Decision Analysis 2

The Value of Information

Quotes of the day

"No sensible decision can be made any longer without taking into account not only the world as it is, but the world as it will be....

Isaac Asimov (1920 - 1992)

A wise man makes his own decisions, an ignorant man follows public opinion.

Chinese Proverb

"It doesn't matter which side of the fence you get off on sometimes. What matters most is getting off. You cannot make progress without making decisions."

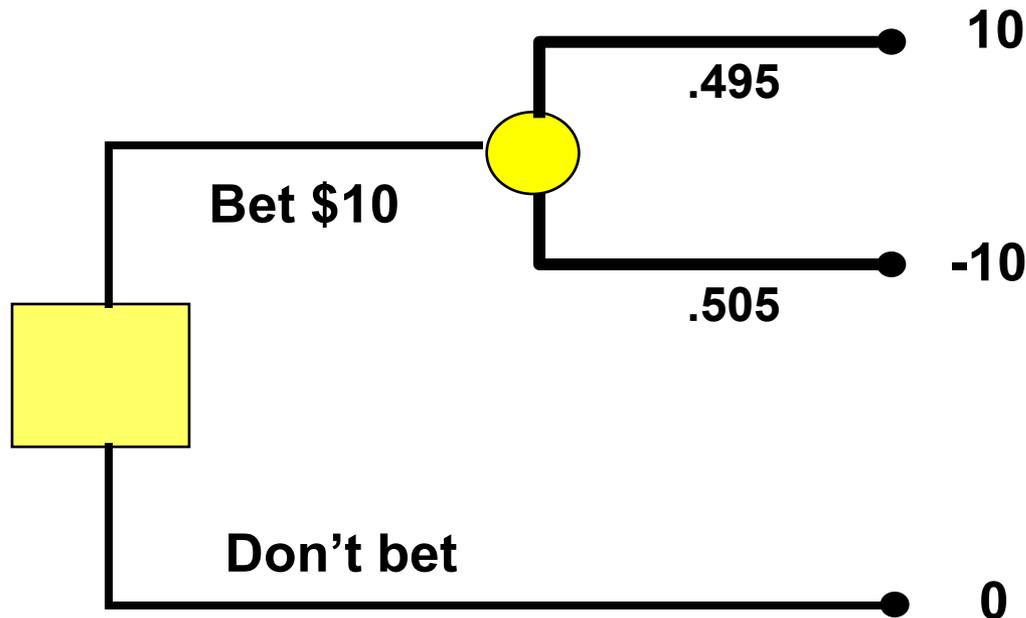Jim Rohn

The value of Information.

- **Using decision analysis to assess the value of collecting information.**

- **The value of perfect information**

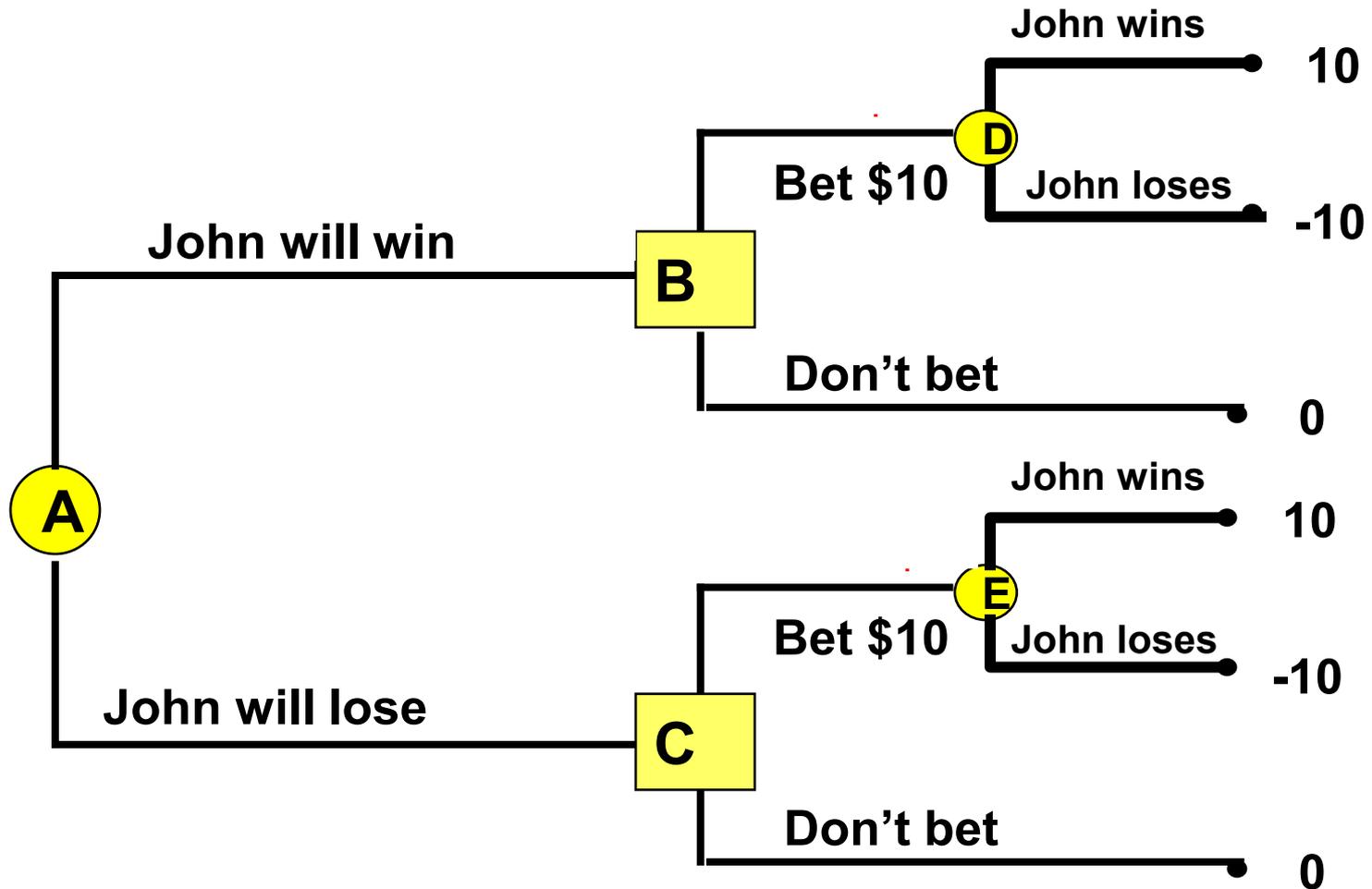- **The value of imperfect information**

A blackjack example.

