Addressing Alternative Explanations: Multiple Regression

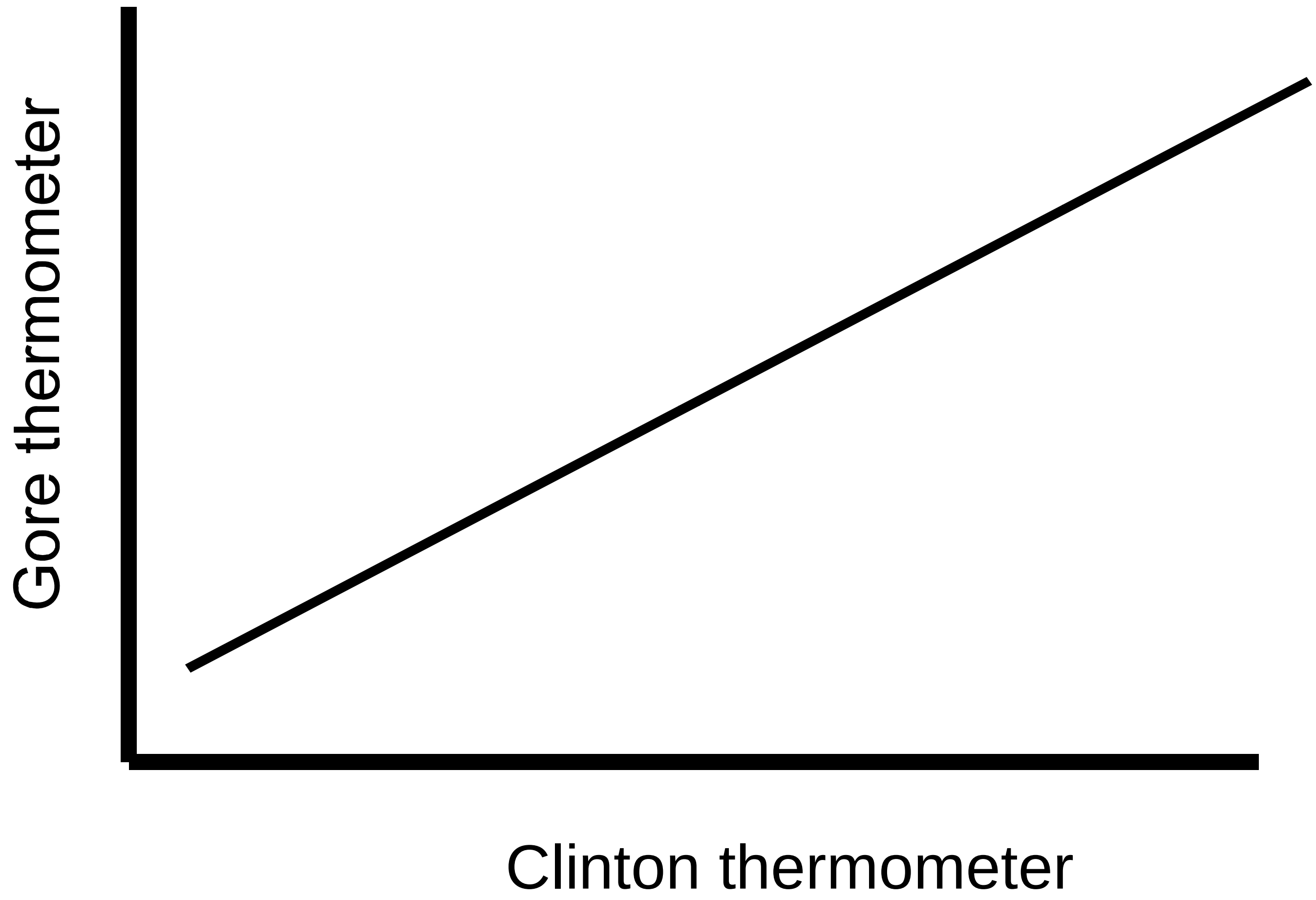
17.871

Spring 2012

Did Clinton hurt Gore example

- Did Clinton hurt Gore in the 2000 election?
  - Treatment is not liking Bill Clinton

