Problem Set 11 Solution

17.881/882

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1 Gibbons 4.1 (p.245)

1.1 Game A

The normal form representation of this game is the following:

	L'	R'
L	$(4, 1)$	$(0, 0)$
M	$(3, 0)$	$(0, 1)$
R	$(2, 2)$	$(2, 2)$

The pure-strategy Nash Equilibria are (L, L') and (R, R'). Since there are no proper subgames, these are also subgame perfect.

Let us now find conditions on p such that (L, L') and (R, R') are perfect Bayesian equilibria.

Requirement 1

Player 2 has *belief* that player 1 has played L with probability p and M with probability $1 - p$

Requirement 2

Given p, player 2's expected payoff from playing L' and R' are

$E(L') = p; \qquad E(R') = 1 - p$

Thus, it is sequentially rational for player 2 to choose L' if and only if $p \epsilon [1/2, 1]$ and R' if and only if $p \epsilon [0, 1/2]$.

Given player 2's belief, player 1's strategy should also be sequentially rational. If player 2 chooses L', player 1 should choose L. If player 2 chooses R', player 1 should choose R.

Requirement 3

Consider the NE (L, L'). Player 2 gets to play on the equilibrium path. Thus, player 2's belief p must be 1. So $(L, L', p = 1)$ represents a pbe.

Consider the NE (R, R'). Player 2 does not have to play on the equilibrium path. Requirement 3 places no restrictions on p

Requirement 4

(R, R') is off the equilibrium path. Requirement 4 does not impose any restriction on p.

To sum up, we have the following two perfect Bayesian equilibria:

$(L, L', p = 1), (R, R', p \epsilon [0, 1/2])$

1.2 Game B

The normal form representation of this game is the following:

	L'	$M\prime$	R'
L	$(1,3)$	$(1,2)$	$(4,0)$
M	$(4,0)$	$(0,2)$	$(3,3)$
R	$(2,4)$	$(2,4)$	$(2,4)$

The only pure-strategy Nash Equilibria is (R, M'). Let us now find conditions on p such that this equilibrium is perfect Bayesian.

Requirement 1

Player 2 has *belief* that player 1 has played L with probability p and M with probability $1 - p$

Requirement 2

Given p, player 2's expected payoff from playing L', M' and R' are

$E(L') = 3p; \qquad E(M') = 2; \qquad E(R') = 3(1-p)$

When is it sequentially rational for player 2 to play M'? M' brings a higher expected payoff than L' if and only if $p\epsilon[0, 2/3]$; it brings a higher expected payoff than R' if and only if $p\epsilon[1/3, 1]$. The intersection of these two conditions is $p\epsilon[1/3, 2/3]$.

Given player 2's belief, player 1's strategy should also be sequentially rational. If player 2 chooses M', player 1 should choose R.

Requirement 3

Player 2 does not have to play on the equilibrium path. Requirement 3 places no restrictions on p.

Requirement 4

(R, M') is off the equilibrium path. Requirement 4, by itself, does not impose any restriction on p. We only require that player 2's belief makes (R, M') the optimal strategy for both players. From requirement 2, we have that $(R, M', p\epsilon[1/3, 2/3])$ is a pure-strategy perfect Bayesian equilibrium.