

# Practice Final: Linear?



```
param T;  
param Demand{1..T};  
var InitialLevel;  
var GrowthRate;  
var Estimate{1..T};  
minimize TotalError:  
    sum{t = 1..T} (Estimate[t] - Demand[t])*  
                  (Estimate[t] - Demand[t]);  
s.t. DefineEstimate {t in 1..T}:  
    Estimate[t] = InitialLevel + GrowthRate*t;
```

# Linear?

```
Set QUESTIONS; /* The set of questions */
Param AvgPctCorrect{QUESTIONS};
Param MinAverage; /* minimum average score */
Param MaxAverage; /* maximum average score */
Var Points{QUESTIONS} >= 0; Var TooHigh >= 0; Var TooLow >= 0;
Minimize Error:
    TooHigh + TooLow;
s.t. Nearly100TotalPoints:
    sum{q in QUESTIONS} Points[q] = 100+TooHigh - TooLow;
s.t. MeetMin:
    (sum{q in QUESTIONS} AvgPctCorrect[q]*Points[q])/
    (100+TooHigh - TooLow) >= MinAverage;
s.t. MeetMax:
    (sum{q in QUESTIONS} AvgPctCorrect[q]*Points[q])/
    (100+TooHigh - TooLow) <= MaxAverage;
```

# Linear?



```
Set POOLS; Set STORES; Set DCS; Set PRODUCTS;  
Param Demand{PRODUCTS, STORES};  
Param CubicCapacity; Param WeightLimit;  
Param MaxFillTime{DCS, POOLS};  
Param Weight{PRODUCTS}; Param Cube{PRODUCTS};  
var WeighOut{DCS, POOLS} binary;  
var UseEdge{DCS, POOLS} binary;  
var Flow[PRODUCTS, DCS, POOLS] >= 0;  
var Assign{POOLS, STORES} binary;
```

# Linear Continued

s.t. ImposeTrailerFillbyWeight{dc in DCS, pool in POOLS}:

$$\begin{aligned} & \text{MaxFillTime}[dc, \text{pool}] * \text{sum}\{\text{prd in PRODUCTS}\} \\ & \text{Weight}[\text{prd}] * \text{Flow}[\text{prd}, dc, \text{pool}] \end{aligned}$$

$\geq$  WeightLimit\*WeighOut[dc, pool]\*UseEdge[dc, pool];

s.t. ImposeTrailerFillbyCube{dc in DCS, pool in POOLS}:

$$\begin{aligned} & \text{MaxFillTime}[dc, \text{pool}] * \text{sum}\{\text{prd in PRODUCTS}\} \text{Cube}[\text{prd}] * \text{Flow}[\text{prd}, \\ & dc, \text{pool}] \end{aligned}$$

$\geq$  CubicCapacity\*CubeOut[dc, pool]\*UseEdge[dc, pool];

s.t. DefineUseEdge{prd in PRODUCTS, dc in DCS, pool in POOLS}:

$$\begin{aligned} & \text{Flow}[\text{prd}, dc, \text{pool}] \leq (\text{sum}\{\text{store in STORES}\} \text{Demand}[\text{prd}, \\ & \text{store}]) * \text{UseEdge}[dc, \text{pool}]; \end{aligned}$$

s.t. WeightOrCube{dc in DCS, pool in POOLS}:

$$\text{WeighOut}[dc, \text{pool}] + \text{CubeOut}[dc, \text{pool}] = 1;$$

# Linear “And”

CubeOut[dc, pool]\*UseEdge[dc, pool] equivalent to  
CubeOut[dc, pool] AND  
UseEdge[dc, pool]

Linear Version:

$(\text{CubeOut}[\text{dc}, \text{pool}] + \text{UseEdge}[\text{dc}, \text{pool}] - 1)$

Or more precisely:

Var Both[dc, pool] binary

$\text{Both}[\text{dc}, \text{pool}] \geq \text{CubeOut}[\text{dc}, \text{pool}] + \text{UseEdge}[\text{dc}, \text{pool}] - 1$

$\text{Both}[\text{dc}, \text{pool}] \leq \text{CubeOut}[\text{dc}, \text{pool}]$

$\text{Both}[\text{dc}, \text{pool}] \leq \text{UseEdge}[\text{dc}, \text{pool}]$

# Sensitivity Analysis

Decisions	Blend A	Blend B	Blend C	Total	Supply
Vintage 1	180.00	-	-		180 180
Vintage 2	246.71	3.29	-		250 250
Vintage 3	-	200.00	-		200 200
Vintage 4	22.46	377.54	-		400 400
Total Produced	449.17	580.83	-		
Total Sold	449.17	580.83	-	Profit	
Sales Price	\$ 70.00	\$ 40.00	\$ 30.00	\$	54,675