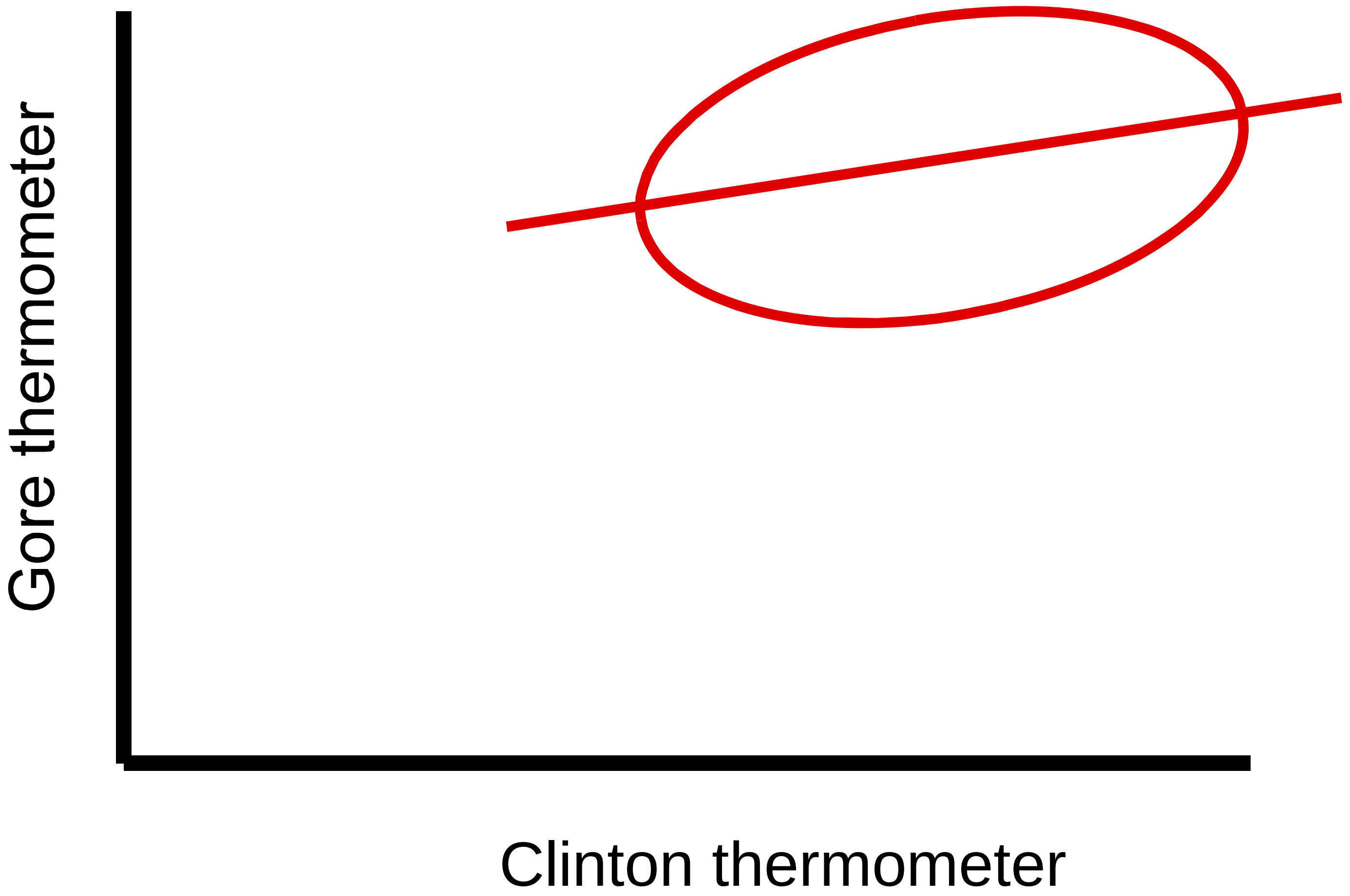
Bivariate regression of Gore thermometer on Clinton thermometer



