Marketable Securities, Time Value of Money

15.501/516 **Accounting**
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Professor S. Roychowdhury
Sloan School of Management
Massachusetts Institute of Technology

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Marketable Securities

- Trading securities (debt and equity)
  - Acquired for short-term profit potential
  - Changes in market value reported in the income statement (net of taxes), investment marked to market in the balance sheet
  - Purchases and disposals reported in operating section of SCF
- Held to maturity (debt only)
  - Acquired with ability and intent to hold to maturity
  - No changes in market value reported in the income statement, thus investment carried at historical cost in the balance sheet
  - Interest income reported in operating section of SCF
- Available for sale (debt and equity)
  - Securities not classified as either of above
  - Changes in market value reported in "**Other Equity**" (net of taxes), instead of the income statement!
  - Purchases and disposals reported in investing section of SCF

2

What Is "Other Equity"?

- So far, what have we seen in class?
- Stockholders' Equity (SE) =
  Contributed capital (CC) + Retained Earnings (RE)
- The above is a simplification! It is known as the "Clean Surplus Equation".
- In fact, Clean Surplus is often violated
- SE = CC + RE + Other Equity
- What causes changes in "Other Equity"?
  - Changes in the market value of AFS securities, for one!

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An Illustration: Acquisition and Dividends (2002)

On Jan. 1, 2002, Ace acquired 500 common shares of security MITCo for $25/share.

	C	MS	$+MS_{adj}$	=	DTL	OE	RE
Trading:							
AFS:							

"Adjunct" account

On Nov. 30, 2002, Ace received $625 in dividends ($1.25/share of MITCo)

	C	MS	$+MS_{adj}$	=	DTL	OE	RE
Trading:							
AFS:							

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An Illustration: Acquisition and Dividends (2002)

On Jan. 1, 2002, Ace acquired 500 common shares of security MITCo for $25/share.

	C	MS	$+MS_{adj}$	=	DTL	OE	RE
Trading:	(12,500)	12,500					
AFS:	Same as above						

On Nov. 30, 2002, Ace received $625 in dividends ($1.25/share of MITCo)

	C	MS	$+MS_{adj}$	=	DTL	OE	RE
Trading:							
AFS:							

5

An Illustration: Acquisition and Dividends (2002)

On Jan. 1, 2002, Ace acquired 500 common shares of security MITCo for $25/share.

	C	MS	$+MS_{adj}$	=	DTL	OE	RE
Trading:	(12,500)	12,500					
AFS:	Same as above						

On Nov. 30, 2002, Ace received $625 in dividends ($1.25/share of MITCo)

	C	MS	$+MS_{adj}$	=	DTL	OE	RE
Trading:	625						625
AFS:	Same as above						

"Investment income" on I/S

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Unrealized Gains and Losses (2002)

On Dec. 31, 2002, MITCo is trading at $30/share. Ace elected to keep the shares, and has a tax rate of 30%.

Trading:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500					
EB:							

AFS:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500					
EB:							

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Unrealized Gains and Losses (2002)

On Dec. 31, 2002, MITCo is trading at $30/share. Ace elected to keep the shares, and has a tax rate of 30%.

Trading:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500					
			2,500				2,500
EB:							

($30 – 25) x 500 "Investment income" on I/S

AFS:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500					
EB:							

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Unrealized Gains and Losses (2002)

On Dec. 31, 2002, MITCo is trading at $30/share. Ace elected to keep the shares, and has a tax rate of 30%.

Trading:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500					
			2,500				2,500
					750		(750)
EB:							

2,500 x 0.3 "Income tax expense" on I/S

AFS:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500					
EB:							

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Unrealized Gains and Losses (2002)

On Dec. 31, 2002, MITCo is trading at $30/share. Ace elected to keep the shares, and has a tax rate of 30%.

Trading:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500					
			2,500				2,500
					750		(750)
EB:		**12,500**	**2,500**		**750**		
AFS:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500					
EB:							10

Unrealized Gains and Losses (2002)

On Dec. 31, 2002, MITCo is trading at $30/share. Ace elected to keep the shares, and has a tax rate of 30%.

