Game Theory
for
Strategic Advantage

15.025

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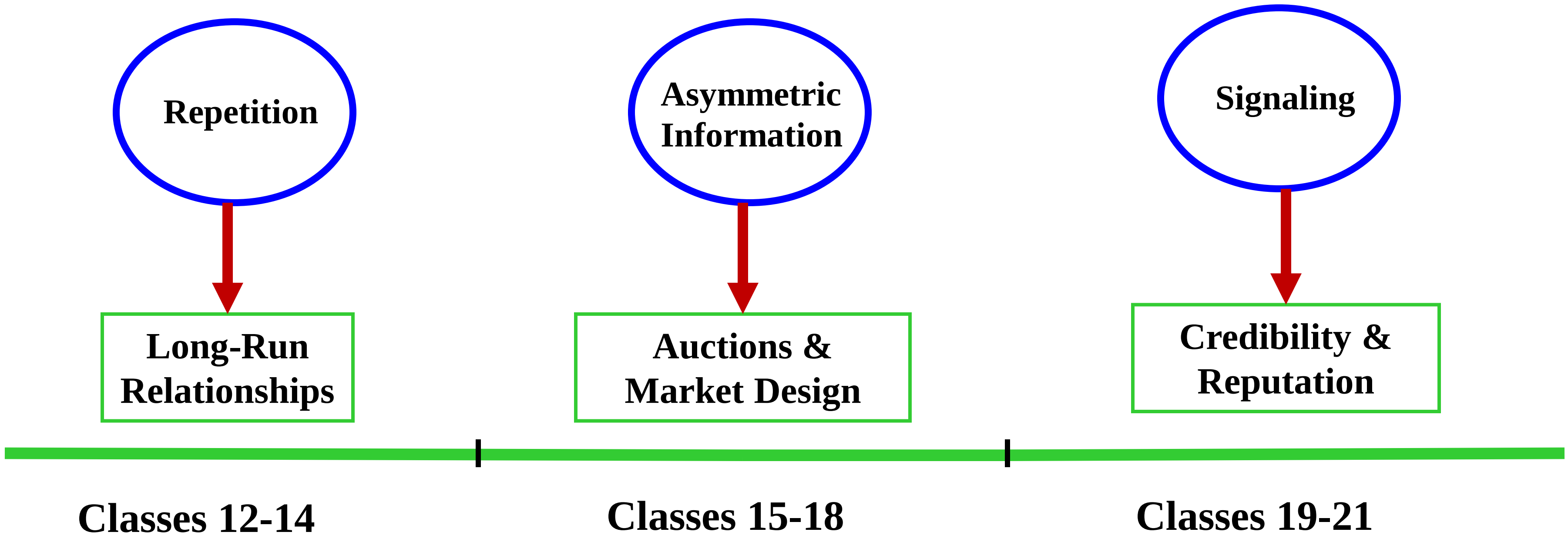
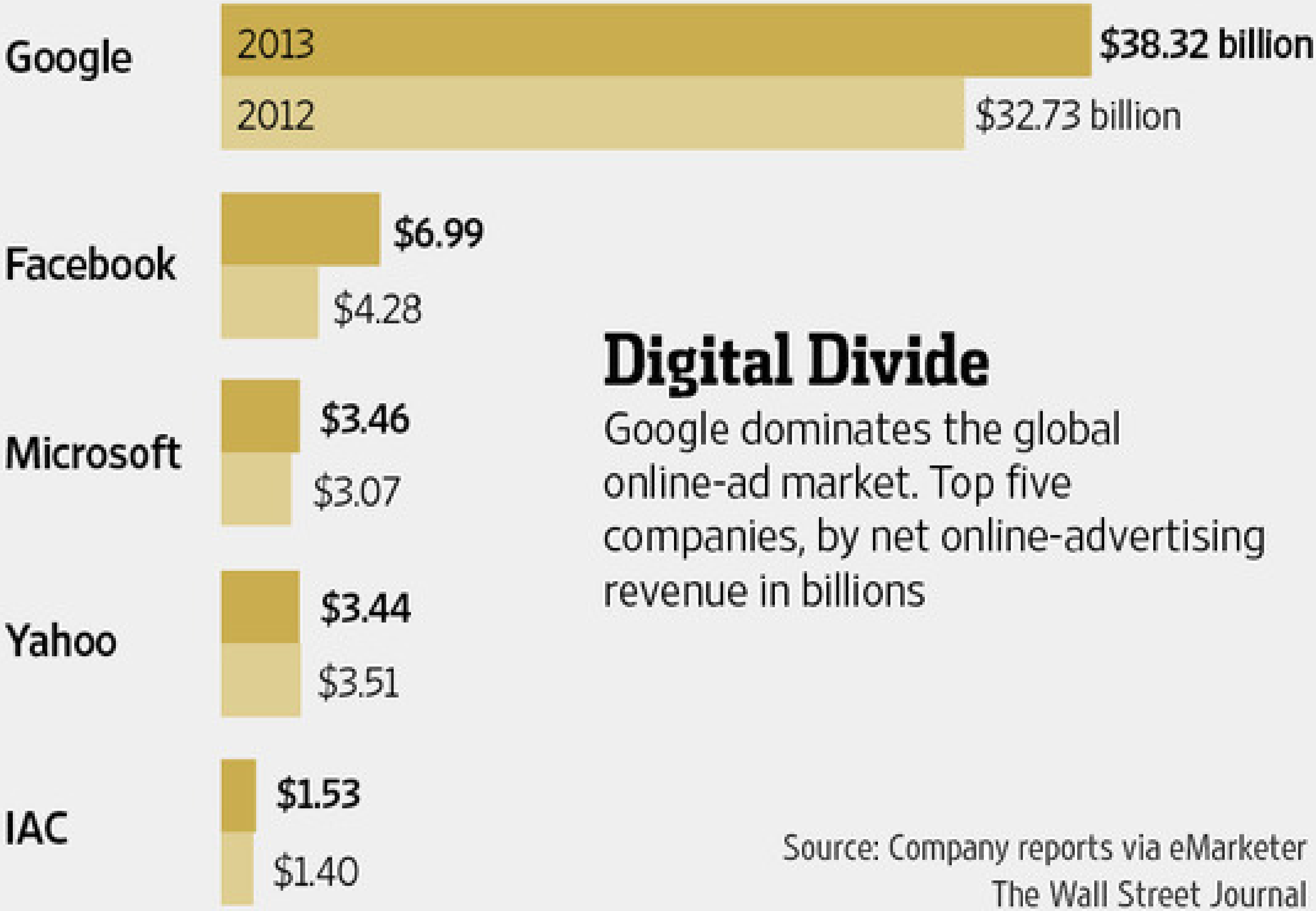
Part III: “Big” Applications