- **John is at a blackjack table. He can place a bet of $10 or do nothing. His odds of winning a bet are 49.5%. What should he do if he wants to maximize expected value?**
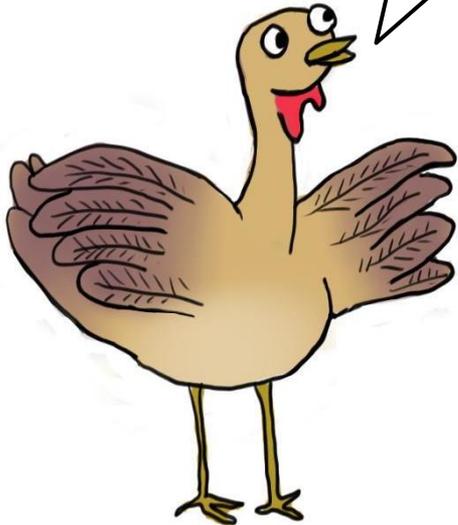
Perfect Information

Suppose that prior to placing a bet, John will be told (with 100% accuracy) whether he will win or lose. How much is this information worth?

A question on probabilities

I understand why the probability that John will win is .495. But why is the probability that he wins 1 at node D of the tree?

Actually, that was the probability that John wins given that he has been told he will win. Since he is always told the truth, it means that there is a 100% chance of winning.

The Expected Value of Perfect Information

- *EVWOI*: Expected value with original information. This is the value of the original tree, which is $$0.

- *EVWPI*: Expected value with perfect information. This is the value of the tree, assuming we can get perfect information (where the type of information is specified.)   = $4.95

- *EVPI*: Expected value of perfect information. = EVWPI – EVWOI = $4.95

Imperfect Information

Suppose John counts cards.

- **If the deck is "good", his odds of winning are 52%.**
- **If the deck is "bad", his odds of winning are 49%.**
- **The deck is good 20% of the time.**

Deck is "good"

Bet $10

10

-10

Don't bet

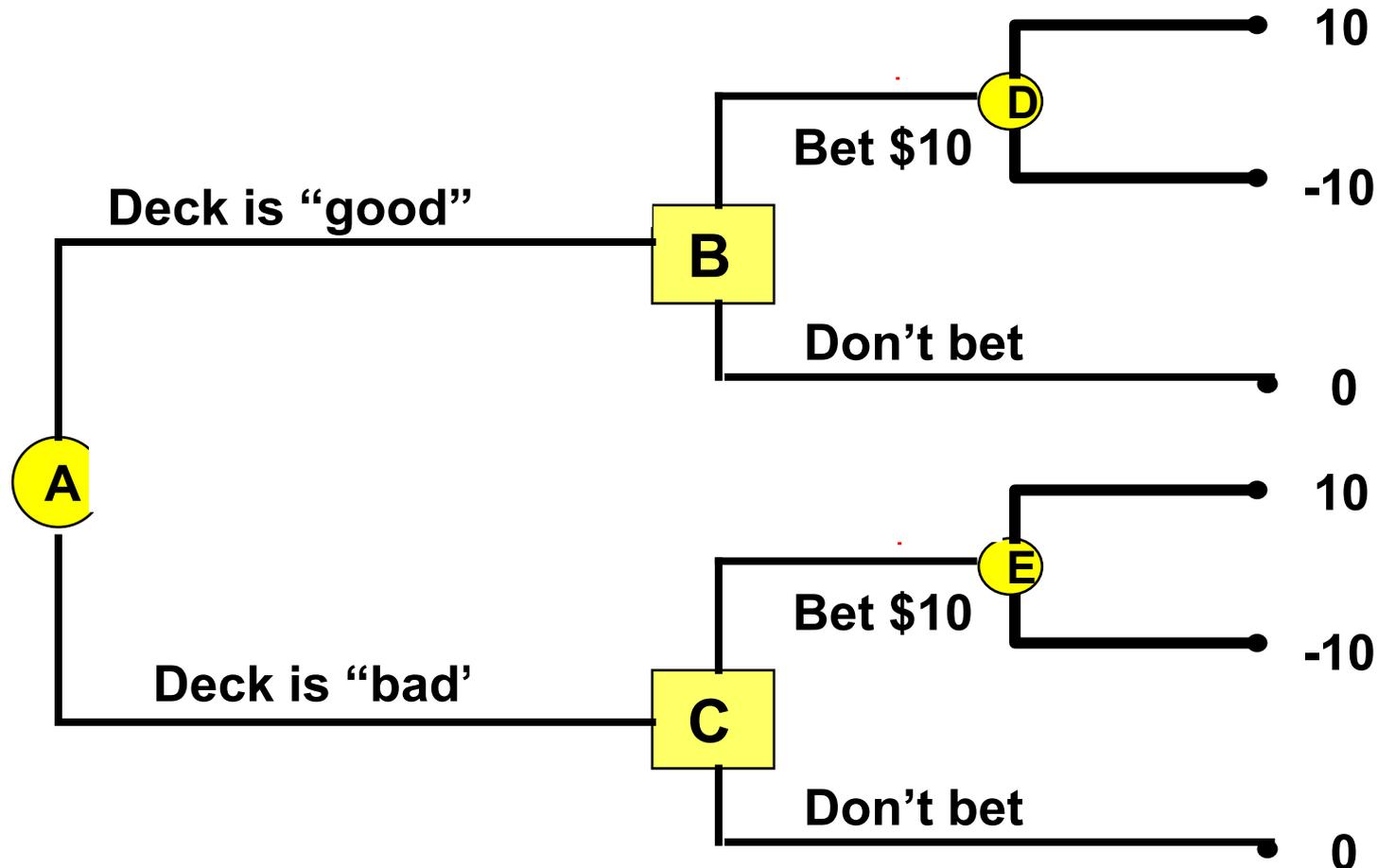0

Deck is "bad'

Bet $10

10

-10

Don't bet

0

The Expected Value of Perfect Information

- *EVWOI*: Expected value with original information. This is the value of the original tree, which is **$ 0**.

