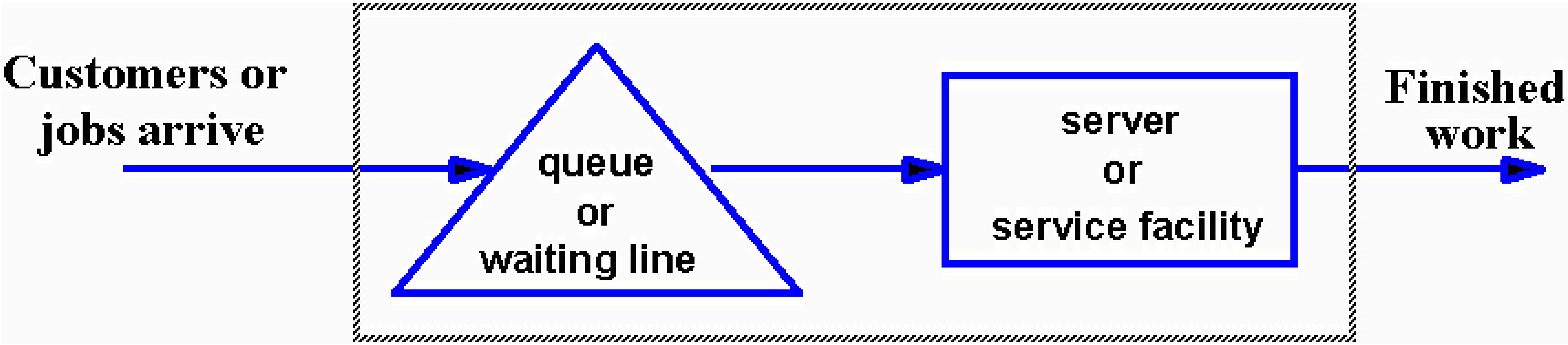
Capacity or Server utilization

$$\rho = \lambda / \mu$$ (**arrival rate/service rate**)

$$\rho = \lambda / (S \times \mu)$$

$$\rho = \lambda \times M$$

Basic Concepts in Queueing: Little's Law



System Performance = f(System parameters)