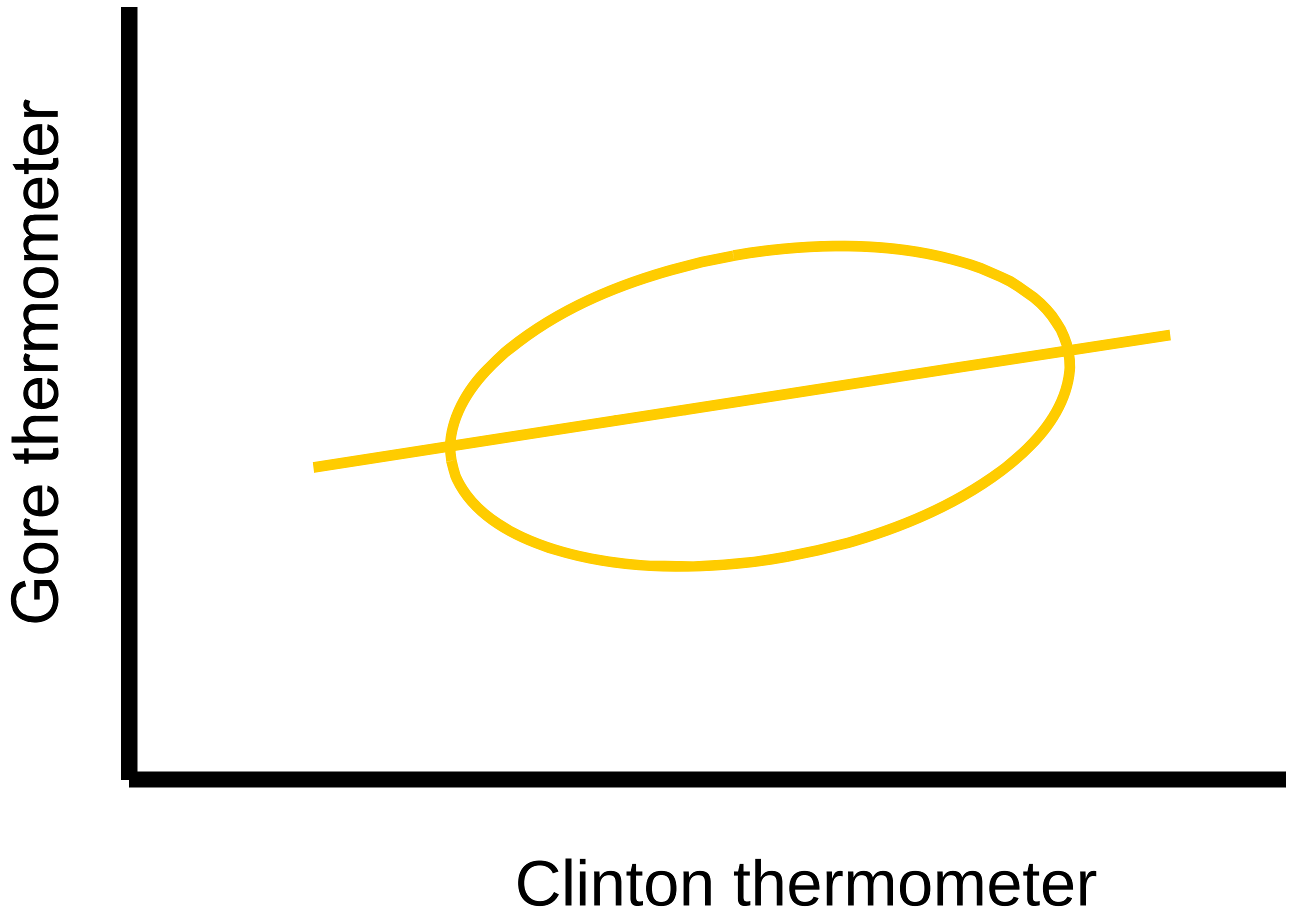
Did Clinton hurt Gore example

- What alternative explanations would you need to address?
- Nonrandom selection into the treatment group (disliking Clinton) from many sources
- Let’s address one source: party identification
- How could we do this?
  - Matching: compare Democrats who like or don’t like Clinton; do the same for Republicans and independents
  - Multivariate regression: control for partisanship statistically
    - Also called multiple regression, Ordinary Least Squares (OLS)
    - Presentation below is intuitive

