Game Theory
for
Strategic Advantage

15.025

Alessandro Bonatti
MIT Sloan

Classic Examples

- Government
  - T-Bills, mineral rights (e.g. oil fields), assets (e.g. privatization)
  - **Electromagnetic spectrum**
  - Public construction contracts (e.g., California Highways)
- Internet
  - Display & **keyword advertising**, personal data (cookies)
- Real Estate
  - Development contracts
  - individual homes
- Stocks
  - IPOs, Repurchases, **M&A**
- Auctions in disguise
  - Patent races, Lobbying, Legal disputes, hiring

First-Price Auction

How *should* you bid?

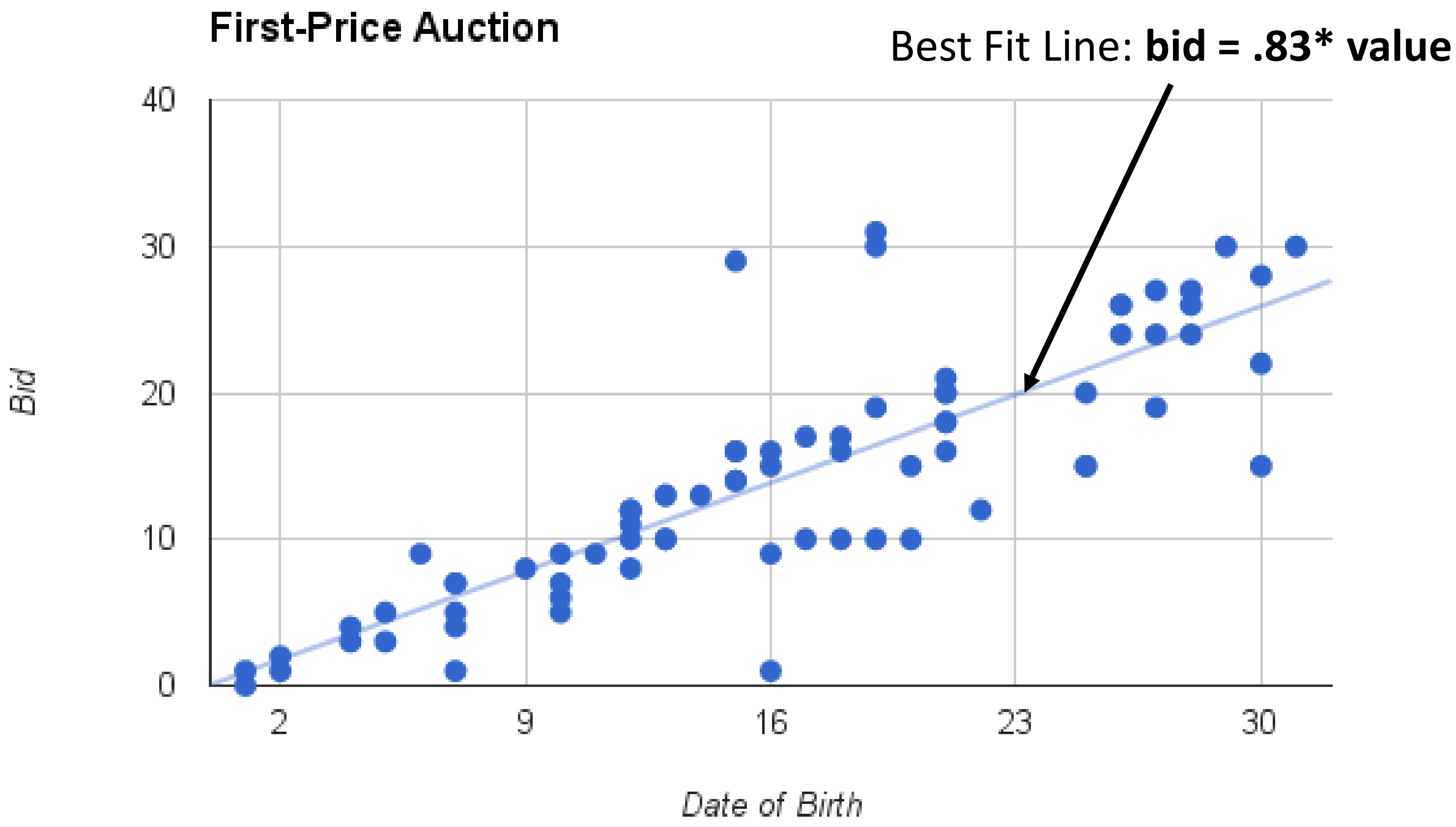
Is bidding your total valuation $\boldsymbol{v_i}$ a good strategy?

