Game Theory for Strategic Advantage

15.025

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What Have We Learned So Far?

- You must account for your own and your opponents' rationality / sophistication

- In some games, it is appropriate (useful!) to exploit the logic of rationalizability

- Having the **right game** in mind is a source of **competitive advantage**. (Recall Epson vs. HP)

Class 3 Game Plan

1. Building a Language: *ask lots of questions!*

2. Nash Equilibrium: *the Good, the Bad…*

3. Prototypical Games: *PD, Coordination, Chicken*

A **Structured** Approach

1) Game theory is a **toolkit for strategic analysis**

2) Specify a game: payoffs represent **total utility**

3) Use **all available information** to describe the game

4) But once we are in the game, ***we are in the game***.

5) Base the **analysis** on the <u>game's elements alone</u>

More formally …

A **game** is a multi-player decision problem:

- players $i = 1, 2, \ldots, n$
- strategies a_i from "feasible set" A_i
- payoff functions (utilities) $u_1(a_1, a_2)$, $u_2(a_1, a_2)$

- typically:

  A_{Row} = {Top, Bottom} A_{Column} = {Left, Right}

  Payoff = <u>total utility</u> in "payoff matrix"

- many more examples (like beauty contest)

Dominated Strategies

Definition: Strategy *x* is ***dominated*** for player *i* if there exists another strategy ***y*** that guarantees a higher payoff to player *i*.

For example:

		Phillip Morris: No Ad	Phillip Morris: Ad
Reynolds	No Ad	50 , 50	20 , 60
Reynolds	Ad	60 , 20	30 , 30

No Ad is dominated by Ad (for both players)

Dominant Strategies

A ***dominant strategy*** for player *i* **always** gives player *i* a **higher payoff** than any other strategy.

For example: placing yourself at the **median voter's** location (if all you care about is winning the election)

Recall: Cigarette Ad Game

		Phillip Morris: No Ad	Phillip Morris: Ad
Reynolds	No Ad	50 , 50	20 , 60
	Ad	60 , 20	30 , 30

Reynolds' best strategy is Ad <u>regardless</u> of what Philip Morris does

➔ Ad is a *"**dominant strategy**"*

Dominance: a Risk-Free Concept

PENALTY KICKS

		Goalie: Left	Goalie: Right
Kicker	Left	2 , 5	5 , 1
	Middle	3 , 3	3 , 3
	Right	5 , 1	2 , 5

Middle is not dominated for the Kicker!

(but it's not a great idea either)

Iterative Elimination of *Strictly* Dominated Strategies

Strategies that survive all rounds of elimination are called ***rationalizable strategies***

Required assumptions:

- Know the game
- Rational player
- Rational opponents
- Knowledge of knowledge of … of rationality

Order of elimination does not matter

Example: Tourists & Natives

- Two bars can charge a price per drink of \$2, \$4, or \$5
  - 6,000 tourists pick a bar randomly
  - 4,000 natives select bar with lowest price

- Example: Both charge \$2
  – each gets 5,000 customers ➔ payoff *= \$ 10,000*

- Example: Bar 1 charges \$4, Bar 2 charges \$5
  – Bar 1 gets 3,000+4,000=7,000 customers (➔ *\$28,000*)
  – Bar 2 gets 3,000 customers (➔ *\$15,000*)

		Bar 2		
		$2	$4	$5
	$2	10 , 10	14 , 12	14 , 15
Bar 1	$4	12 , 14	20 , 20	28 , 15
	$5	15 , 14	15 , 28	25 , 25

For each Bar, $2 is dominated by both $4 and $5

In the reduced game (with only 4 and 5), $5 is dominated by $4

($4,$4) is the only **rationalizable** strategy profile

Traffic Game

		Column	
		Drive	Stop
Row	Drive	(-2, -2)	(1, 0)
	Stop	(0, 1)	(-1, -1)

Rationalizable Outcomes = all four!!

Beliefs and Best Responses

Definition: Player i's ***belief*** about the strategy that i's opponents will play is a ***probability distribution*** over their actions.

Definition: Strategy $\boldsymbol{x}$ for player i is a ***best response*** if $\boldsymbol{x}$ maximizes i's ***expected payoff,*** given i's beliefs.

Example: In the traffic game, "drive" is a best response if player *1* believes player *2* "stops" with probability >50%.

Best Responses: more advanced examples

- “Guess 0.75*Average” ➔ best response = 75% of your estimate of your opponents’ average.

- Hide and Seek ➔ best response = hide where you think your opponents are *least likely to search*, seek where you think they are *most likely to hide*.