Min Blend %	Blend A	Blend B	Blend C
Vintage 1	180.00		-
Vintage 2	246.71	3.29	
Vintage 3		200.00	-
Vintage 4			
Least % of Total	75%	35%	50%
Balance	89.83	-	-
Must Be	0	0	0

Max Blend %	Blend A	Blend B	Blend C
Vintage 1			
Vintage 2			
Vintage 3			
Vintage 4	22.46		-
Least % of Total	5%	0%	40%
Balance	0	0	0
Must Be	0	0	0

# Questions

⌘ What is the minimum amount by which the selling price of C would have to change before it would be attractive to produce Blend C?

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Vintage 3	-	200.00	-	200	200
Vintage 4	22.46	377.54	-	400	400
Total Produced	449.17	580.83	-		
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⌘ What are the Shadow Prices of the 4 vintages?

⌘ What are the units of these prices?

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Vintage 2	246.71	3.29	-		
Vintage 3	-	200.00	-		
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Balance	89.83	-	-		
Must Be	0	0	0		
Max Blend %	Blend A	Blend B	Blend C		
Vintage 1					
Vintage 2					
Vintage 3					
Vintage 4	22.46		-		
Least % of Total	5%	0%	40%		
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Must Be	0	0	0		





⌘ What would be the impact of losing 100 gals of Vintage 3?

# True or False

- ⌘ In solving an integer programming problem, Solver and CPLEX employ genetic algorithms.
- ⌘ In solving an integer programming problem, Solver and CPLEX employ the simplex method.
- ⌘ The set of optimal solutions to an integer program describes a convex set.
- ⌘ The function  $f(x) = x^2$  (that's  $x \cdot x$ ) is a convex function

# True or False



- ⌘ The function  $f(x,y) = x^2 + y^2$  (that's  $x*x + y*y$ ) a convex function
- ⌘ All polynomials are convex functions.
- ⌘ If a real valued function  $f$  is a convex function then the function  $g(x) = -f(x)$  is a convex function

# Restrictions & Relaxations

$$\boxed{\times} \text{ \_\_\_\_\_\_ } \leq \text{ \_\_\_\_\_\_ } \leq \text{ \_\_\_\_\_\_ } \leq \text{ \_\_\_\_\_\_ }$$

- A. Optimal objective function value of the linear programming relaxation with integrality restrictions on some, but not all, variables.
- B. Optimal objective function value of the integer program.
- C. Optimal objective function value of the complete linear programming relaxation (no integrality restrictions).
- D. Optimal objective function value of the integer program if we fix the values of some of the variables.

# Restrictions & Relaxations

✘ Minimizing:  $\underline{\quad C \quad} \leq \underline{\quad A \quad} \leq \underline{\quad B \quad} \leq \underline{\quad D \quad}$

- A. Optimal objective function value of the linear programming relaxation with integrality restrictions on some, but not all, variables.
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- D. Optimal objective function value of the integer program if we fix the values of some of the variables.

# IP Modeling

- ⌘ There are  $N$  items numbered  $1, 2, \dots, N$ . If we choose item  $i$ , we earn  $r[i]$ . If we select exactly one of the items, we must pay  $b$ . You may introduce other variables if you wish. You will certainly need to add an objective (minimize total cost) and constraints.
- ⌘ param  $N$ ; # the number of items.
- ⌘ param  $r\{1..N\}$ ; # the revenue on each item
- ⌘ param  $b$ ; # the cost incurred if we choose exactly one item
- ⌘ var  $x\{1..N\}$  binary; #  $x[i]$  is one if we choose item  $i$  and 0 otherwise.

# IP Modeling

- ⌘ param N; # the number of items.
- ⌘ param r{1..N}; # the revenue on each item
- ⌘ param b; # the cost incurred if we choose exactly one item
- ⌘ var x{1..N} binary; # x[i] is one if we choose item i and 0 otherwise.
- ⌘ Var All binary; # is 1 if we choose all
- ⌘ maximize Revenue :  $\sum\{k = 1..N\} r[k]*x[k] - b*All$ ;
- ⌘ s.t. NotNecessaryButForCompleteness{k in 1..N}:
  - ⊠ All  $\leq x[k]$ ;
- ⌘ s.t. DidWeSelectAll:
  - ⊠  $N*All \geq \sum\{k in 1..N\} x[k]$ ;