15.433 INVESTMENTS

Class 14: The Fixed Income Market

Part 2: Time Varying Interest Rates and Yield Curves

Spring 2003

Time-Varying Interest Rates
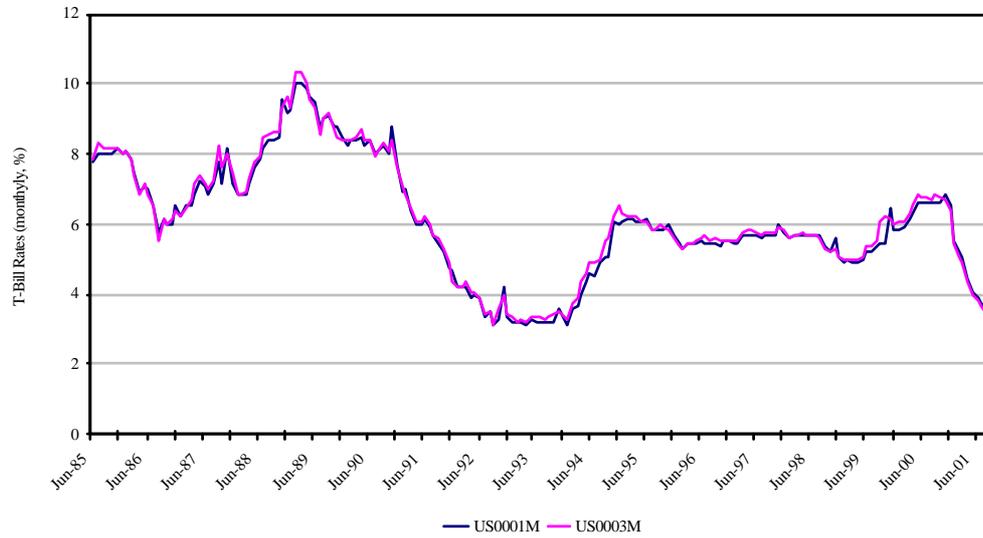

Figure 1: Time varying interest rates, Source: Bloomberg.

A Model for Stochastic Interest Rates

Let r_t be the time-t one-period interest rate:

$$r_{t+1} = r_t = k\left(\bar{r} - r_t\right) + \sigma \cdot \varepsilon_{t+1} \tag{1}$$

To be consistent with our earlier notation, $r_t = r_{t,1}$!

Important:

- ε_t is the random shock that occurs at time t.

- The shocks follow a standard normal distribution.

- The shocks are independent across time.

- k, r, and σ are constant coefficients.

• The Coefficients and the Moments

r is the long-run mean of the interest rate.

$$E\left(r_t\right) = ? \tag{2}$$

σ is related to the volatility of the interest rate.

$$var(r_t) = ? \tag{3}$$

k captures the rate at which the interest rate reverts to its long-run mean, $\bar{r}, 0 < k <:$

$$cov(r_t, r_{t+1}) = ? \tag{4}$$

The Risk of Rolling Over

At time t, let r_t be zero-coupon interest rate with maturity n. Consider the value of dollar by following the two investment strategies:

1. Long-term: invest in the (2) two-period bond.

$$e^{2 \cdot r_{t,2}} \tag{5}$$

2. Short term: invest in the (1) one-period bond and roll over at time t + 1.

$$e^{r_{t,1} \cdot r_{t+1,1}} \tag{6}$$

The long-term investment is riskless, while the short-term investment is risky. In particular, it is exposed to the random shock ε_t that is unknown at time t.

Implication for the Yield Curve

The time-t value of $ 1 to be collected at t + 2:

1. The short-term strategy:

$$E\left(e^{-[r_{t,1} \cdot r_{t+1,1}]}\right) \tag{7}$$

2. The long-term strategy:

$$e^{-2 \cdot r_{t,2}} \tag{8}$$

If investors are risk-neutral, then they are indifferent between the two strategies, implying

$$r_{t,2} - r_{t,1} = \frac{1}{2}k\left(\bar{r} - r_{t,1}\right) - \frac{1}{4}\sigma^2 \tag{9}$$

When do we have an upward/downward sloping term structure?
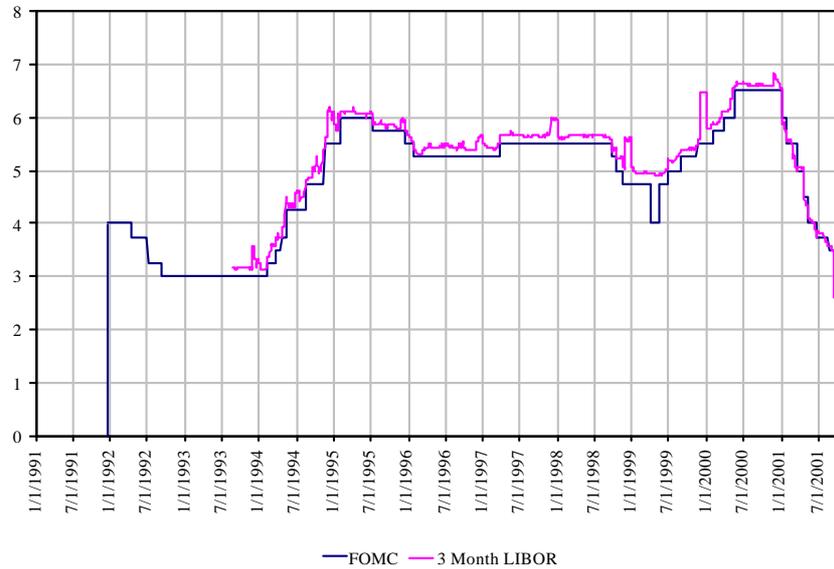
Term Structure of Interest Rates