Trading:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500					
			2,500				2,500
					750		(750)
EB:		**12,500**	**2,500**		**750**		
AFS:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500					
			2,500		750	1,750	
EB:						No I/S effect	11

Unrealized Gains and Losses (2002)

On Dec. 31, 2002, MITCo is trading at $30/share. Ace elected to keep the shares, and has a tax rate of 30%.

Trading:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500					
			2,500				2,500
					750		(750)
EB:		**12,500**	**2,500**		**750**		
AFS:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500					
			2,500		750	1,750	
EB:		**12,500**	**2,500**		**750**	**1,750**	12

Unrealized Gains and Losses (2003)

On Dec. 31, 2003, MITCo is trading at $27/share. Ace elected to keep the shares.

Trading:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500	2,500		750		
EB:							
AFS:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500	2,500		750	1,750	
EB:							

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Unrealized Gains and Losses (2003)

On Dec. 31, 2003, MITCo is trading at $27/share. Ace elected to keep the shares.

Trading:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500	2,500		750		
			(1,500)				**(1,500)**
EB:							
AFS:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500	2,500		750	1,750	
EB:							

($27 - $30) x 500
"Investment income" on I/S

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Unrealized Gains and Losses (2003)

On Dec. 31, 2003, MITCo is trading at $27/share. Ace elected to keep the shares.

Trading:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500	2,500		750		
			(1,500)				**(1,500)**
					(450)		**450**
EB:							
AFS:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500	2,500		750	1,750	
EB:							

($1,500) x 0.30 "Income tax
benefit" on I/S

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Unrealized Gains and Losses (2003)

On Dec. 31, 2003, MITCo is trading at $27/share. Ace elected to keep the shares.

Trading:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500	2,500		750		
			(1,500)				**(1,500)**
					(450)		**450**
EB:		**12,500**	**1,000**		**300**		
AFS:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500	2,500		750	1,750	
EB:							

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Unrealized Gains and Losses (2003)

On Dec. 31, 2003, MITCo is trading at $27/share. Ace elected to keep the shares.

Trading:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500	2,500		750		
			(1,500)				**(1,500)**
					(450)		**450**
EB:		**12,500**	**1,000**		**300**		
AFS:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500	2,500		750	1,750	
			(1,500)		**(450)**	**(1,050)**	
EB:							

No I/S effect

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Unrealized Gains and Losses (2003)

On Dec. 31, 2003, MITCo is trading at $27/share. Ace elected to keep the shares.

Trading:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500	2,500		750		
			(1,500)				**(1,500)**
					(450)		**450**
EB:		**12,500**	**1,000**		**300**		
AFS:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500	2,500		750	1,750	
			(1,500)		**(450)**	**(1,050)**	
EB:		**12,500**	**1,000**		**300**	**700**	

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Realized Gains and Losses (2004)

On Feb. 14, 2004, Ace sold all of its investment in MITCo, then trading at $36/share.

Trading:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500	1,000		300		
EB:							

AFS:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500	1,000		300	700	
EB:							

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Realized Gains and Losses (2004)

On Feb. 14, 2004, Ace sold all of its investment in MITCo, then trading at $36/share.

Trading:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500	1,000		300		
	18,000	(12,500)	(1,000)				4,500
EB:							

$36 x 500

Remove existing accounts

"Plug" that equals ($36 - $27) x 500 "Investment income" on I/S

AFS:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500	1,000		300	700	
EB:							

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Realized Gains and Losses (2004)

On Feb. 14, 2004, Ace sold all of its investment in MITCo, then trading at $36/share.

Trading:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500	1,000		300		
	18,000	(12,500)	(1,000)				4,500
	(1,650)				(300)		(1,350)
EB:							

Pay tax on the full gain ($36 - $25) x 500 x 0.30

Recognize tax expense for gain in this year's I/S, $4,500 x 0.30

AFS:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500	1,000		300	700	
EB:							

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Realized Gains and Losses (2004)

On Feb. 14, 2004, Ace sold all of its investment in MITCo, then trading at $36/share.

Trading:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500	1,000		300		
	18,000	(12,500)	(1,000)				4,500
	(1,650)				(300)		(1,350)
EB:							

AFS:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500	1,000		300	700	
	18,000	(12,500)	(1,000)		(300)	(700)	5,500
EB:							

$36 x 500

Remove existing accounts

"Plug" that equals ($36 - $25) x 500 "Investment income" on I/S

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Realized Gains and Losses (2004)

On Feb. 14, 2004, Ace sold all of its investment in MITCo, then trading at $36/share.