Conservation of Flows in Stochastic Systems

$$L = \lambda \times W$$

Avg Length of the Queue = Arrival rate x Avg Waiting time

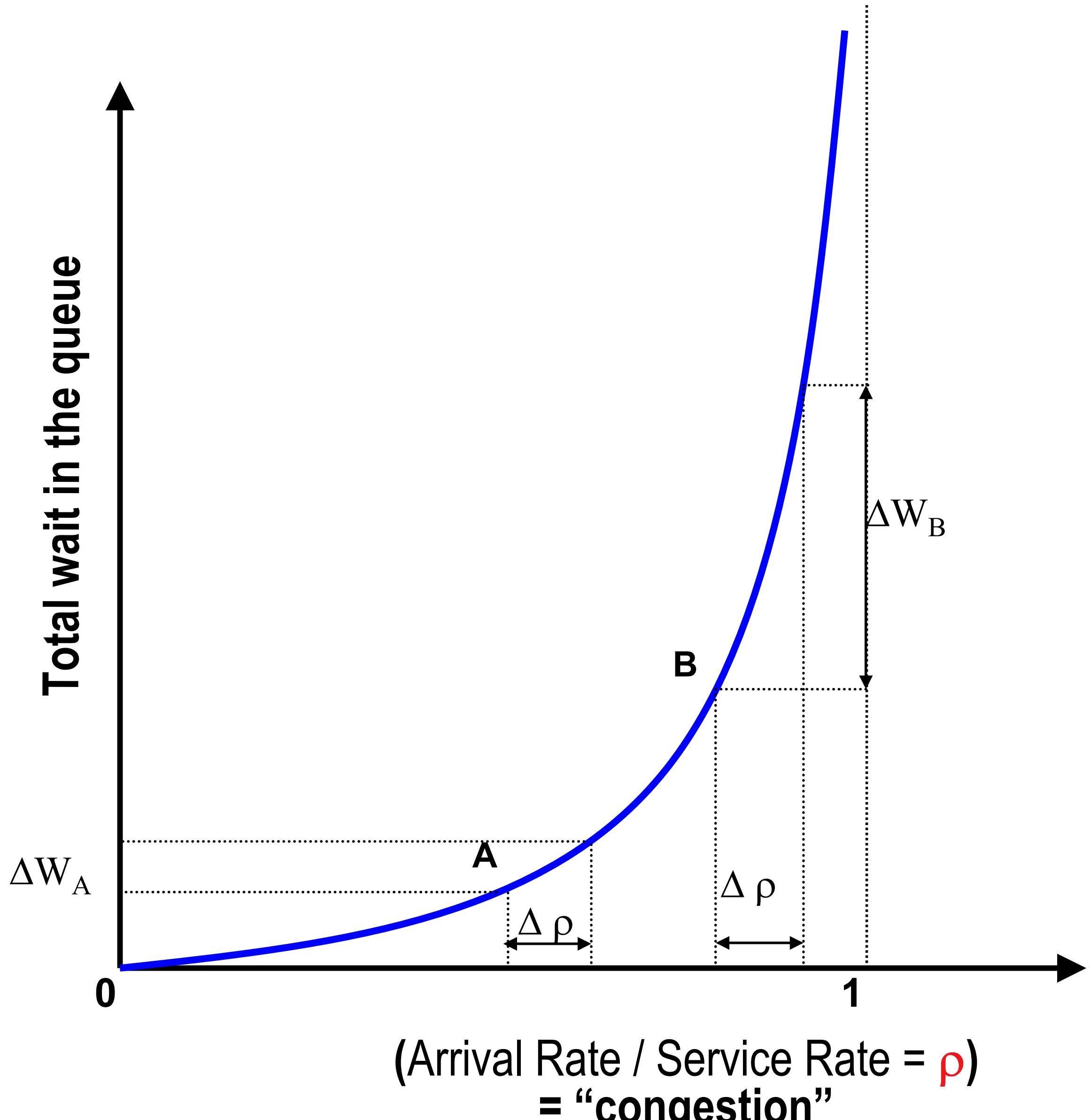
600 MBA's = 300/year x 2 years

Basic Concepts in Queueing: Nonlinearities in Congestion in Stochastic Systems

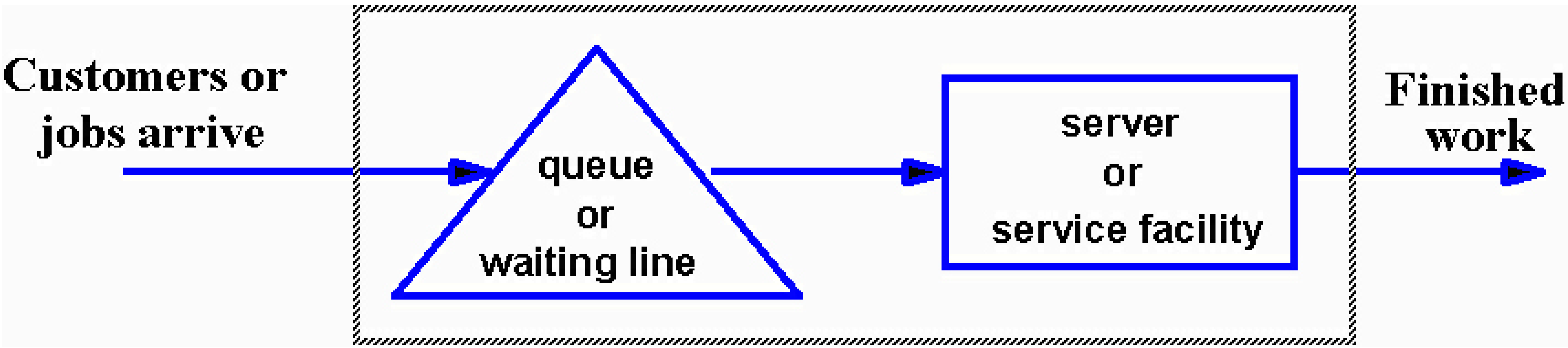
If service times and interarrival times have exponential distributions, then