Figure 2: Term-structure of Fed-Rates and 3-month Libor-interest rates, source: Bloomberg Professional.
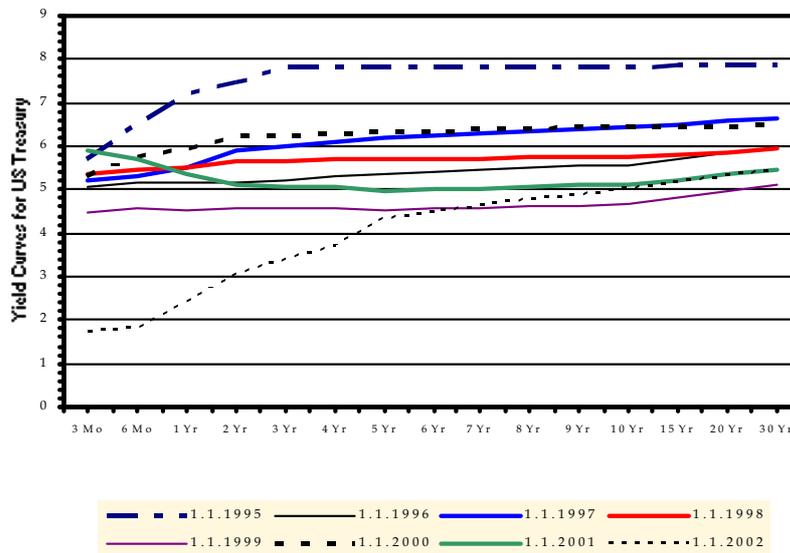

Figure 3: Term-structure of interest rates across time, source: Bloomberg Professional.

Forward Rates

A forward interest rate is the interest rate implied by current zero rates for a specified future time period.

Let r_{t,n_1} and r_{t,n_2} be the zero rates for maturities n_1, and n_2, respectively. The forward rate for the period of time between $t + n_l$ and $t + n_2$:

$$f_t^{n_1,n_2} = \frac{n_2 \cdot r_{t,n_2} - n_1 {}_{t,n_1}}{n_2 - n_1} \tag{10}$$

It is important to note that the forward rate $f_t^{n_1,n_2}$ is known at time t.

Yield Curve and Expectations Hypothesis

The expectations hypothesis can be stated in the following two equivalent ways:

1. The N-period yield is the average of expected future one-period yields:

$$r_{t,N} = \frac{1}{N} \cdot E_t \left(r_{t,(1)} + r_{t+1,(1)} + \cdots + r_{t+N-1,(1)} \right) \qquad (11)$$

2. The forward rate equals the expected future spot rate:

$$f_t^{N,N+1} = E_t \left(r_{t+N,(1)} \right) \qquad (12)$$

Implications: a positive yield spread must be associated with rising interest rates.

Liquidity Preference and Yield Curve

Investors prefer to preserve their liquidity and invest funds for short periods of time.

Borrowers, on the other hand, usually prefer to borrow at fixed rates for long periods of time.

In practice, to match depositors with borrowers and avoid interest rate risk, financial intermediaries raise long-term interest rates relative to expected future short-term interest rates.

This strategy reduces the demand for long-term fixed-rate borrowing and encourages investors to deposit their funds for the long-term.

Market Segmentation and Yield Curve

Different institutions invest in bonds of different maturities and do not switch maturities.

The short-term interest rate is determined by supply and demand in the short-term bond market; the medium-term interest rate is determined by supply and demand in the medium-term bond market; ...

There need be no relationship between short-, medium-, and long-term interest rates.

Repo Market

Repurchase agreement - a sale of a security with a commitment to buy the security back at a specified price at a specified date.

- overnight repo (1 day)

- term repo (longer)

Repo Example: You are a dealer and you need $ 10 mio. to purchase some security.

Your customer has $ 10 mio. in his account with no use. You can offer your customer to buy the security for you and you will repurchase the security from him tomorrow. Repo rate 6.5%.

Compute: What is the profit for the client allowing such a transaction?

Solution: Then your customer will pay $ 9'998'185 for the security and you will return him $ 10 mio. tomorrow.

$$\$9'998'185 \cdot \frac{0.065}{360} = \$1'805 \tag{13}$$

This is the profit of your customer for offering the loan.

Note that there is almost no risk in the loan since you get a safe security in exchange.

Focus:

BKM Chapter 15

- p. 456-460

- p. 461-469

Style of potential questions: Concept check questions, p. 474 ff. question 8, 10, 14

Preparation for Next Class

Please read:

- BKM Chapter 16, 22 & 23.