Trading:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500	1,000		300		
	18,000	(12,500)	(1,000)				4,500
	(1,650)				(300)		(1,350)
EB:							

AFS:	C	MS	+MSadj	=	DTL	OE	RE
BB:		12,500	1,000		300	700	
	18,000	(12,500)	(1,000)		(300)	(700)	5,500
	(1,650)						(1,650)
EB:							

Pay tax on the full gain ($18,000 - $12,500) x 0.30

Recognize tax expense for the gain in this year's I/S, $5,500 x 0.30

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Marketable Securities: Income patterns

Trading	*Available for Sale*

5,500
4,500
2,500
-1,500

02   03   04

5,500
4,500
2,500
-1,500

02   03   04

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Marketable Securities: Income patterns

| *Trading* | *Available for Sale* |
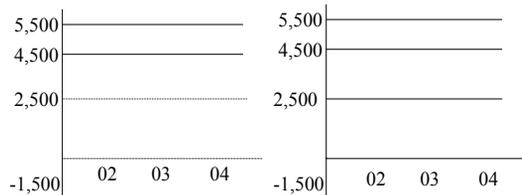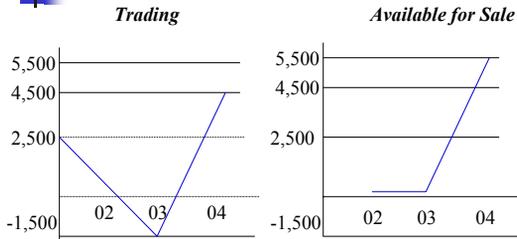
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Marking to market

- For both Trading and AFS securities
  - Book value of security investments tracks market value
  - Balance sheet effects are the same! Marketable Securities and *total* Stockholders Equity will be the same.
- What is different?
  - Income statement effects
  - For trading securities, gains or losses recognized in Income Statement track changes in market value
  - For AFS securities, gains or losses net of taxes are cumulatively recorded in "Other Equity" till securities are sold
  - At that time, all gains and losses are recognized in Income Statement and the "Other Equity" account as well as "Deferred Tax Liability" account is cleared off.

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Reclassifications of Marketable Securities

- Trading to Available for sale
  - Gains or losses of the period recognized on reclassification date
  - Subsequent market value changes reported in "Other Equity"

- Available for sale to Trading
  - Cumulative gains or losses, including those of current period, recognized on reclassification date
  - Subsequent market value changes reported in the income statement

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Why does recognition of gains/losses matter?

Former SEC Chairman Breeden, on mark-to-market (ca 1990):
If you are in a volatile business, then your balance sheet and income statement should reflect that volatility. Furthermore, we have seen significant abuse of managed earnings. Too often companies buy securities with an intent to hold them as investments, and then miraculously, when they rise in value, the companies decide it's time to sell them. Meanwhile, their desire to hold those securities that are falling in value grows ever stronger. So companies report the gains and hide the losses.

Current SEC Chairman Arthur Levitt, Jr (1997):
it is unacceptable to allow American investors to remain in the dark about the consequences of a $23 trillion derivatives exposure. We support the independence of the FASB as they turn on the light.

Federal Reserve Chairman Greenspan, on derivatives (ca 1997):
Putting the unrealized gains and losses of open derivatives contracts onto companies' income statements would introduce "artificial" volatility to their earnings and equity. Shareholders would become confused; management might forego sensible hedging strategies out of purely window dressing concerns.

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A compromise in GAAP?

- Recognize all unrealized gains/losses for "trading securities" in Net Income

- Mark "available for sale" securities to market value, but don't report changes in the income statement
  - Reduces earnings volatility
  - Managers dislike income volatility
    - They complain similarly about other accounting method changes that increase reported earnings volatility even though underlying cash flows are unaffected

- Ignore value changes for "held to maturity" category

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Marketable Securities in other countries

- Canada: LCM for investments classified as current assets; historical cost for noncurrent assets, but recognize "permanent" declines in value
- Mexico: Carry marketable securities at net realizable value, report gains/losses in the income statement; LCM for other investments
- Japan: LCM for marketable securities
- Others: Typically either LCM or mark-to-market, exclusively
- International Accounting Standards: Similar to US GAAP

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Summary

- Valuation adjustment necessary when changes in market values are objectively measurable
- Lower of cost or market applied to inventory valuation
- New GAAP in marketable securities: mark-to-market treats gains and losses equally
- Disclosure vs. Recognition in mark-to-market accounting:
    - Not all gains and losses are reported in the income statement
- A compromise!