- Product Positioning ➔ best response = locate to the left/right of your competitor’s expected position***

Nash Equilibrium

Definition: A profile of strategies (i.e., one for each player) is a ***Nash Equilibrium*** if each player's strategy is a ***best response*** to the other players' strategies.

Examples:

- both firms locating their product at the center of the line;
- everyone choosing the number 1;
- player 1 driving and player 2 stopping.

In the movie

Two pharma firms choose which compound to pursue…

Two friends at a bar…

		Pfizer		
		P1	P2	P3
Merck	P1	(0 , 0)	(5 , 1)	(5 , 1)
	P2	(1 , 5)	(0 , 0)	(1 , 1)
	P3	(1 , 5)	(1 , 1)	(0 , 0)

			Bar 2	
		\$2	\$4	\$5
	\$2	10 , 10	14 , 12	14 , 15
Bar 1	\$4	12 , 14	20 , 20	28 , 15
	\$5	15 , 14	15 , 28	25 , 25

(\$4,\$4) is the only **rationalizable** strategy profile

(\$4,\$4) is also the unique **nash equilibrium**

Iterated elimination of dominated strategies **might yield** a Nash equilibrium and **cannot eliminate** one.

Nash Equilibrium: the Good

- It always exists (John Nash, 1950)
- Easy to find
  - For us
  - For firms (given enough time)
- It is "stable"
- A tool for out-of-sample predictions
- A criterion for investment decisions (next class)
  - **What if demand ↗? ↘?**
  - **What if one firm cuts its costs?**

Nash Equilibrium: the Bad

- Equilibrium does not mean optimal!
- Think of the prisoners' dilemma:
- Unique Nash equilibrium, but "Pareto-inefficient"

Column

Row	L	R
T	(2, 2)	(0, 3)
B	(3, 0)	(1, 1)

- Many interesting games have >1 Nash Equilibrium!
- Stability not-so-great anymore!

Nash Equilibrium: the Many

Column

Row		L	R
	T	(2, 2)	(0, 0)
	B	(0, 0)	(1, 1)

Coordination Game

- No dominated strategy for either player
- Two Pareto-ranked Nash equilibria
- Could have path-dependence!

Selecting Nash Equilibrium

- How to “steer the game”?

- Commitment tactics! (Cigarettes, ice-cream vendors)

- Before then…. Do we have the **right game?**

The Last Chocolate

Column

		Split	Steal
Row	Split	(2, 2)	(0, 4)
	Steal	(4, 0)	(1, 1)

- Dominant strategy for each player
- Seemingly self-evident solution to the game

Tiny Details Matter

Column

		L	R
Row	T	(5 , 5)	(0 , 10)
	B	(10 , 0)	(0 , 0)

- Is B a dominant strategy for Row?
- Is R a dominant strategy for Column?
- Find all Nash Equilibria

Youtube ➔ "Best Split or Steal Ever"

Stag Hunt

(J.-J. Rousseau, 1754)

Column

Row		L	R
	T	(1, 1)	(1, 0)
	B	(0, 1)	(2, 2)

- Hunting stag vs. rabbit
- No dominated strategy for either player
- Two Pareto-ranked Nash equilibria
- The role of <u>trust</u>

Example: Technology Adoption

One upstream and one downstream firm

- Currently **old** inventory-management system ➔ profit = \$1M each
- Each can independently invest \$1M in upgrading to a **new** system
- If only one upgrades, no one benefits
- If both upgrade, they exploit synergies worth \$3M (gross)

What's the game?

- Dominated strategies? **No**
- What's the Prediction?

Two Nash Equilibria:

- (*Old,Old*) and (*New,New*)

Downstream \ Upstream	*Old*	*New*
Old	(1,1)	(1,0)
New	(0,1)	(2,2)

- This is a coordination game that may explain some interesting real world phenomena (mergers to coordinate on synergies)

Takeaways: Building a Language

1. Dominant Strategy: *performs better than all other strategies, regardless of opponents' behavior*

2. Dominated Strategy: *an alternative strategy always performs better, regardless of opponents' behavior*

3. Rationalizable Strategies: *survive the iterated elimination of dominated strategies*

4. Best-Responses

5. Nash Equilibrium

Building a New Approach

1) Once in the game…

2) Assumptions ➔ Conclusions

3) Do we like the conclusions?

YES

NO

Can we trust the assumptions?

Were the assumptions wrong?
Or did we just learn something?

- **Who is the opponent?**
- **What is the *actual* game?**