Notes on the Theories of Growth

Economic Growth & Development

These recitation notes cover basic concepts in economic growth theory relevant to the cases seen in class (Singapore, Indonesia, Japan, and Mexico). Robert Solow from the MIT Department of Economics, pioneered the subject of growth theory in the article "A Contribution to the Theory of Economic Growth" (Quarterly Journal of Economics, Feb. 1956, pp. 65-94.)

The Solow Growth Model

Solow's model explains the growth in an economy by breaking down the aggregate output (Y or GDP) into contributions of growth inputs (labor, capital, and technology). That is, the model explains how much of the growth in an economy is explained by changes in the amount of labor or by changes in the amount of output.

The general model is described by the following formula:

$$Y(t) = A(t) \times K(t)^{\alpha} \times L(t)^{1-\alpha}$$

where,

- Y is aggregate output of the economy in year (t) usually measured by GDP,
- A is an index of the level of technology,
- K is the stock of capital in the economy,
- L is the amount of labor in the economy usually measured by hours worked by an index of labor efficiency
- α is the contribution of capital to aggregate output Y.

All of these variables con be observed by looking at economic indices of each country except A. Therefore, we can solve in the equation for A and find the contribution technology improvements had in the economy!!!! α, or alpha, is the share of output that were paid to owners of capital in the form of rents. Capital includes machinery, equipment, land and natural resources. Whereas the remainder $1-\alpha$, is the share of output paid to workers in the form of wages.

A is also known as Total Factor Productivity (TFP) and includes generally changes in the level of technology, presence of strong institutions, and regulatory environment. The equation above is a production function which can be applied to an individual business. The equation has constant returns to scale, which means that the contribution of capital and labor to aggregate output will always add up to one.

The above equation can be transformed by taking the logarithm and partial derivatives to arrive to a simpler form:

$$\%\Delta Y = \%\Delta TFP + (\alpha) \times (\%\Delta K) + (1-\alpha) \times (\%\Delta L),$$

which simply expresses changes in output, technology, capital, and labor in percentage changes. The table below shows the factors contribution to growth for 18 countries.

Growth Accounting Sample

	GDP Gr. Rate	Contrib. Capital	Contrib. Labor	Contrib. TFP	Alpha
1960-1990					
Canada	4.10%	2.29%	1.35%	0.46%	45%
France	3.50%	2.03%	0.02%	1.45%	42%
Germany	3.20%	1.88%	-0.25%	1.58%	40%
Italy	4.10%	2.02%	0.11%	1.97%	38%
Japan	6.81%	3.87%	0.97%	1.96%	42%
U.K.	2.49%	1.31%	-0.10%	1.30%	39%
U.S.	3.10%	1.40%	1.29%	0.41%	41%
1940-1980					
Argentina	3.60%	1.55%	0.95%	1.10%	54%
Brazil	6.40%	3.25%	1.30%	1.85%	45%
Chile	3.80%	1.30%	1.00%	1.50%	52%
Colombia	4.80%	2.05%	1.55%	1.20%	63%
Mexico	6.30%	2.55%	1.45%	2.30%	69%
Peru	4.20%	2.85%	1.35%	0.00%	66%
Venezuela	5.20%	2.95%	1.75%	0.50%	55%
1966-1990					
Hong Kong	7.30%	3.09%	2.00%	2.20%	37%
Singapore	8.50%	6.20%	2.68%	-0.40%	53%
South Korea	10.32%	4.77%	4.35%	1.20%	32%
Taiwan	9.10%	3.68%	3.62%	1.80%	29%

Capital and Investments

We can also express the original equation in terms of per capita output and capital per worker by simply dividing by L. Capital is a stock variable which means that it accumulates through time, while investments is the amount of capital plugged into the economy <u>in a given year</u>. Recall the aggregate income or GDP equation of: $Y = C + I + G + (Exports - Imports)$. We will re-write it ignoring government expenditures G, and assuming a close economy (Exports-Imports=0), to get:

$$Y = C + I.$$

Which can be expressed in per capita terms by dividing by L and get:

$$y = c + i,$$

where lower caps is the notation for "per capita". At the same time consumption is assumed in the model to only depend on the savings rate and income y. Therefore,
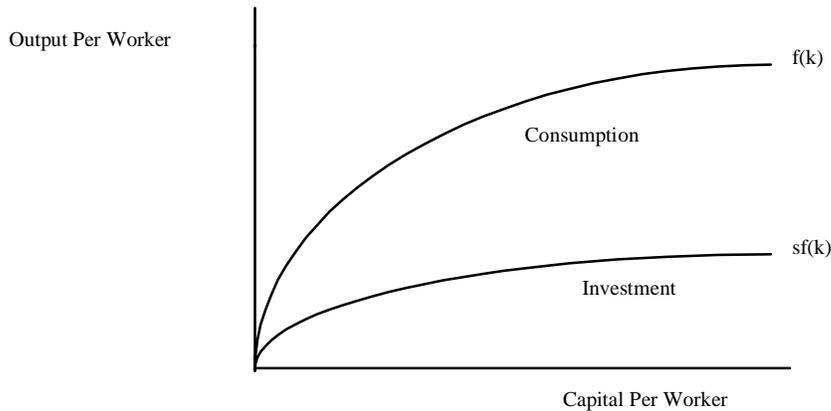
$$c = (1-s) \, y.$$

By substituting this equation in the previous equation and solving for i, we get:

$$y = (1-s)y + i,$$
$$i = sy.$$

We now graph these equations to see that output is decomposed into consumption and investment (which in turn is equal to savings).

Graph 1

Notice that since the equations are expressed in per capita terms, the slope of the function y=f(k), or output per capita, is the Marginal Product of Capital. That is, for poorer countries, which have low levels of capital per worker and are located closer to the origin on the horizontal axis, then the MPC must be higher. This is consistent with the fact that business seek higher return on investments (roe) projects in poorer countries.

Remember that capital is expressed in cumulative terms. But, capital depreciates with time, which means that with no new investments, the capital stock per worker will decrease. We can take care of this by depreciating capital at a constant rate δ. Therefore, the capital stock will be affected by two factors:
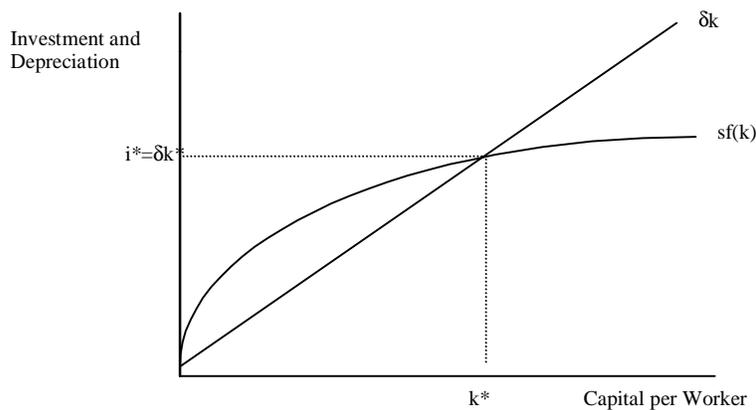
- new investments made in a given year, and
- depreciation of the capital stock in that year.

And can be expressed as:

$$\Delta k = i - \delta k,$$

which means that the change in the capital stock per worker is equal to new investment per worker minus the depreciation of the capital stock per worker.

Graph 2

The level of investment per worker which exactly offsets depreciation of the capital stock per worker is called the **steady-state-level of capital**. If capital k is to the left of k* it means that investment will be higher than depreciation and thus the capital stock per worker will rise to k*. Similarly, if k is to the right

of k*, depreciation will be higher than investment (the capital stock will be depleted by more than the new investment put in) and k will fall to k*.

You can combine the two graphs to get the steady state of output per capita or growth rate per capita.

Graph 3

Output Per Worker

f(k)

y*

Consumption

δk

sf(k)

Investment

k*
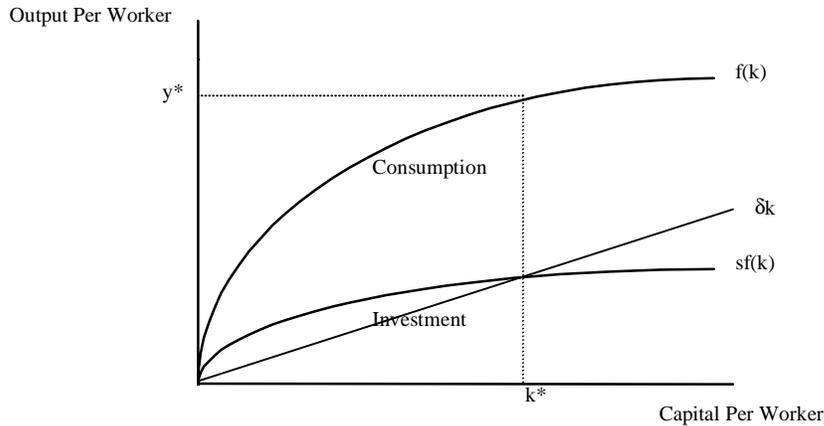
Capital Per Worker

Labor and Population Growth

Population growth has two effects:
- increases the labor force and thus the amount of labor hours, and
- decreases the amount of capital stock per worker.