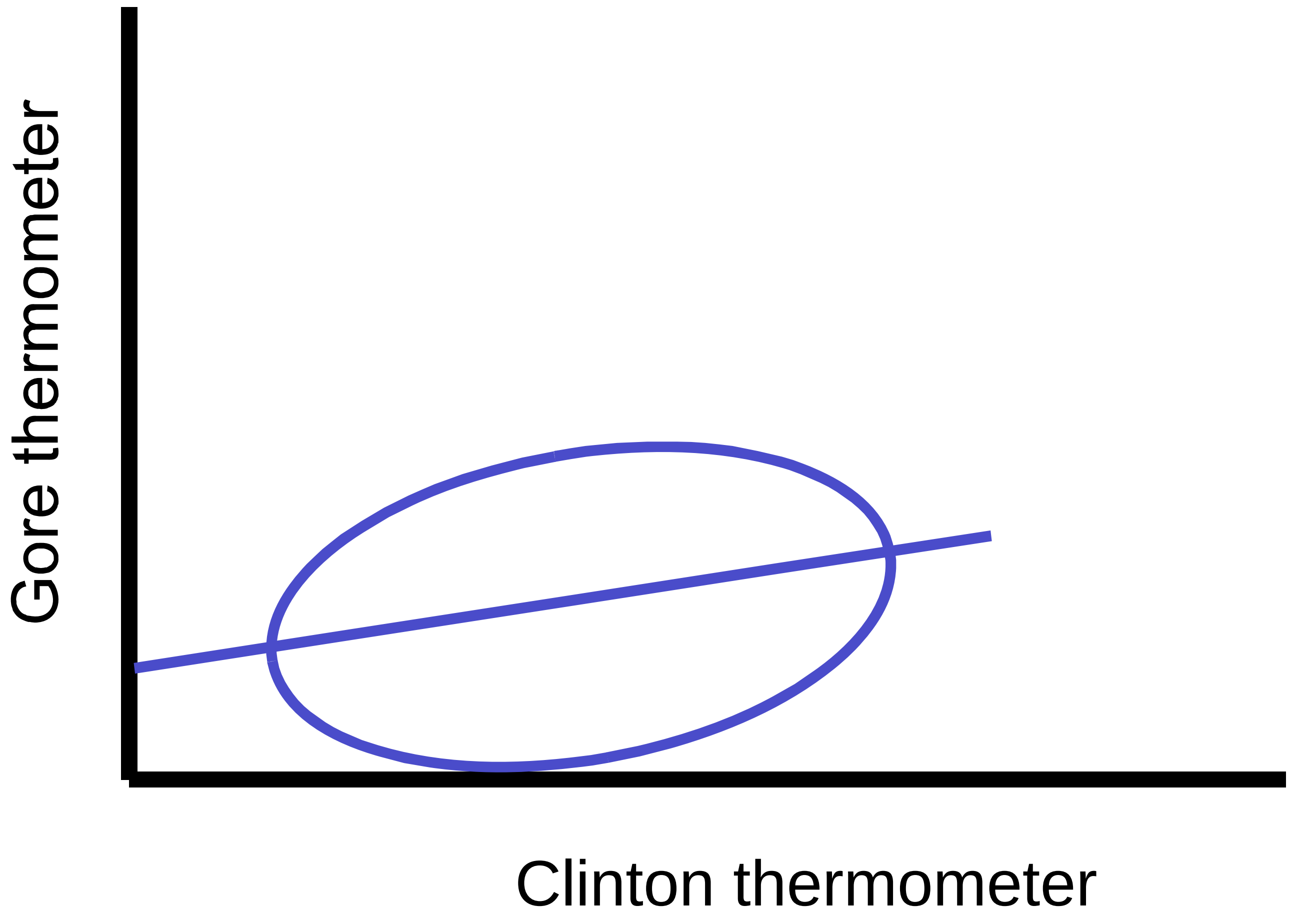
Democratic picture