- *EVWII*: Expected value with imperfect information. This is the value of the tree, assuming we can get perfect information (where the type of information is specified.)   = **$.08**

- *EVII*: Expected value of imperfect information.
      = EVWPI – EVWOI = **$.08**

Which of the following is false about the value of information?

1. EVII ≥ 0

2. EVPI ≥ EVII

3. EVPI > 0

4. If all payoffs (values at endpoints) are doubled and if everything else stays the same, then EVPI and EVII are also doubled.

Imperfect Information

Suppose John has to bet between $10 and $25. If the deck is good, he bets $25. If the deck is bad, he bets $10.
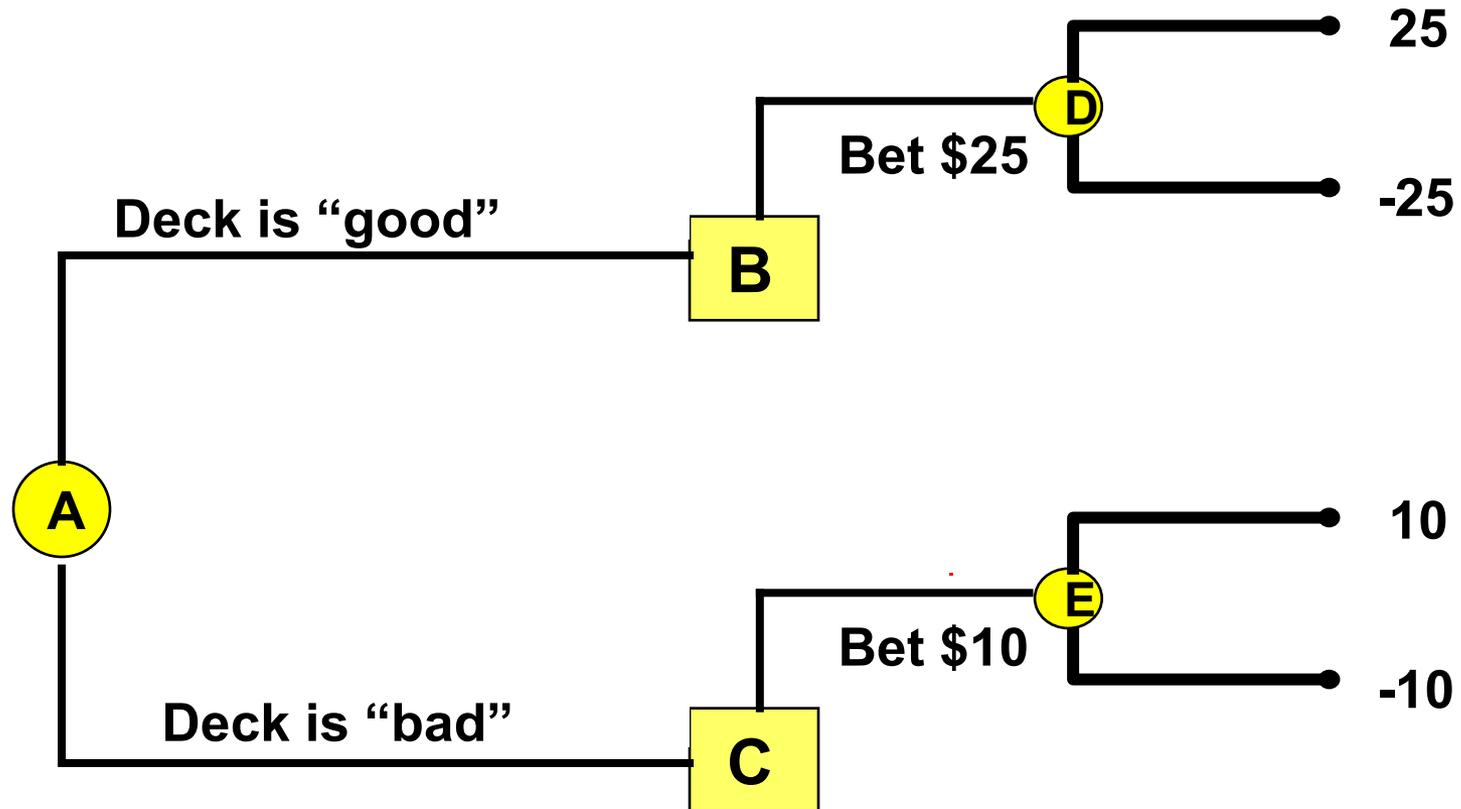
What is the probability that you would be winning after 200 bets?