$$L = \rho^2/(1-\rho)$$

$$W = \rho^2/\lambda(1-\rho)$$



Basic Concepts in Queueing: Nonlinearities in Congestion in Stochastic Systems



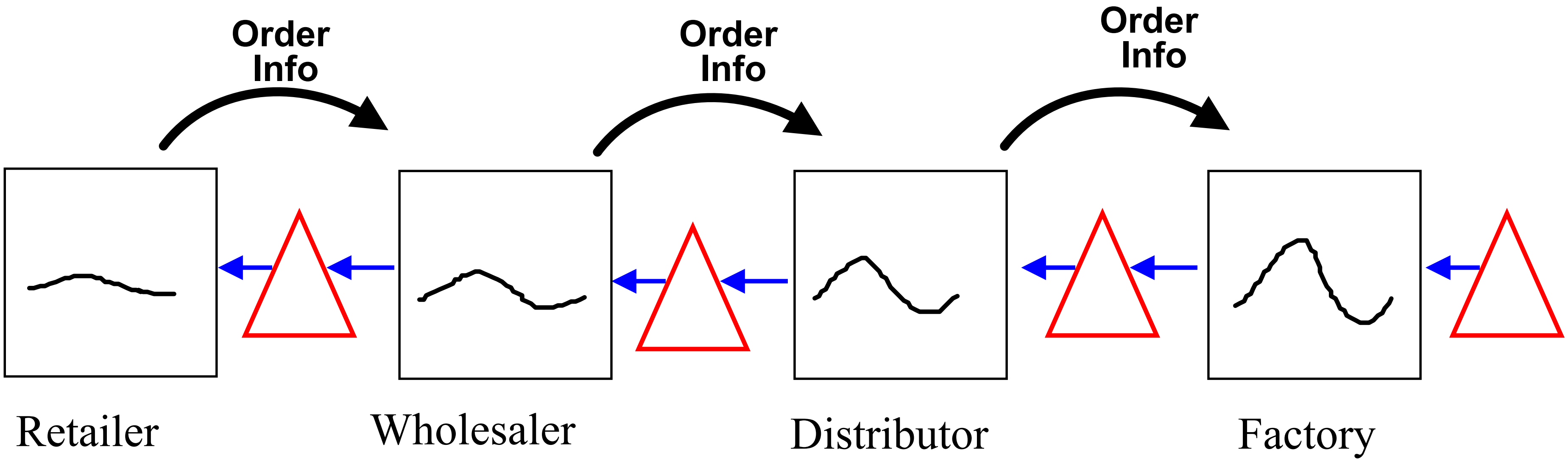
System Performance = f(System parameters)

With exponential (λ) interarrivals, and service times with mean = M and std dev = σ,

Then

$$W = \frac{\lambda (M^2 + \sigma^2)}{2(1-\rho)}$$

Volatility Amplification in the Supply Chain:
"The Bullwhip Effect"



How does production control work in the Beer Game?

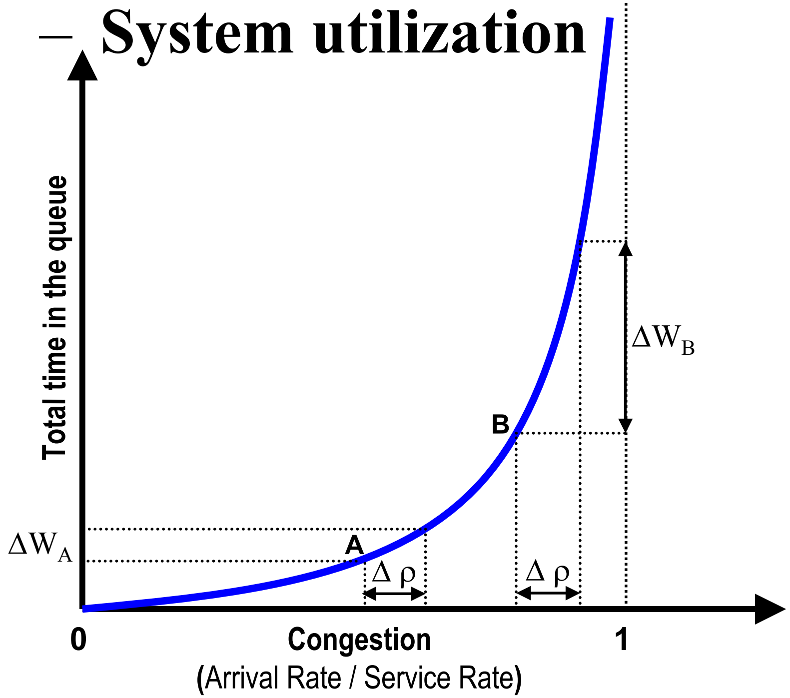
Information lags
Delivery lags
Over- and underordering
Misperceptions of feedback
Lumpiness in ordering
Chain accumulations