Google
2013
$38.32 billion
2012
$32.73 billion
Facebook
$6.99
$4.28
Microsoft
$3.46
$3.07
Yahoo
$3.44
$3.51
IAC
$1.53
$1.40
Digital Divide
Google dominates the global online-ad market. Top five companies, by net online-advertising revenue in billions
Source: Company reports via eMarketer
The Wall Street Journal

Uncertainty Example: an Auction

- Two firms (GE vs. W) bid for a contract.
- The value of the contract to ***GE*** is v_{GE} **= *$65M***.
- Say you are ***GE***: *h*ow much do you bid?
- Do you have all the information you'd like?
- ***GE*** doesn't know v_W.
- ***W*** doesn't know v_{GE}.

Today’s Class

1. Uncertainty in games
2. New equilibrium notion
3. Applications: basic auctions

Looking Ahead

1. Reserve prices & winners’ curse
2. Online auctions
3. Designing auctions and markets

Uncertainty in Canonical Games

Game Type	**Source of Uncertainty**
Prisoners' Dilemma	Gain from defection
Chicken / Entry	Cost of acting tough / entry
Stag Hunt	Go-it-alone value
…	…
Coordination	Strength of common interest
Beauty contest	Opponents' sophistication

What game is my opponent seeing?

Our Old Entry Game

- The (gross) value of winning the market alone is 50.
- Each player *i={1,2}* has a cost 30 of investing.
- If both enter, price competition erases all (gross) profits

		Player 2	
		In	Out
Player 1	In	(-30, -30)	(20, 0)
	Out	(0, 20)	(0, 0)

- **No dominated strategies**

Entry Game Revisited

- The (gross) value of winning the market alone is 50.
- Each player *i={1,2}* has a cost c_i of entering.
- If both enter, price competition erases all (gross) profits

		Player 2: In	Player 2: Out
Player 1	In	$(-c_1,-c_2)$	$(50-c_1,0)$
	Out	$(0,50-c_2)$	(0,0)