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LIABILITIES: Current Liabilities

- Obligations that must be discharged in a short period of time (generally less than one year)
- Reported on balance sheet at **nominal value**
- Examples:
    - Accounts payable
    - Short-term borrowings
    - Current portion of long-term debt
    - Deposits
    - Warranties
    - Deferred Revenues / Income

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LIABILITIES: Long-term Liabilities

- Obligations spanning a longer period of time (generally more than one year)
- Generally reported on the balance sheet at **present value** based on interest rate when initiated
- Examples:
    - Bonds
    - Long-term loans
    - Mortgages
    - Capital Leases
- How do we compute present values? And interest expense?

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Time Value Of Money - Refresher

Interest = 10%

Time 1
$1.00

Time 0
$1.00

Time 1
$1.10
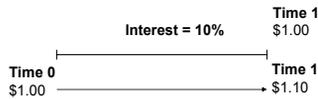
Future value of $ 1.00 today = $1.00 (1+10%) = $1.10 at the end of one year.

What is the present value of $1.10 to be received one year from now?

Present value of $1.10 one year from now = $1.10/(1+10%) = $1.00

What is the present value of $1.00 to be received one year from now?

Present value of $1.00 one year from now = $1.00/(1.10) = $0.91

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Time Value Of Money – Refresher

$0.83 ← $0.91 ← $1.00

T=0   T=1   T=2
$1.00 $1.10 $1.21
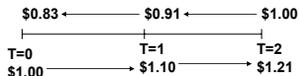
Future value of $1.00 two years from now = $1.00*(1+10%)*(1+10%)
= $1.00*(1.10)^2 = $1.21

Present value of $1.00 to be received two years from now
= $1.00/[(1.10)^2] = $0.83

RECALL: PV of $1.00 to be received a year from now = $0.91

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Calculating present values: An example

You have just won a lottery. The lottery board offers you three different options for collecting your winnings:

(1) Payments of $500,000 at the end of each year for 20 years.

(2) Lump-sum payment of $4,500,000 today.

(3) Lump-sum payment of $1 million today, followed by $2,100,000 at the end of years 5, 6, and 7.

Assume all earnings can be invested at a 10 percent annual rate. Ignoring any tax effects, which option should you choose and why?

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Future Value of Option 1:
$500,000 at the end of each year for 20 years.
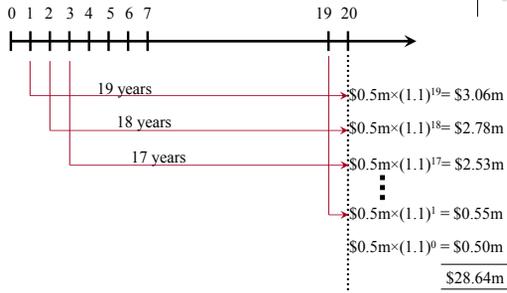
0 1 2 3 4 5 6 7   19 20

19 years $0.5m \times (1.1)^{19} = \$3.06m$

18 years $0.5m \times (1.1)^{18} = \$2.78m$

17 years $0.5m \times (1.1)^{17} = \$2.53m$

$0.5m \times (1.1)^{1} = \$0.55m$

$0.5m \times (1.1)^{0} = \$0.50m$

$\$28.64m$

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Future Value of Option 2:
Lump-sum payment of $4,500,000 today

0 1 2 3 4 5 6 7   19 20

20 years $4.5m \times (1.1)^{20} = \$30.27m$

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Future Value of Option 3: $1m today, and $2.1m at the end of years 5, 6, and 7.