1. A little over 50%

2. Around 60%

3. Around 75%

4. Close to 98%

Net winnings after 200 hands each
A histogram of 400 simulations

Winnings after 240,000 bets.
(Expected value = $9600)
9 simulations

$8,740

$6,260

$15,310

$7,200

$6,050

$7,045

$16,305

$8,990

$7,221
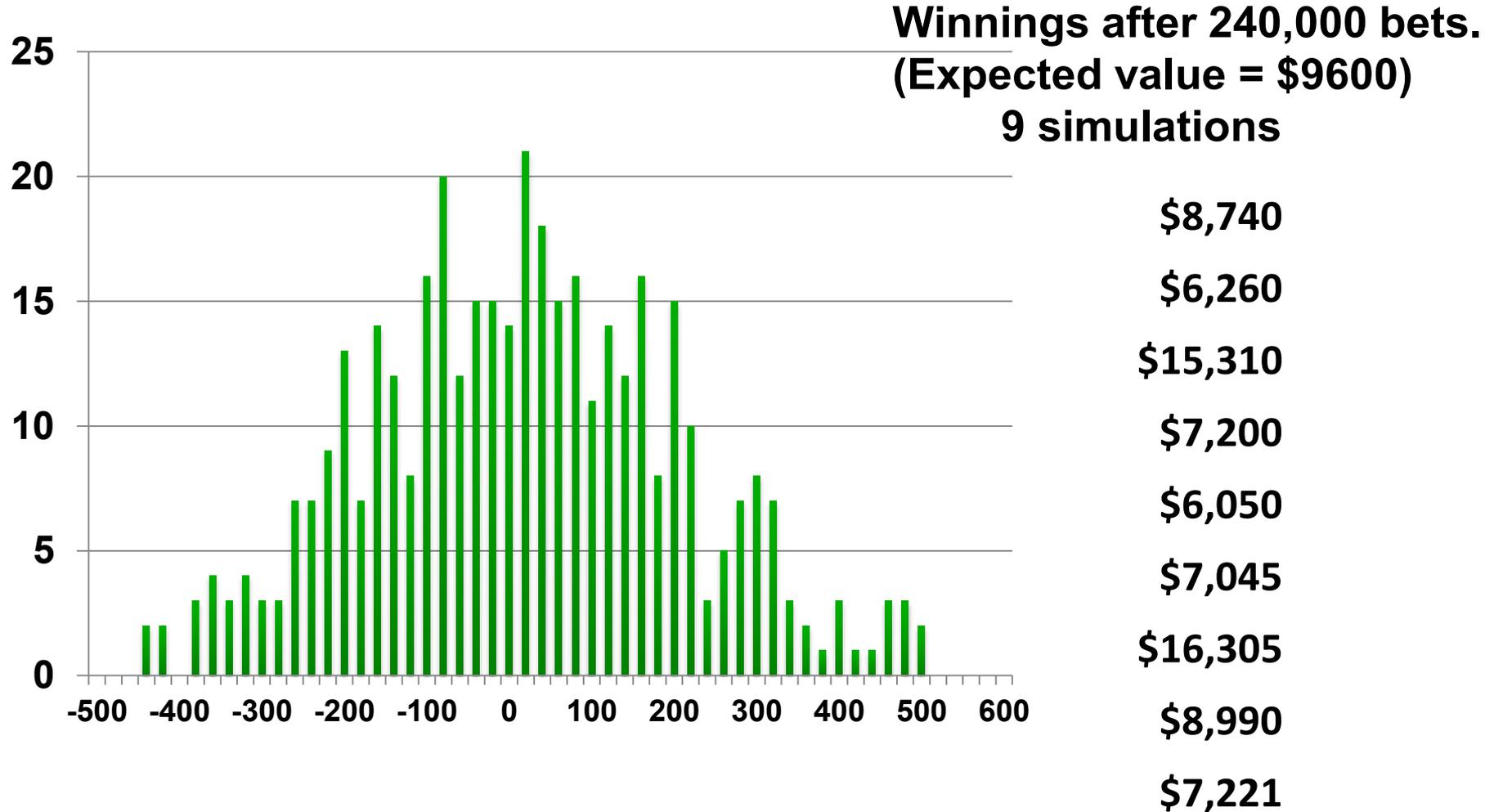
Next: Medical diagnosis

- **Bayes rule**

- **Application to AIDS testing**

- **Applications to ALAS, a fictional, and relatively uncommon, viral disease.**

Example for Bayes Rule

There are 100 students in subject 15.ABC

- 60 males,   40 females

- 30 seniors, 70 sophomores or juniors

- Proportion of females who are seniors is 35%.

- What is the proportion of seniors who are females?

Bayes Rule (in disguised form)

	Male	Female	Total
Not senior	9	8	3
Senior	7	6	4
Total	1	2	5

Students in 15.ABC

[ ] **out of 30 seniors are female**

Males: 60%   **Seniors: 30%**

Prob(Senior | female) = 35%

What is Prob(female | senior)?

Bayes Rule: Prob(X AND Y) = Prob(X) Prob(Y | X)

Prob(Y | X) = Prob (X and Y) / Prob(X)

	Male	Female	Total
Not senior	.44	.26	.70
Senior	.16	.14	.30
Total	.60	.40	1

Prob(female | senior) = 14/30.

Mental Break

The 1991 Ig Nobel Prize in Education

http://www.improbable.com/ig/winners/#ig1991

Bayes Rule and AIDS Testing

Approximately 1 out of 10,000 citizens develop AIDS each year.

Suppose that everyone in the US were tested for AIDS each year. (Approximately 300 million people).

AIDS Test

Prob(test is Positive | Person has AIDS) = .98

Prob(test is Negative | Person does not have AIDS) = .99

False Negative Rate: 2%

False Positive Rate: 1%

Question 1.   If a randomly selected person tests positive, what is the probability that the person has AIDS?

1.   99%.

2.   98%

3.   approx.   80%

4.   approx.   1%

Question 2.   If a randomly selected person tests negative, what is the probability that the person does not have aids?

1.   more than 99.99%.

2.   between 98% and 99%

3.   approx. 80%

4.   approx. 1%

	Test Pos.	Test Neg.	Total
Has AIDS	.000098	.000002	.0001
No AIDS	.010099	.989901	.9999
Total	.010197	.989903	1

Prob(No AIDS and Neg) = Prob(Neg |No AIDS) Prob(No AIDS)

Prob(AIDS and Pos) = Prob(Pos | AIDS) Prob(AIDS)

Prob(AIDS | Pos) = .000098/.010197 ≈ .0001/.01 = 1/100

Prob(No AIDS | Neg) = .989901/.989903 ≈ .999998

Tests and treatment for ALAS

ALAS is a type of viral disease. Untreated ALAS can result in severe brain damage, possibly leading to death.   Approximately 1 in 10,000 persons have ALAS. The treatment of ALAS is an new anti-viral that costs $100,000.

If discovered in time, 100% of patients can be cured. If not then only 10% will survive.

The government has ordered testing of the entire population. What test(s) should be administered?

Tests for ALAS

There are three tests for ALAS.

TEST 1. 100% accurate test.

Cost = $500; test results take 24 hours.