- **Any dominated strategies?**
- **What if I'm not sure about Pl. 2's cost?**

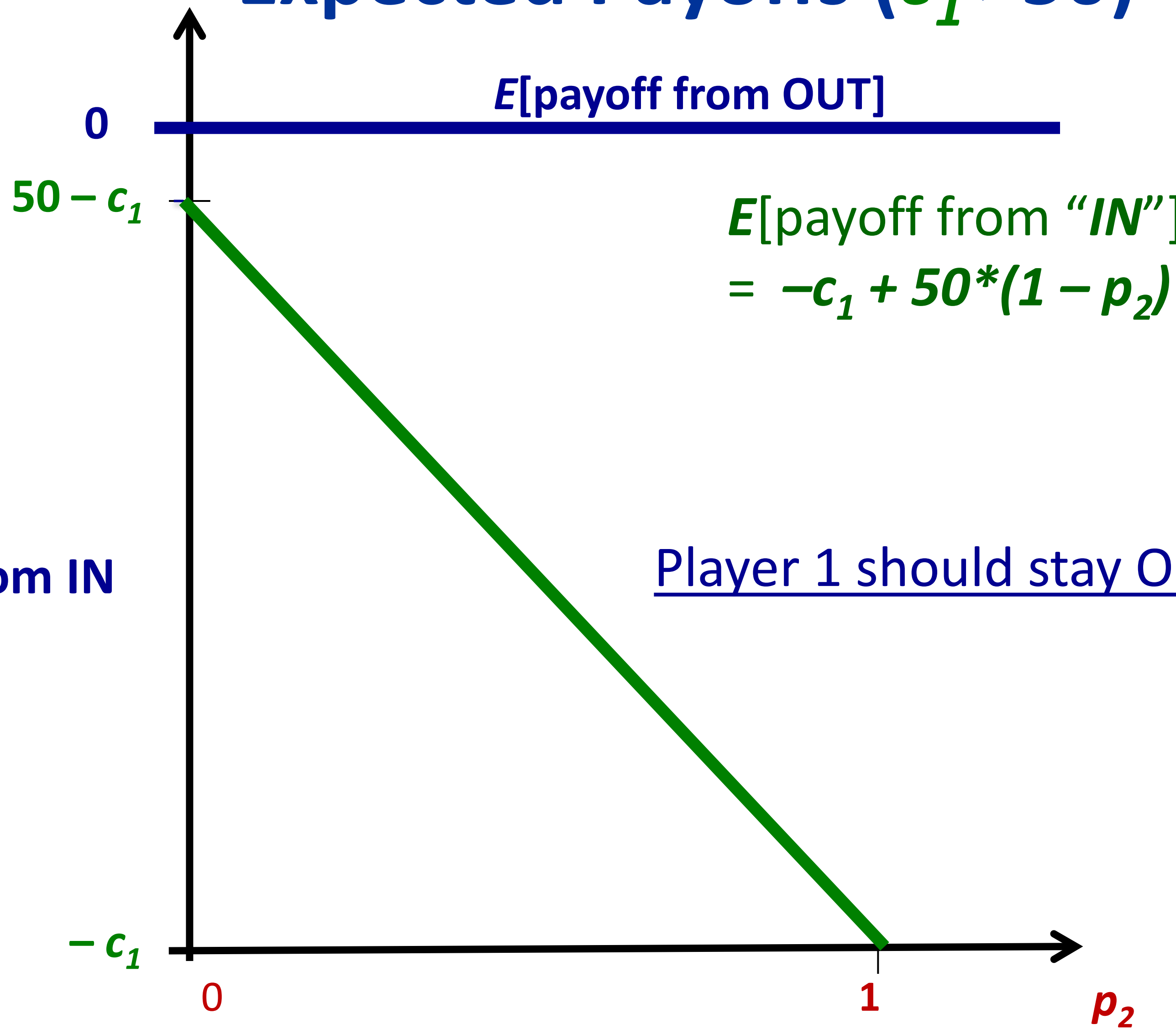
Information Structure

Each player's c_i is uniformly drawn from [0, 100].
The two draws are independent.
Players <u>know their own cost only</u>.

		Player 2	
		In	Out
Player 1	In	$(-c_1,-c_2)$	$(50-c_1,0)$
	Out	$(0,50-c_2)$	(0,0)

- How to proceed? Let's play!!

Expected Payoffs ($c_1 > 50$)