0 1 2 3 4 5 6 7   19 20

20 years $1.0m \times (1.1)^{20} = \$3.36m$

15 years $2.1m \times (1.1)^{15} = \$8.77m$

14 years $2.1m \times (1.1)^{14} = \$7.97m$

13 years $2.1m \times (1.1)^{13} = \$7.25m$

$\$30.71m$

39
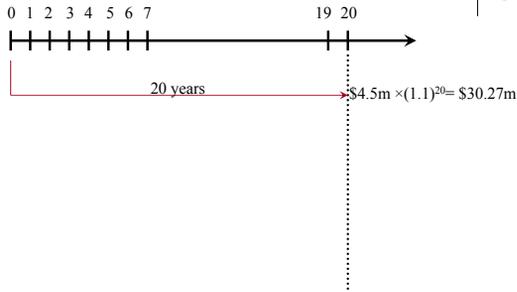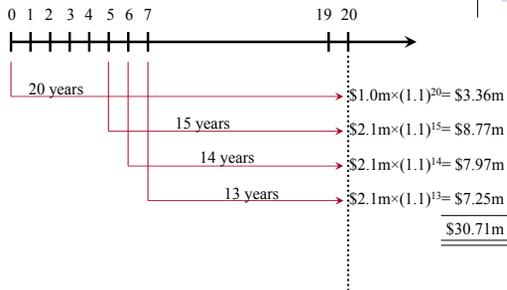
Future Values

- If you invest all lottery receipts at 10% peryear, how much will you have in 20 years?

  1. $500K $\times$ (1.10)^{19} + $500K $\times$ (1.10)^{18} + ...
     ... + $500K $\times$ (1.10)^{1} + $500K = $28.64m

  2. $4,500,000 $\times$ (1.10)^{20} = $30.27m

  3. $1m $\times$ (1.10)^{20} + $2.1m $\times$ (1.10)^{15} +
     $2.1m $\times$ (1.10)^{14} + $2.1m $\times$ (1.10)^{13} = $30.71m

  → FV(Option 1) < FV(Option 2) < FV(Option 3)

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Present Value of Option 1: $500,000 at the end of each year for 20 years.

```
0  1  2  3  4  5  6  7        19 20
+--+--+--+--+--+--+--+--...--+--+-->
```

$0.5m×(1.1)$^{-1}$ = $0.45m ← 1 year

$0.5m×(1.1)$^{-2}$ = $0.41m ← 2 years

$0.5m×(1.1)$^{-3}$ = $0.38m ← 3 years

$0.5m×(1.1)$^{-19}$ = $0.08m ← 19 years

$0.5m×(1.1)$^{-20}$ = $0.07m ← 20 years

$4.26m
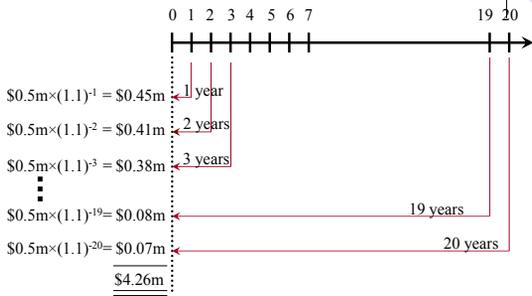
41

Present Value of Option 2:
Lump-sum payment of $4,500,000 today

```
0  1  2  3  4  5  6  7        19 20
+--+--+--+--+--+--+--+--...--+--+-->
```

$4.5m

42

Present Value of Option 3: $1m today, and $2.1m at the end of years 5, 6, and 7.

```
                0  1  2  3  4  5  6  7              19 20
                |--+--+--+--+--+--+--|---...---|----+--+-->
```

$\$1.0m \times (1.1)^{0} = \$1.00m$

$\$2.1m \times (1.1)^{-5} = \$1.30m$ — 5 years

$\$2.1m \times (1.1)^{-6} = \$1.19m$ — 6 years

$\$2.1m \times (1.1)^{-7} = \$1.08m$ — 7 years

$\underline{\$4.57m}$

43
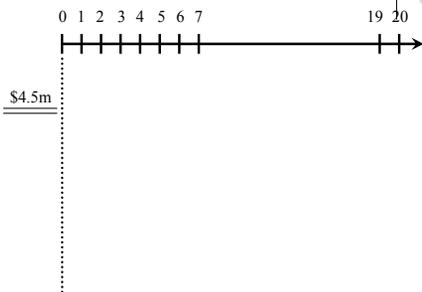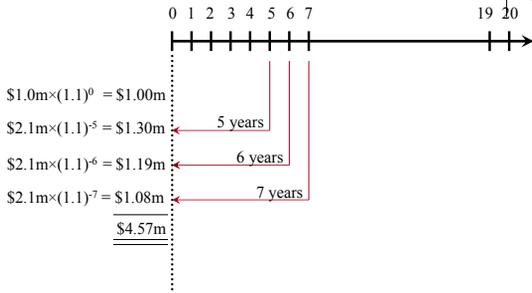
Present Values

- If all lottery receipts can be invested at 10% per year, what is the *present value* of each option?

