15.063 Communicating with Data
Summer 2003

Solutions to Homework Assignment #2

Issued: Lecture 7. **Due: Lecture 11, before Lecture.**

The Exercises are from the book, *Data, Models, and Decisions: The Fundamentals of Management Science* by Dimitris Bertsimas and Robert M. Freund, Southwestern College Publishing, 2000.

Problem 2.22: Selling Umbrellas

Solution:

Let U1 be the umbrellas sold at the department store, U2 be the umbrellas sold at the outlet, and S be the total sales revenue. Then, S = 17 U1 + 9 U2. The expectation can be computed as:

E(S) = 17 x 147.8 + 9 x 63.2 = $3,081,

and the variance as

VAR(S) = 289 VAR(U1) + 81 VAR(U2) + 2 x 17 x 9 x 51 x 37 x Corr(U1, U2)
= 1,266,773,

from where the standard deviation is σ(S) = $1,125.51.

Problem 2.25: Defective Microchips

Solution:

Let A denote the event "the inspector accepts the microchip" and D denote the event "the microchip is defective".

(a) We note that "the number of chips that are not defective" is a binomial distribution with n=10 and p=0.95. Then the desired answer is $(0.95)^{10}$ = 0.6.
(b) Drawing a decision tree or a probability table (as you prefer), we can derive that P(A) = P(A | D) P(D) + P(A | not D) P(not D) = 0.1 x 0.05 + 1 x 0.95 = 0.955.
(c) As in part (a), "the number of accepted chips" is binomial with n=10 and p=0.955. Then, p(Accepts 9 out of 10) = 10 x $(0.955)^9$ x 0.045 = 0.2973.

(d) Using conditional probabilities, we derive that
P(not D | A) = 0.95 / 0.955 = 0.995.

(e) Now, we use conditional probabilities again:
p(no defects 10 | accepts 10) =
= p(no defects 10 and accepts 10) / p(accepts 10)
= p(accepts 10 | no defects 10) x p(no defects 10) / p(accepts 10)
= $(0.995)^{10}$ (because p(accepts 10 | no defects 10) = 1)
= 0.95.

Problem 2.27: Overbooking Flights

Solution:

Let X be the number of persons (out of 11) who show up. X is binomial with parameters n = 11 and p = 0.8.

(a) Using the binomial distribution, P(X<=5) = 0.012.

(b) P(X = 10) = 11 x $(0.8)^{10}$ x 0.2 = 0.236.

(c) E(X) = 11 x 0.8 = 8.8.
E(Profit) = 1200E(X) – 3000 p(X=11) = $10,302.
To arrive to this answer you could have done a decision tree or a table with the profits and probabilities as well.

(d) Again, we have a binomial but now n=10. Then the expectation is
1200 x 0.8 x 10 = $9,600.

(e) Yes, because if one person shows up, then it is very likely that his/her companions will also show up. Therefore, the event a person shows up is not independent of the event the next person shows up.

Problem 3.8: Pension Funds

Solution:

Let T = 0.3 X + 0.7 Y denote the total annual return.

(a) E(T) = E (0.3 X + 0.7 Y) = 0.3 x 7 + 0.7 x 13 = 11.2%.

(b) VAR(T) = VAR(0.3 X + 0.7 Y) = 0.3^2 x 2^2 + 0.7^2 x 8^2 – 2 x 0.3 x 0.7 x 2 x 8 x 0.4
= 29.03,
SD (T) = $\sqrt{(29.03)}$ = 5.4%.

(c) T has a normal distribution (because it is the sum of two normals) with mean μ = 11.2% and standard deviation σ = 5.4%.

(d) Can be done using the normal distribution table or Excel:
P(10 < T < 15) = P(-0.22 < Z < 0.70)
= F(0.70) – F(-0.22)
= 0.7580 – 0.4129
= 0.3451.

Problem 3.11: MBA salaries

Solution:

Let A and B denote the initial salary and the bonus, respectively.

(a) The compensation the first year is the salary plus the bonus. Then
E(A + B) = E(A) + E(B) = $90,000 + $25,000 = $115,000.

(b) VAR(A + B) = $20000^2 + 5000^2$ = 425,000,000 and SD(A + B) = $20,616.

(c) The compensation the second year is the salary with a 20% increase plus the bonus. Then,
E(1.2 A + B) = 1.2 x $90,000) + $25,000 = $133,000.

(d) VAR(1.2 A + B) = $(1.2)^2 \times 20000^2 + 5000^2$ = 601,000,000 and
SD(1.2 A + B) = $24,515.

(e) As A and B are normal, we know that the compensation the second year is normal too. Then,
p(1.2 A + B > $140,000) = p(Z > 0.29) = 1 – P(Z < 0.29) = 1 – 0.6141 = 0.3859.

Problem 3.20: Painting Cars

Solution:

(a) p(Car returned) = 1 - P(Car is not defective) = 1 - 0.80 x 0.90 = 0.28 (A car is not defective if both processes went OK. Note that the two processes are independent).

(b) Let N denote the number of cars returned for rework. As N is a binomial RV,
E(N) = 1000 x 0.28 = 280 and
SD(N) = $\sqrt{(1000 \times 0.28 \times 0.72)}$ = 14.2.

(c) Noticing that np>5 and n(1-p)>5, we can use the normal approximation to binomials. Then, P(N <= 200) ≈ P(Z < -5.63) = 0.

(d) X has a binomial distribution with parameters n = 1000 and p = 0.20. Y has a binomial distribution with parameters n = 1000 and p = 0.10.

(e) As X and Y can be approximated by normals and the sum of normals is normal,
X + Y can be approximated by a normal random variable with parameters
μ = 1000 x 0.20 + 1000 x 0.10 = 300 and standard deviation
$\sigma = \sqrt{(1000 \times 0.2 \times 0.8 + 1000 \times 0.1 \times 0.9)}$ = 15.8.
Using that X+Y is approximately normal, we can now use the table to compute
P(X + Y <= 300) ≈ P(Z < 0) = 0.5.