How much to shade?

New approach: types of your opponent

(i.e., when to win and when to lose)

Your Bids


First-Price Auction
Best Fit Line: **bid = .83* value**
40
30
20
10
0
Bid
2
9
16
23
30
Date of Birth

Setting Up the Problem

- You bid to maximize your expected payoff

- Make a **projection** about the other bidder's strategy

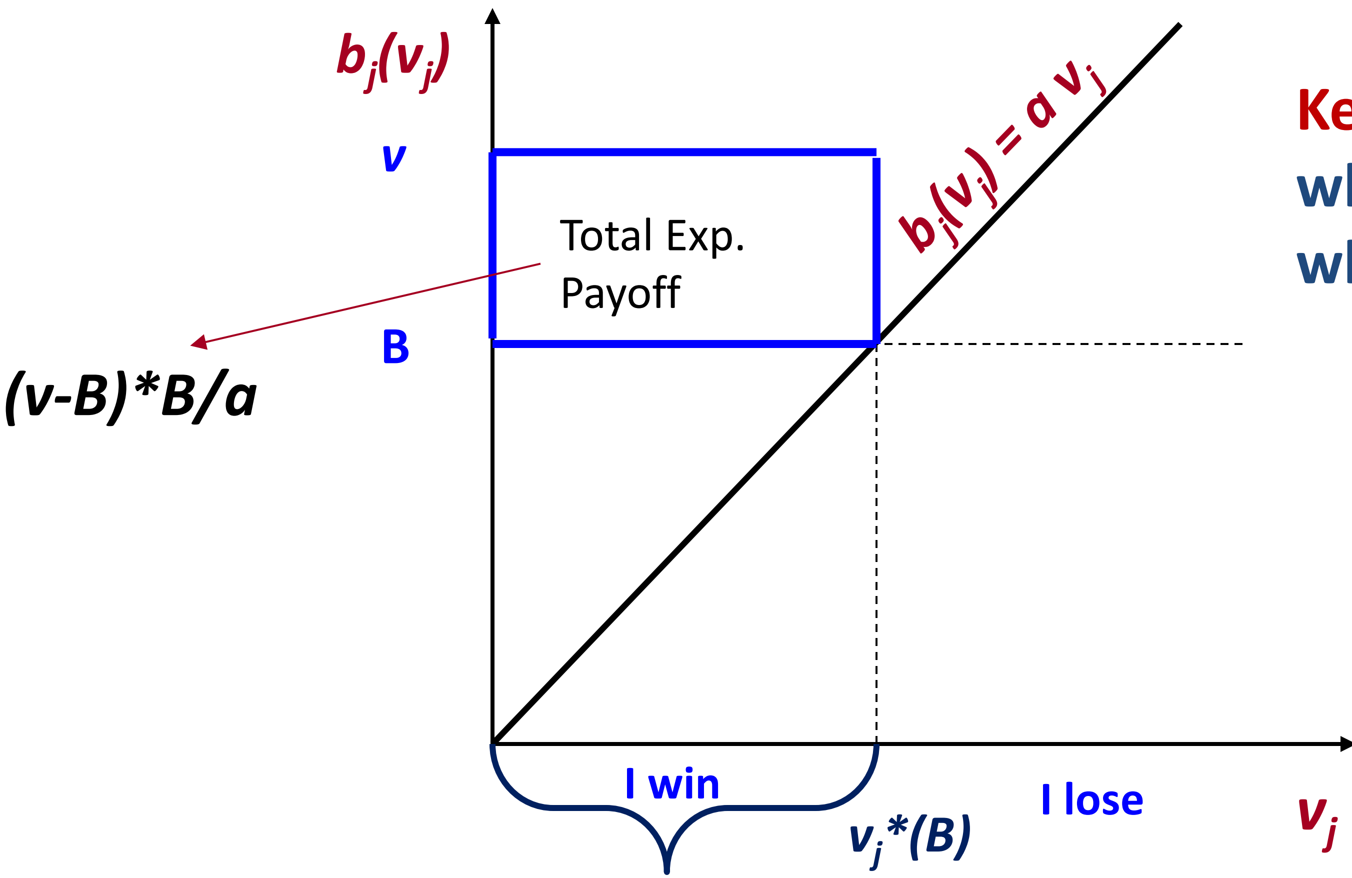
- Presumably this strategy depends on the valuation that bidder has.