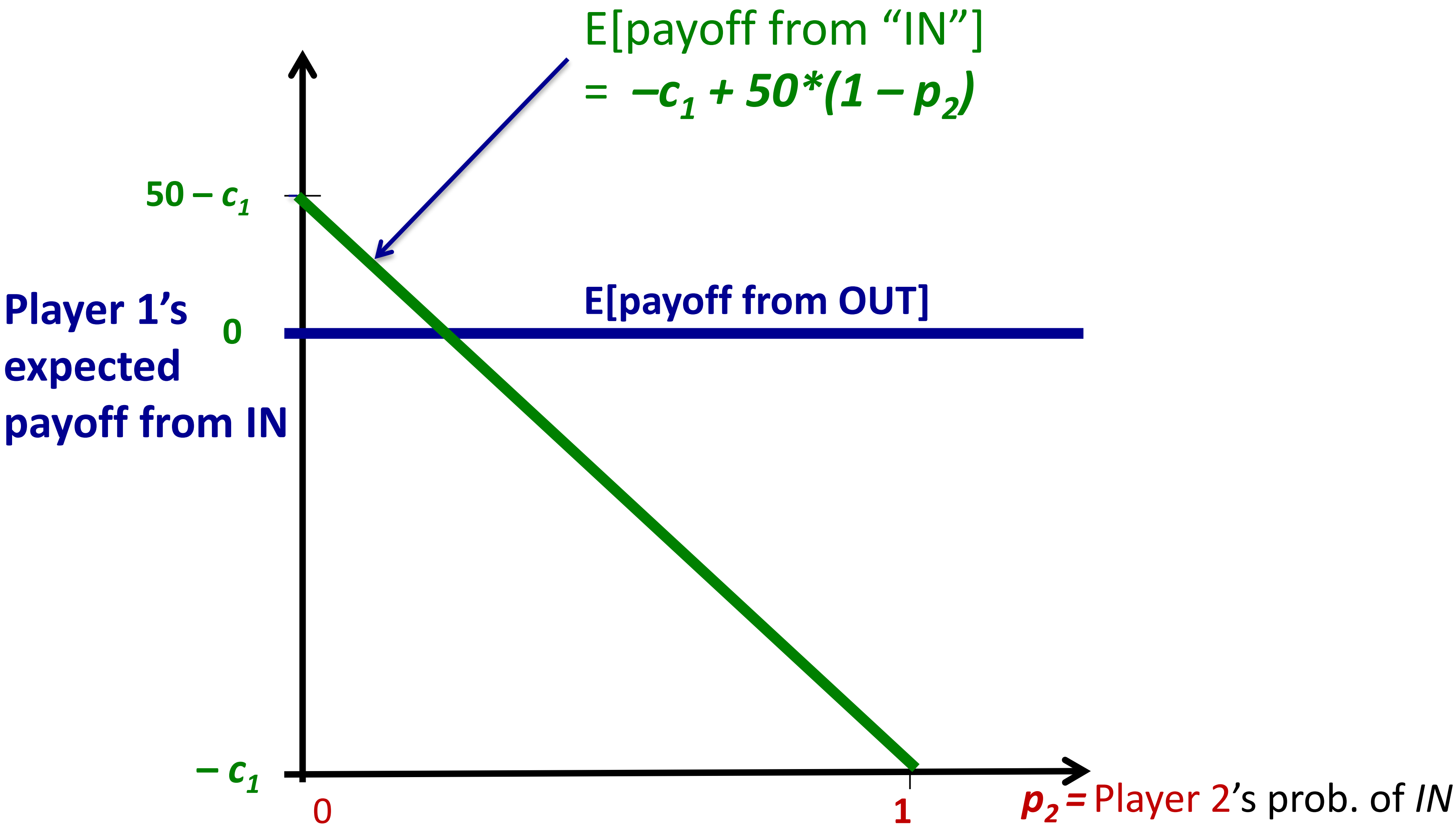


Player 1's expected payoff from IN

Player 1 should stay OUT if $c_1 > 50$

p_2 = Player 2's prob. of *IN* (from 1's point of view)