TEST 2. Prob(Pos test | ALAS) = .98   **False Neg Rate 2%**

Prob(Neg test | No ALAS) = .99   **False Pos Rate 1%**
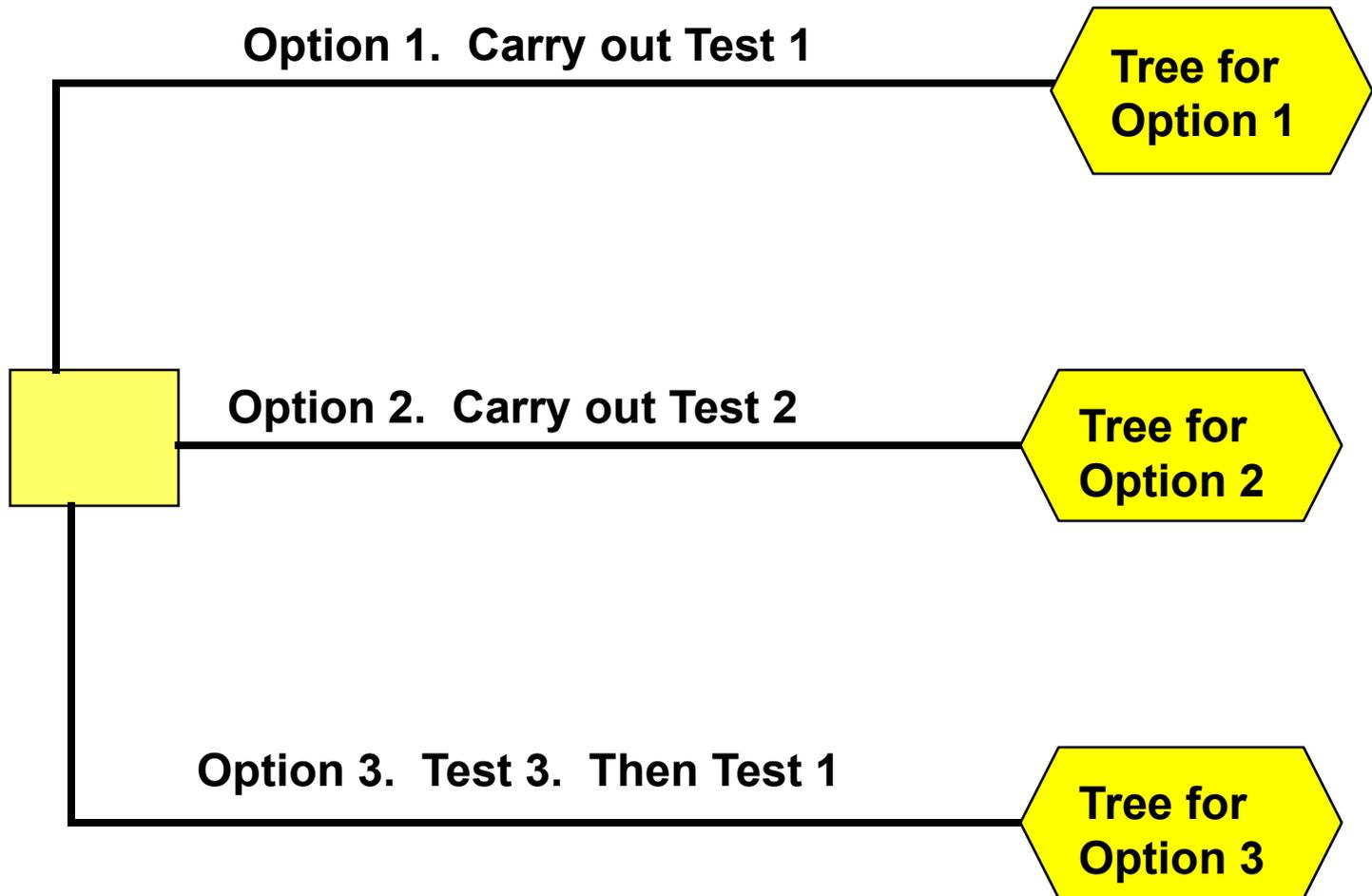
Cost = $100; test results take 5 minutes.

TEST 3. Prob(Pos test | ALAS) = 1   **False Neg Rate 0%**

Prob(Neg test | No ALAS) = .90   **False Pos Rate 10%**
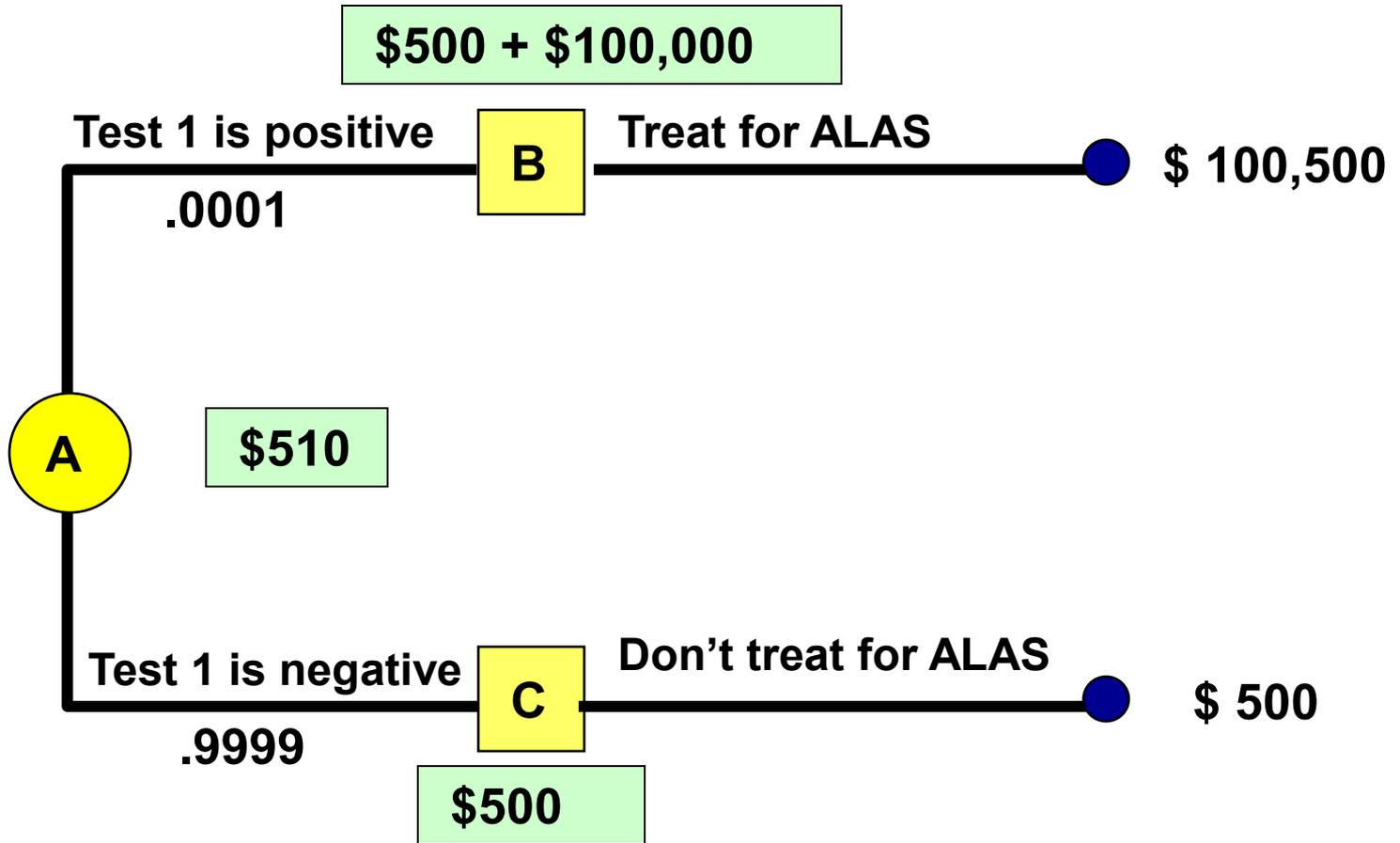
Cost = $100; test results take 5 minutes.