- Let $\mathbf{b}_j(\boldsymbol{v}_j)$ be your projection for the bid of the other bidder when their valuation is $\boldsymbol{v}_j$.

Bidders Problem

- Suppose your value is $v_i = v$.
- Choose a bid, B, to maximize expected profits.
- ***E[**Profit**] = (v – B) * Pr(B** is the highest bid**)***
- ***Pr(B** is the highest bid**) = Pr(B > b_j(v_j))***

What is My Optimal Bid?



$$v_j*(B) = B / a$$

Key Calculation:

who do I beat?

who do I lose to?

Bidder's Problem Revisited

- So now you must choose ***B*** to maximize
  - ***E[Profit] = (v – B)*B/a***
- Differentiate with respect to ***B***
  - $(v - 2B)/a = 0$
  - $B = \boldsymbol{v_i}/2$
- If your opponent shades <u>proportionally</u> to his value ➔ bid <u>half</u> your value.

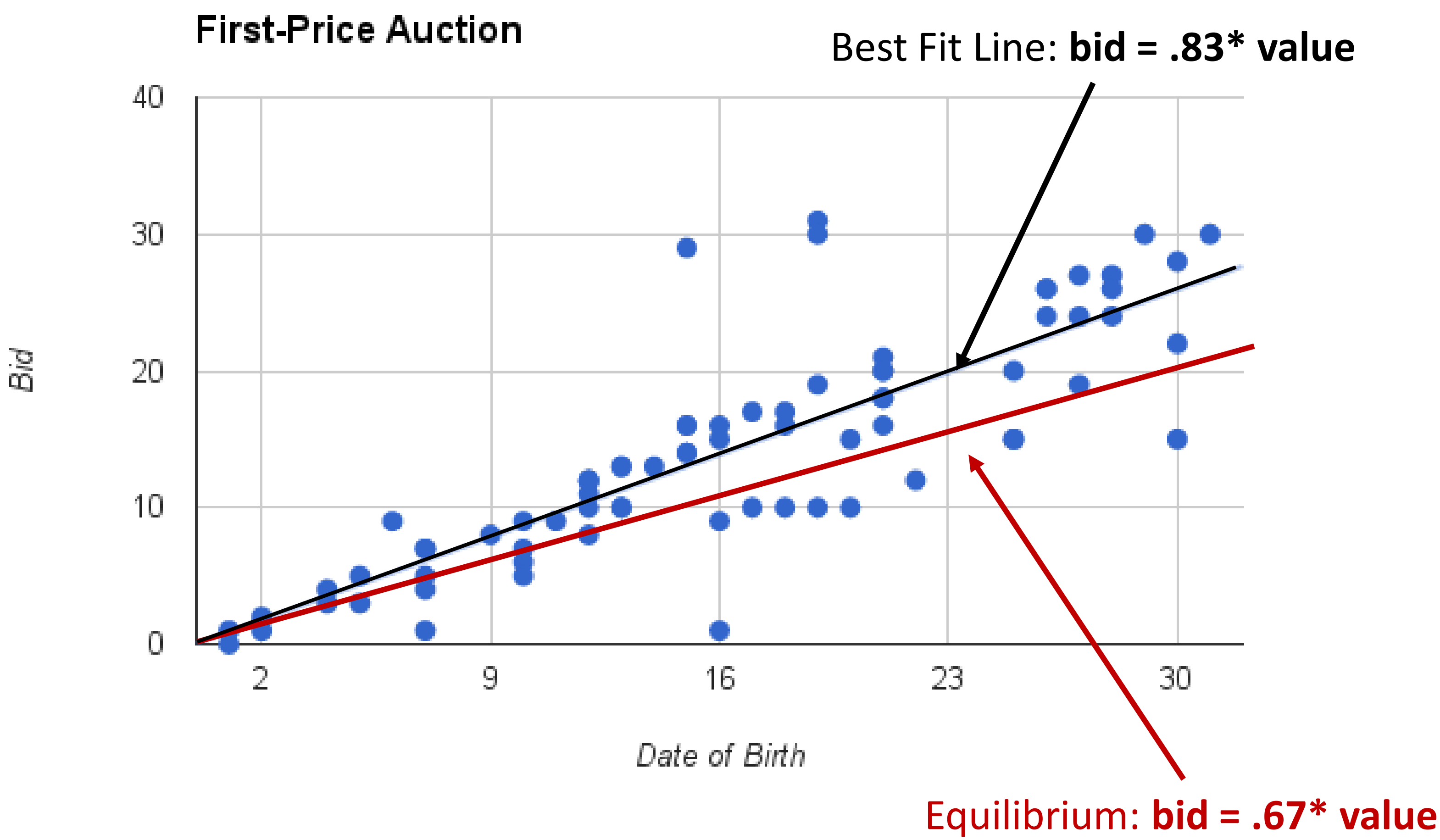
Equilibrium

- My rival is doing the same calculation as me.
  - If he conjectures that I bid ½ my value
  - He should bid ½ his value (for the same reasons)

- Therefore, in **equilibrium**, we each bid half our value.

- More generally, with N bidders, bids = **v*(N-1)/N**

Your Bids (3 bidders)



Bayesian Nash Equilibrium

- Uncertainty over rival's payoffs in this game
- Best-respond to *expectation* of your rival's strategy
- Your rival does likewise
- Mutual best responses in this setting are called *Bayes-Nash Equilibrium*.

M&A Auction Game

1) Want to acquire a large (2-division) company

2) You will bid for the **company's stock**

3) The company's **true value = sum of two divisions' values**

4) Your firm has expertise in one area

5) Can estimate the value of one division / sector

6) Uncertain about the rest of the company

Wallets Game