Expected Payoffs (c_1 <50)



Player 1 should enter if $c_1 < 50*(1-p_2)$

How Do I *Know* p_2?

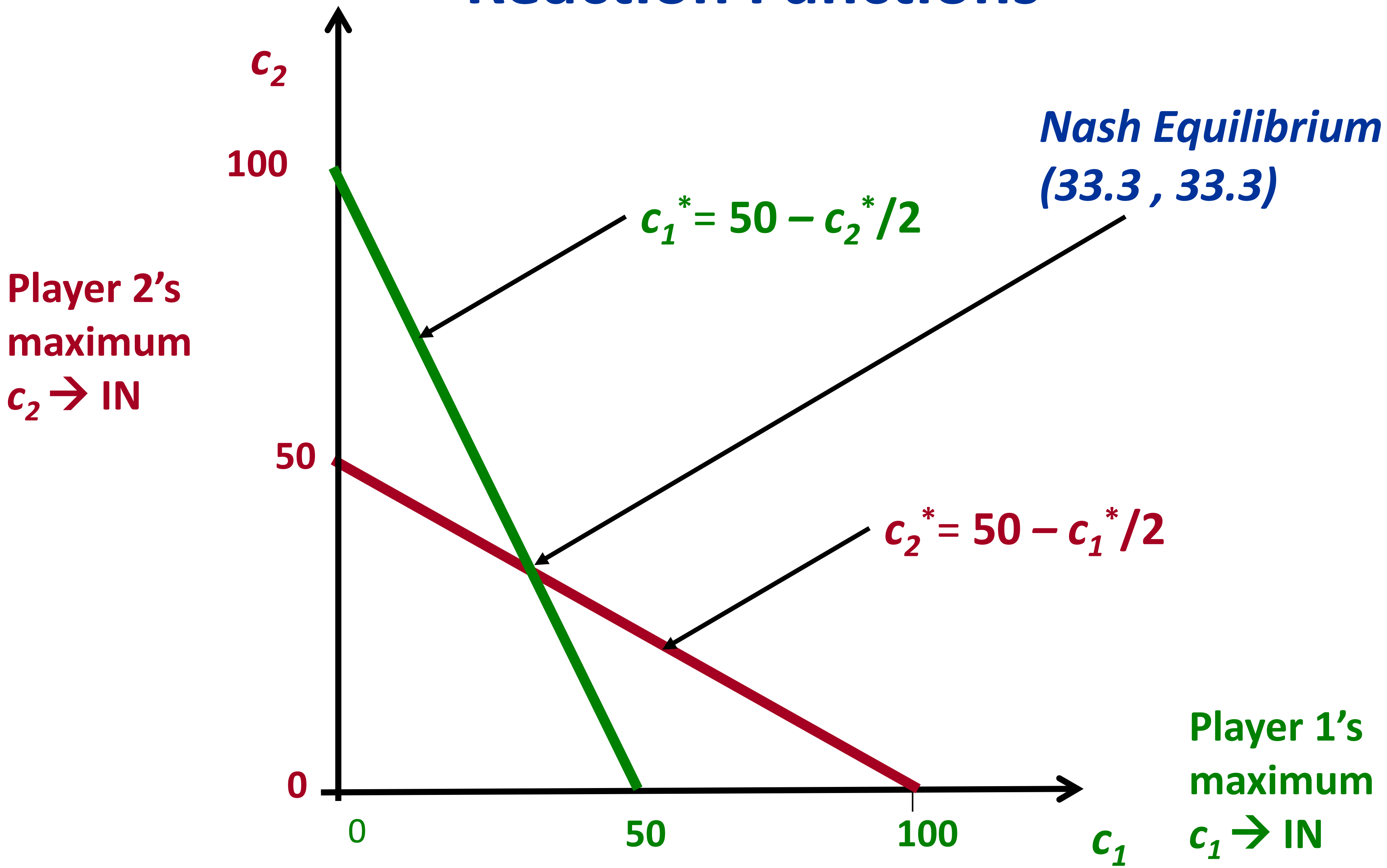
For which cost levels does Pl. 2 choose IN?