1. $\$500K \times (1.10)^{-20} + \$500K \times (1.10)^{-19} + \ldots$
   $\ldots + \$500K \times (1.10)^{-2} + \$500K \times (1.10)^{-1} = \$4.26m$

2. $\$4,500,000 \times (1.10)^{0} = \$4.5m$

3. $\$1m \times (1.10)^{0} + \$2.1m \times (1.10)^{-5} +$
   $\$2.1m \times (1.10)^{-6} + \$2.1m \times (1.10)^{-7} = \$4.57m$

➔ PV(Option 1) < PV(Option 2) < PV(Option 3)

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Converting Present and Future Values

```
              0  1  2  3  4  5  6  7          19 20
Present       |--+--+--+--+--+--+--|---...---|--+-->    Future
Values                                                  Values
```

Present Values		Future Values
Option 1 $4.26m	$FV = 4.26 \times 1.1^{20} = 28.64$	Option 1 $28.64m
	$PV = 28.64 \times 1.1^{-20} = 4.26$	
Option 2 $4.50m	$FV = 4.50 \times 1.1^{20} = 30.27$	Option 2 $30.27m
	$PV = 30.27 \times 1.1^{-20} = 4.50$	
Option 3 $4.57m	$FV = 4.57 \times 1.1^{20} = 30.71$	Option 3 $30.71m
	$PV = 30.71 \times 1.1^{-20} = 4.57$	

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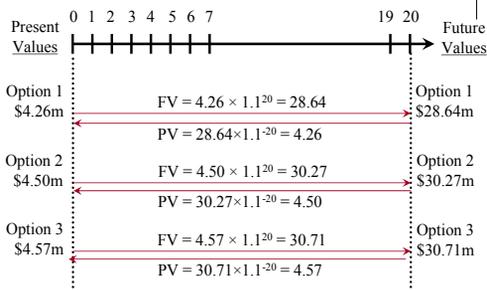
Using PV and FV Tables

- Tbl 1: Future Value of $1
  - A one-time payment to be received now and held (reinvested) for n periods
  - Compounded at interest rate r
  - Multiply the dollar amount received by the factor in Row n, Column r
  - Formula: $FV(\$1) = (1+r)^n$

Pratt, Jamie. *Financial Accounting in an Economic Context.* 5th ed. John Wiley & Sons, 2003. p. 705.

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Using PV and FV Tables

- Tbl 4: Present Value of $1
  - A one-time payment to be received n periods from now
  - Discounted at interest rate r
  - Multiply the dollar amount to be received by the factor in Row n, Column r
  - Formula: $PV(\$1) = 1/(1+r)^n$
    $= (1+r)^{-n}$

Pratt, Jamie. *Financial Accounting in an Economic Context.* 5th ed. John Wiley & Sons, 2003. p. 708.

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Some PV Terminology

- Annuity: a stream of fixed-dollar payments made at regular intervals of time
  - *Ordinary Annuity* (annuity in arrears): payments occur at the <u>end</u> of the period
  - *Annuity due* (annuity in advance): payments occur at the <u>beginning</u> of the period

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Using PV and FV Tables

- Tbl 2: <u>Future</u> Value of $1 <u>ordinary</u> annuity
  - Regular payments to be received at end of year for n years and held (reinvested) until time n
  - Compounded at interest rate r
  - Multiply the dollar amount received by the factor in Row n, Column r

Pratt, Jamie. *Financial Accounting in an Economic Context*. 5th ed. John Wiley & Sons, 2003. p. 706.