The Decision Tree

Option 1. Carry out Test 1

Tree for Option 1

Option 2. Carry out Test 2

Tree for Option 2

Option 3. Test 3. Then Test 1

Tree for Option 3

Option 1.   Do the 100% reliable test

$500 + $100,000

Test 1 is positive   **Treat for ALAS**

.0001

B   ● $ 100,500

A

$510

Test 1 is negative   **Don't treat for ALAS**

.9999

C   ● $ 500

$500

Option 2.   Do the 99% reliable test

Test 2 : Pos   **B**   **Treat for ALAS**   ● $ 100,100

A

Test 2: Neg   **C**   **Don't treat**   **D**

No ALAS   ● $100
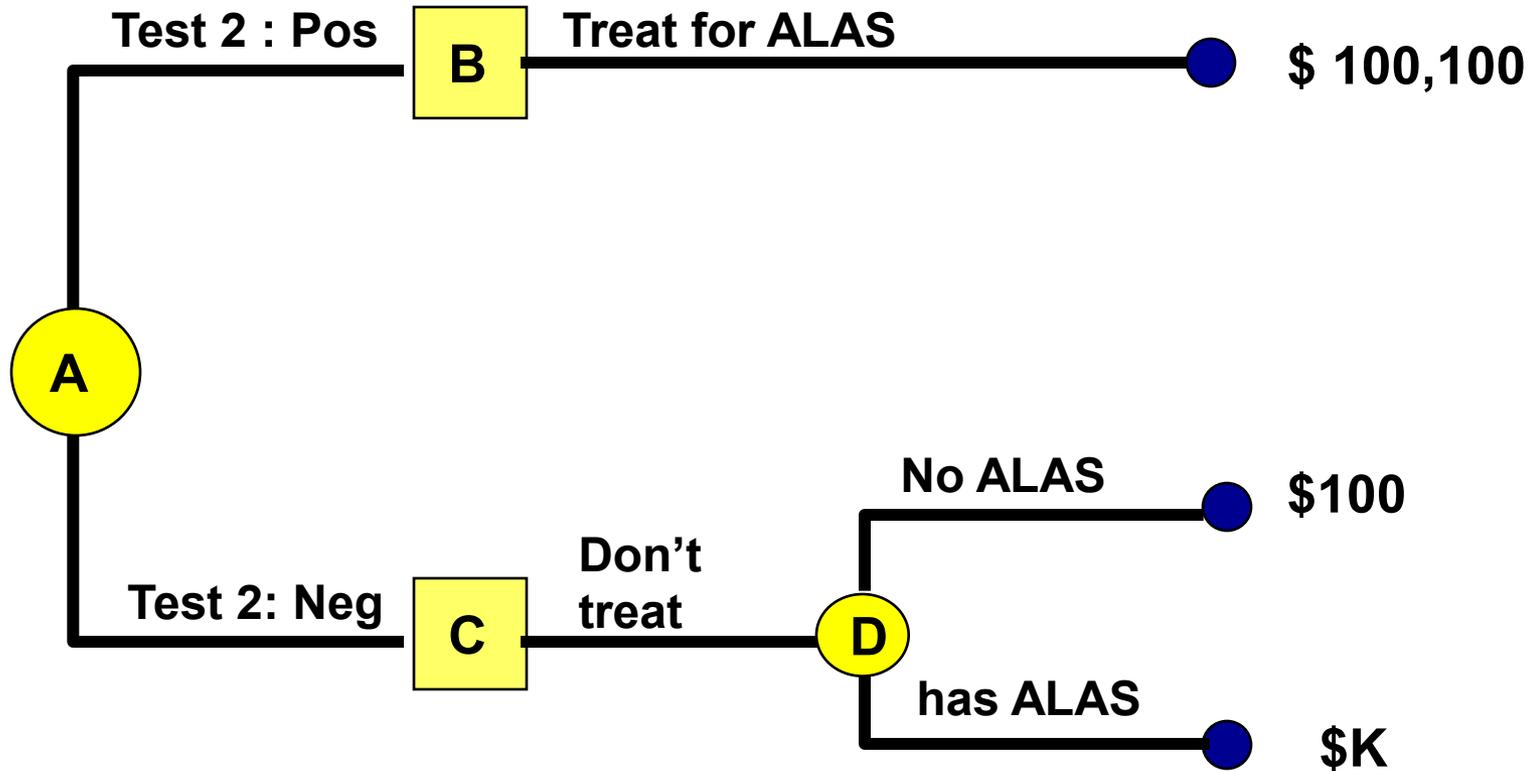
has ALAS   ● $K

We are assuming that the cost of treating a person with false positive is also $100,000.