1) Check **how much cash is in your wallet**.

2) That is your (perfect) estimate of 1 division.

3) I will randomly match you with **1 other bidder**.

4) **Bid** for the company's stock (= **sum of wallets**)

The Bidder's Problem

- Your wallet contains **v** dollars.
- The other bidder's wallet contains **x** dollars.

- You don't know **x**, but it is randomly (uniformly) drawn from **0** to **100**.
- The company is worth **v + x**.

- You conjecture a bidding strategy **b(x)**
- Choose a bid, **B**, to maximize expected profits:

  u = **v+x-B** if you win and loser's value is **x**

  u = **zero** if you lose

Cautious Opponents

- Suppose your opponent thinks as follows:
  1. *"I am afraid the other wallet is empty."*
  2. *"I will never bid more than my wallet's content."*
  3. *"So I'll just bid **b(x) = x**."*
- How do you respond to **b(x)=x**?
- What are your profits if you win against opponent **x**?

v+x-B

How Should you Bid?

- **$\Pr[\text{win} \mid B] = B / 100$**
- Maximize (v+x-B)B?
- Choose B = (v+x)/2 … Don't know *x*
- …. so I should bid (v+50)/2 = 25 + v/2? Right?

When you win, **x < B**!

- Maximize (v+B/2-B)B

→v – B = 0 ➔ B = v!!

→Bid just your wallet's content!

Lessons from Wallets

- Suppose your opponent bids aggressively (a>1)
- Avoid the winner's curse
- Suppose your opponent is overly cautious (a<1)
- Take advantage of it!!

Seller Revenues

- Common-value auctions: revenue equivalence holds only under very special circumstances (symmetry)
- Open- or sealed bid? Are SPA and English auction still strategically equivalent?
- In general, winner's curse ➔ **English > SPA > FPA**
- Instructive for the history of online ad auctions…