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Using PV and FV Tables

- Tbl 3: <u>Future</u> Value of $1 <u>annuity due</u>
  - Regular payments to be received at beginning of year for n years and held (reinvested) until time n
  - Compounded at interest rate r
  - Multiply the dollar amount received by the factor in Row n, Column r
  - Table is redundant

Pratt, Jamie. *Financial Accounting in an Economic Context*. 5th ed. John Wiley & Sons, 2003. p. 707.

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Using PV and FV Tables

- Tbl 5: Present Value of $1 ordinary annuity
  - Regular payments to be received at end of year for n years
  - Compounded at interest rate r
  - Multiply the dollar amount received by the factor in Row n, Column r

Pratt, Jamie. *Financial Accounting in an Economic Context.* 5th ed. John Wiley & Sons, 2003. p. 709.

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Using PV and FV Tables

- Tbl 6: <u>Present</u> Value of $1 <u>annuity due</u>
  - Regular payments to be received at beginning of year for n years
  - Compounded at interest rate r
  - Multiply the dollar amount received by the factor in Row n, Column r
  - Also redundant

Pratt, Jamie. *Financial Accounting in an Economic Context*. 5th ed. John Wiley & Sons, 2003. p. 710.

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Annuity Formulas

$$\text{Sum of infinite series}: \sum_{n=1}^{\infty} \frac{1}{k^n} = \frac{1}{k-1} \ e.g., \ \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ... = 1$$

$$\text{PV of a perpetuity}: \sum_{n=1}^{\infty} \frac{1}{(1+r)^n} = \frac{1}{(1+r)-1} = \frac{1}{r}$$

A perpetuity is a constant amount received at the end of each period.

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Present Values of Lottery Options - Revisited

- If all lottery receipts can be invested at 10% per year, what is the *present value* of each option?

  - $500K × PVFOrdinaryAnnuity$_{20 \text{ years, } 10\%}$ = $4.26m

  - $4,500,000 × PVF$_{0 \text{ years, } 10\%}$ = $4.5m

  - $1m × PVF$_{0 \text{ years, } 10\%}$ + Present Value of ordinary annuity of $2.1m for three years starting Year 4
    = $1m +
         ($2.1m x PVFOrdinaryAnnuity$_{3 \text{ years, } 10\%}$)
                                                         x PVF$_{4 \text{ years, } 10\%}$
    = $1m + (2.1 x 2.48685) x 0.68301
    = $4.57m

  - PV(Option 1) < PV(Option 2) < PV(Option 3)

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Problem With Time Value of Money

- The Croziers want to provide for three-year-old son Ryan's future college education. They estimate that it will cost $40,000/year for four years when Ryan enters college 15 years from now. Assuming a 10% interest rate, and college payments due at the start of each year:

    (a) How much would the parents have to invest today?

    (b) How much would they have to invest at the end of each year for the next 14 years?

    (c) How would your answers change if the expected rate of return is only 8% per year?

Part (a): How much would the parents have to invest today?

- Step 1: Calculate how much money they will need 15 years from now.
    - PV_{15} = $40,000 × $PVFactorAnnuityDue_{4yrs, 10\%}$
        = $40,000 × 3.4869
        = $139,476
- Step 2: Calculate the present value of the amount from Step 1.
    - PV_0 = $139,476 × $PVFactor_{15yrs, 10\%}$
        = $139,476 × 0.2394
        = $33,389

Part (b): How much would they have to invest at the end of each year for the next 14 years?

- The PV of the annuity must equal the PV calculated in part (a).
    - $33,389 = Annuity Payments × $PVFactorAnnuity_{14yrs, 10\%}$
        = Annuity Payments × 7.3667
    - Annuity Payments = $4,532

Part (c): How would your answers change if the expected rate of return is only 8%p/a?

- Step 1: Calculate how much money they will need 15 years from now.
  - PV_{15} = $40,000 × PVFactorAnnuityDue$_{4yrs, 8\%}$
    = $40,000 × 3.5771 = $143,084
- Step 2: Calculate the PV at time 0.
  - PV_0 = $143,084 × PVFactor$_{15yrs, 8\%}$
    = $143,084 × 0.3152 = $45,106
- The PV of the annuity must equal $45,106
  - $45,106 = Annuity Pmts × PVFA$_{14yrs, 8\%}$
    = Annuity Pmts × 8.2442
  - Annuity Pmts = $5471

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