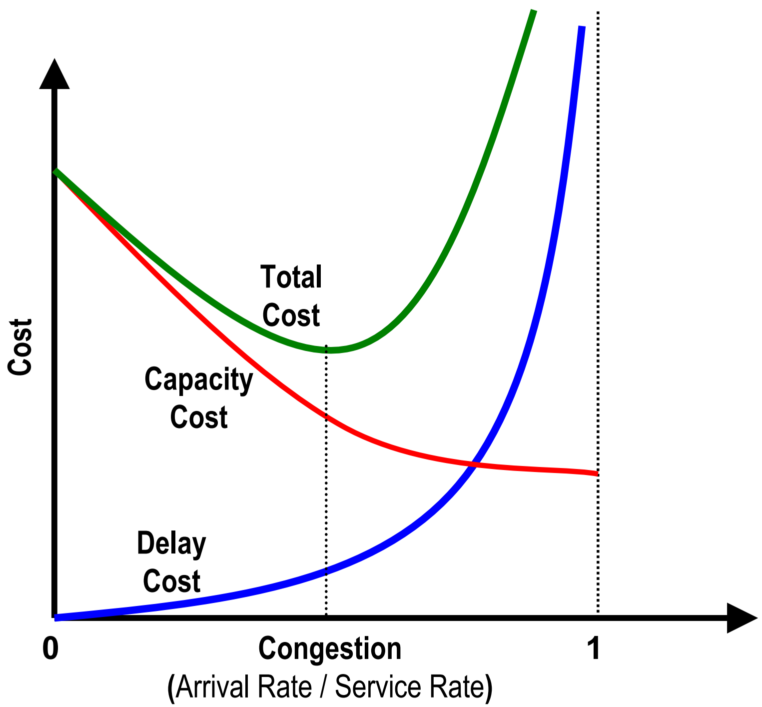
SOLUTIONS:
Countercyclical Markets
Countercyclical Technologies
Collaborative channel mgmt.
(Cincinnati Milacron & Boeing)

Management of Queues

The Physics of Waiting Lines

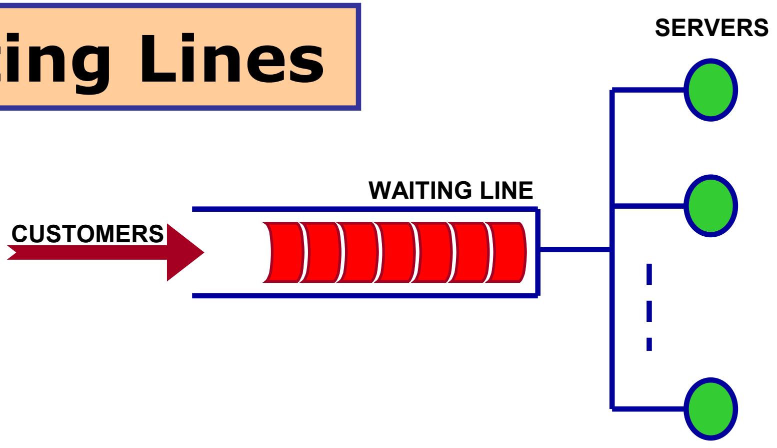
– Number and type of servers
– Waiting time, service time, and system time
– Queue discipline
– Number of people in queue
– System utilization





Management of Queues

The Psychology of Waiting Lines



Propositions

1. Unoccupied time feels longer than occupied time
2. Process waits feel longer than in process waits
3. Anxiety makes waits seem longer
4. Uncertain waits seem longer than known, finite waits
5. Unexplained waits are longer than explained
6. Unfair waits are longer than equitable waits
7. The more valuable the service, the longer the customer will wait
8. Solo waits feel longer than group waits