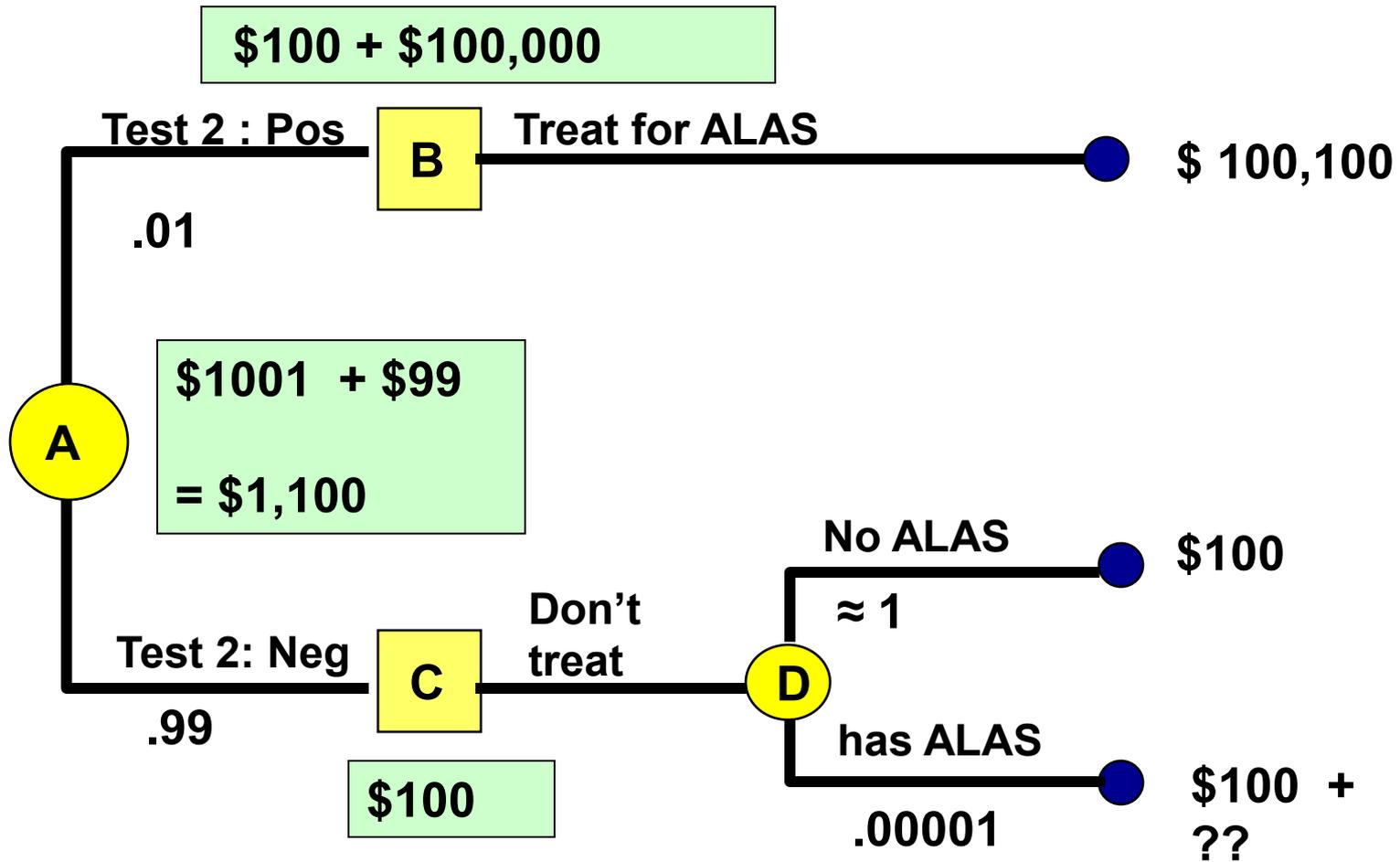|	Test Pos.	Test Neg.	Total
Has ALAS	.000098	.000002	.0001
No ALAS	.010099	.989901	.9999
Total	.010197	.989903	1

Prob(No ALAS and Neg) =

Prob(Neg |No ALAS) Prob(No ALAS)

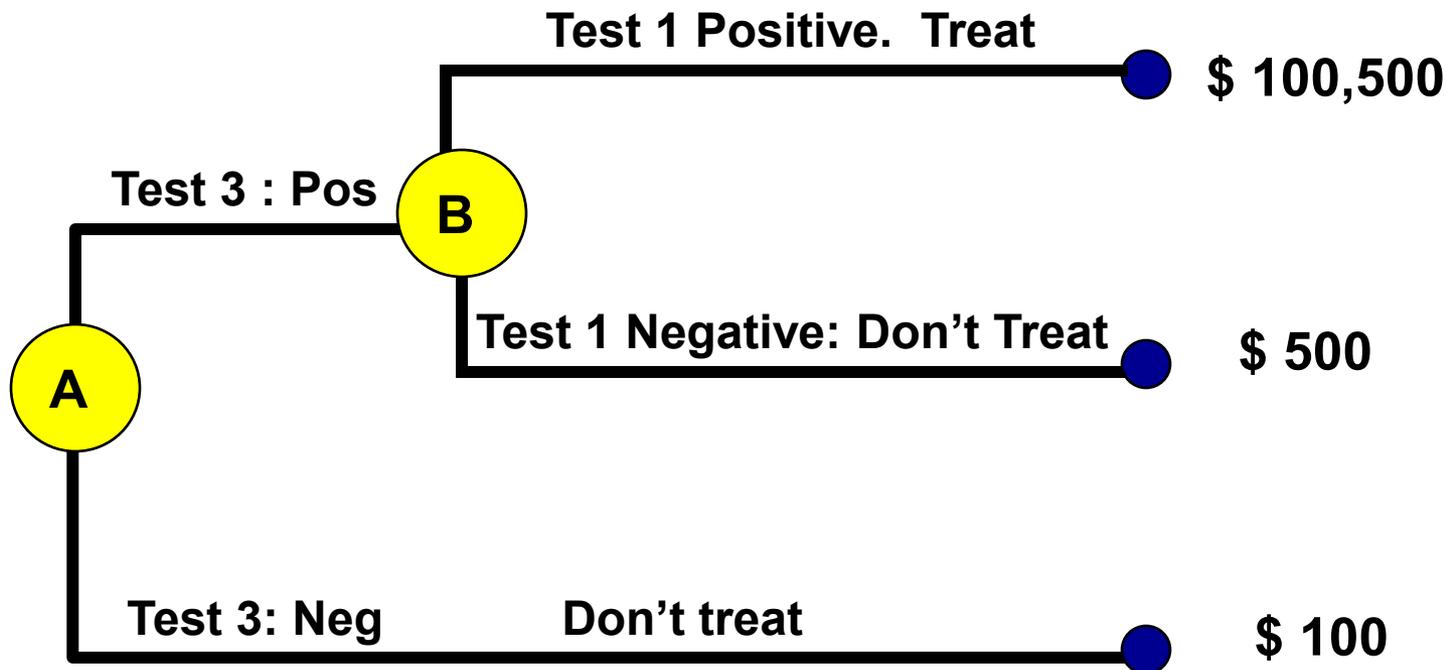**Prob(ALAS and Pos) = Prob(Pos | ALAS) Prob(ALAS)**