We can include population growth in the model quite easily. First, we assume that the rate of growth of population is equal to the rate of growth of the labor force. The rate of growth of the labor force is called n. Since we expressed our original equation in per capita terms, we now include the labor growth rate as:
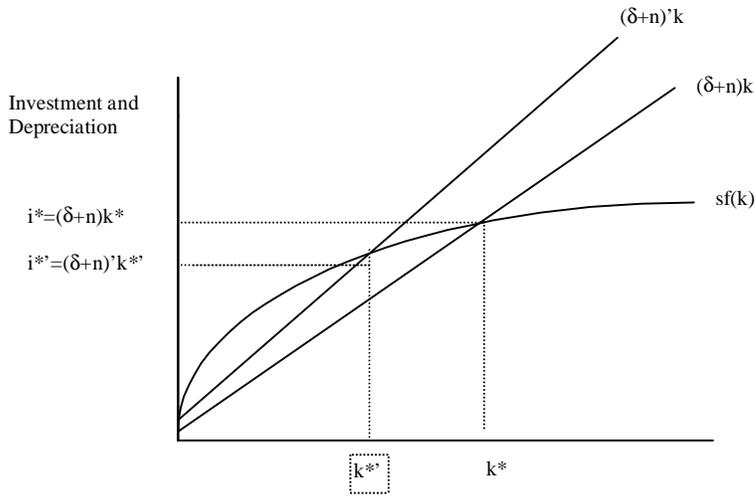
$$\Delta k = i - (\delta + n)k,$$

and the corresponding graph below.

Graph 4

Investment and Depreciation

$(\delta + \nu)\kappa$

$i^* = (\delta + \nu)\kappa$

sf(k)

k*
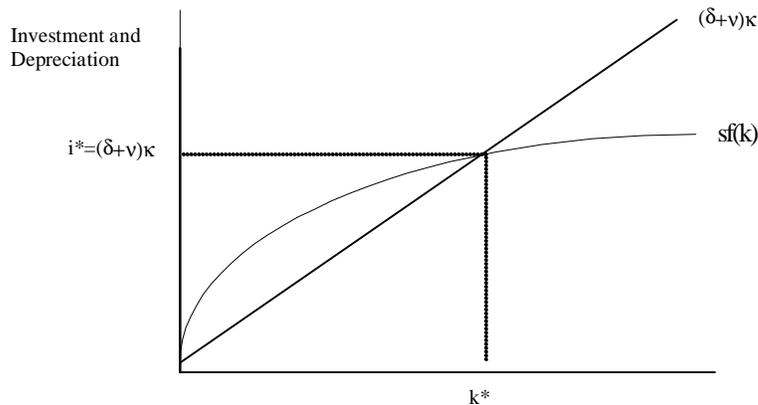
We can reflect an increase in the rate of depreciation or an increase in labor force growth in the same way; by shifting the diagonal up.

Graph 5

Investment and
Depreciation

$(\delta+n)'k$

$(\delta+n)k$

$sf(k)$

$i^*=(\delta+n)k^*$

$i^{*'}=(\delta+n)'k^{*'}$

$k^{*'}$ k^*

Factors Influencing Growth

In summary, the model above states that increases in output can be achieved by increases in:
- capital
- labor
- total factor productivity (TFP)

The first two can be mostly categorized as an expansion of the economy. That is, if the economy increases its capital stock and its labor hours by 2%, then aggregate output will expand by 2% as well. However, the people (as measured by the amount of labor) will still have the same income per capita. Their standard of living is unchanged!!!!! The more difficult task is increasing the total factor productivity, which will raise the standard of living of your population. Increasing TFP takes a longer time horizon because it involves increasing the technological level, transforming institutions, deregulating the economy, and improving education. For example, if these factors improve by 1%, then an increase in capital stock and labor hours of 2% will instead result in a 3% growth of the economy.

Robert Barro (from Harvard) has extensively studied the factors influencing growth. His studies are geared at finding measurable variables which help explain growth rates in an economy. Here are the results of his regression model which uses per capita growth rate as the dependent variable. The numbers in the second column are the coefficients found in the regression. Notice that negative coefficients mean that, for example, when fertility rates increases, then per capita growth rate decreases.

Variables Influencing Growth	
Log (GDP)	-0.025
Male Secondary and Higher Education	0.012
Log (Life Expectancy)	0.042
Log (GDP)* Male Schooling	-0.006
Log (fertility)	-0.016
Government Consumption	-0.136
Rule of Law Index	0.029
Terms of Trade Index	0.137
Democracy Index	0.090
Democracy Index Squared	-0.088
Inflation Rate	-0.043