- Suppose player 2 chooses IN if $c_2 < 50$.
- Then p_2 = Prob (*IN*) = Prob ($c_2 < 50$) = 0.5,
- Then Pl. 1 → IN if $c_1 < 25$. Which means $p_1 = 0.25$
- But then Pl. 2 should go IN if and only if $c_2 < 37.5$.
- … which means Pl. 1 → IN if $c_1 < 31.25$.

More general criterion: Reaction Functions

$$c_1 = 50(1-p_2) = 50(1-c_2/100) = 50 - c_2/2$$

"Reaction Functions"



Solving for Equilibrium

- Equilibrium = two cut-offs (c_1^*, c_2^*) such that
  - c_1^* = max(c_1)→*IN* <u>given that</u> Pl. 2→*IN* if $c_2 < c_2^*$
  - c_2^* = max(c_2)→*IN* <u>given that</u> Pl. 1→*IN* if $c_1 < c_1^*$

- $c_1^*(c_2^*) = 50 - c_2^*/2$ and $c_2^*(c_1^*) = 50 - c_1^*/2$
- $c_1^* = c_2^* = 100/3 = 33.3\ldots$
- $p_1 = p_2 = 1/3$
- E[payoff(IN)] $= -c_i + 50*(1-1/3) = 33.3 - c_i$

(Bayesian) Nash Equilibrium

- A Nash equilibrium of this (Bayesian) game is:

1) A critical value c_1 for Pl. 1 such that playing *IN* for costs below c_1 is a best response to Pl. 2's play

2) A critical value c_2 for Pl. 2 such that playing *IN* for costs below c_2 is a best response to Pl. 1's play

- Best response = maximize expected payoff!

Right and Wrong Information

- In the BNE, entry is profitable only if c<33.3
- Cost distribution: uniform [0, 100]
- Expected cost = 50
- On average, my opponent's dominant strategy is **OUT**
- Best response to expected cost = **IN** !! (given c<50)

- This uses the **wrong information**!! (expected cost)
- **Right information**: **expected action** (**IN with Pr=1/3**)
- **Correct strategy:** IN if c<33.3