Prob(Pos) ≈ .01

Prob(No ALAS | Neg) ≈ .999998

Probabilities and Expected Values for Option 2

$100 + $100,000

Test 2 : Pos — **B** — Treat for ALAS — ● $ 100,100

.01

A

$1001 + $99

= $1,100

No ALAS ≈ 1 — ● $100

Don't treat — **D**

Test 2: Neg — **C**

.99

$100

has ALAS
.00001 — ● $100 + ??

Option 3. Do Test 3. If it comes out positive, then do test 1.

Test 1 Positive. Treat — $ 100,500

Test 3 : Pos — **B**

Test 1 Negative: Don't Treat — $ 500

A

Test 3: Neg   Don't treat — $ 100

The three endpoints have been simplified. We treat a person if they definitely have ALAS. We don't treat them if they definitely don't have ALAS.

	T3: Neg.	T3: Pos T1: Neg	T3: Pos T1: Pos	Total	
ALAS	0	0	.0001	.0001	
No ALAS	.89991	.09999	0	.9999	
Total	.89991	.09999	.0001	1	

T1 is 100% accurate

T1 and T3 are both Pos iff the person has ALAS

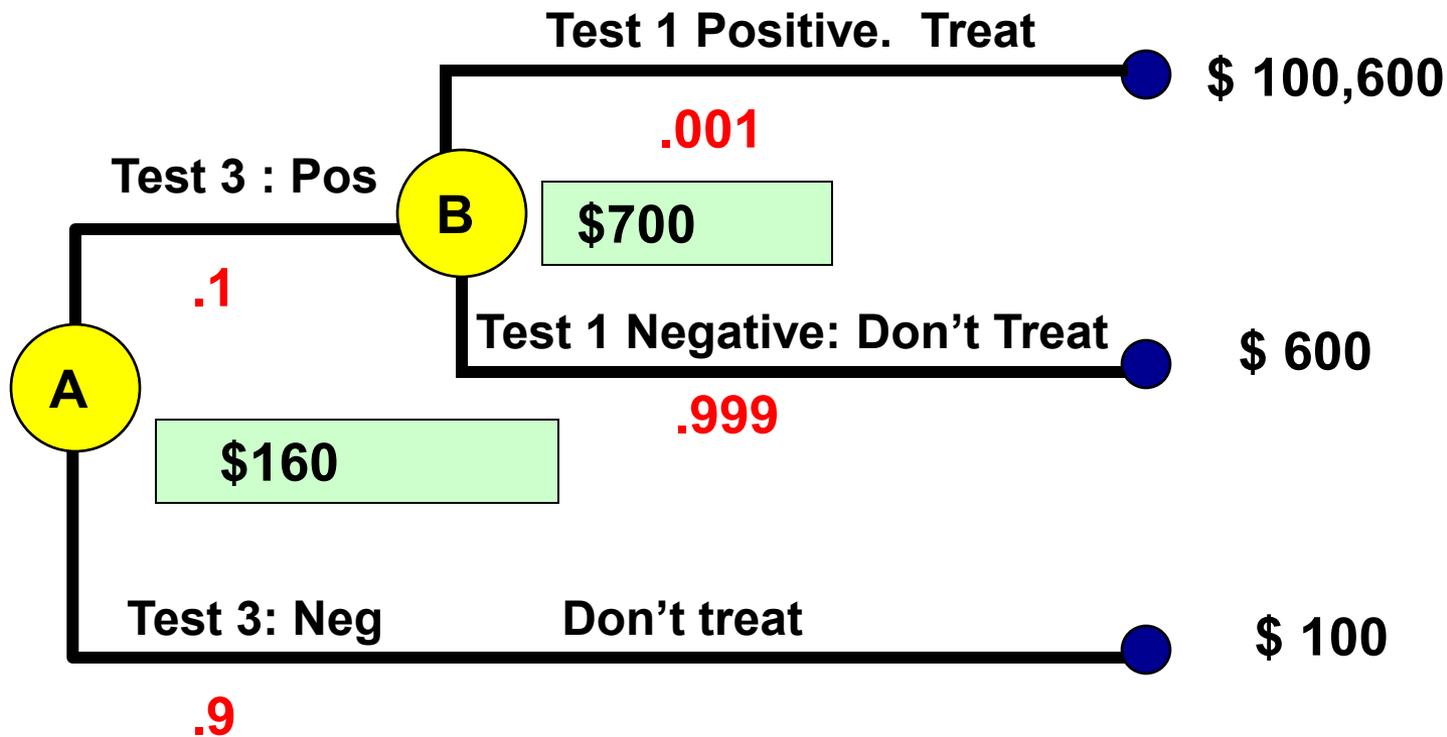**If T3 is Neg, the person does not have ALAS**

Prob(No ALAS AND T3 is Neg) =
Prob(No ALAS) × Prob(T3 is Neg | No ALAS) = .9999 × .9

Prob(T1 is Pos | T3 is pos) = $\dfrac{\text{Prob(T1 is Pos AND T3 is Pos)}}{\text{Prob(Test 3 is Pos)}}$

$\approx .0001/.1 = .001$

Option 3. With approx. probabilities

Savings per person: $510 - $160 = $350

× 300 million people = $105 billion.

Summary

- **The value of information is the increase in the value of the decision problem if new information is provided.**

- **The value depends on what information is available in the original decision problem, and what information is introduced.**

- **In testing for diseases, rare diseases can result in many more false positives than real positives. It is important to know how many more.**

- **Decision trees often incorporate decisions about whether to gather information.**

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