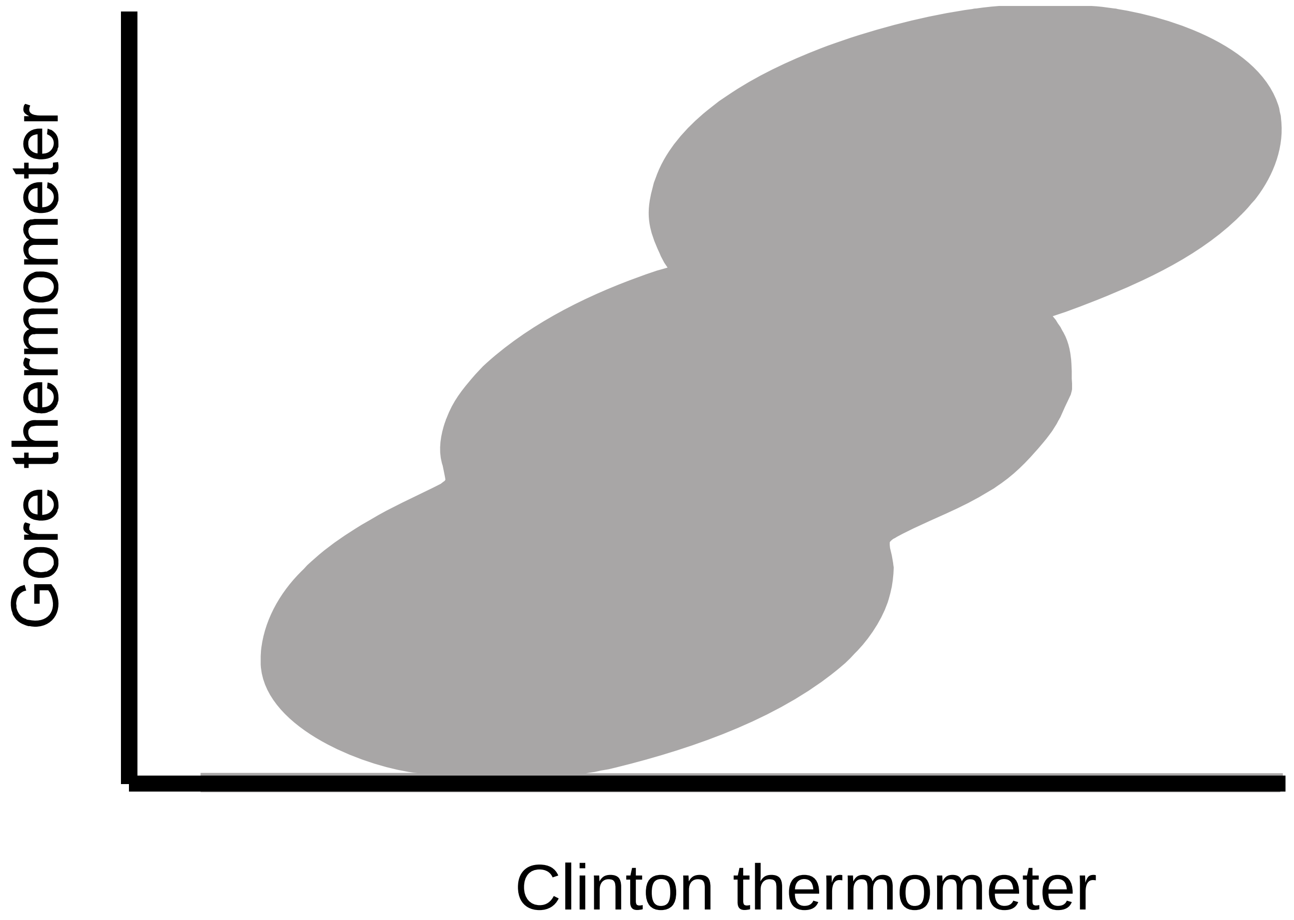
Independent picture



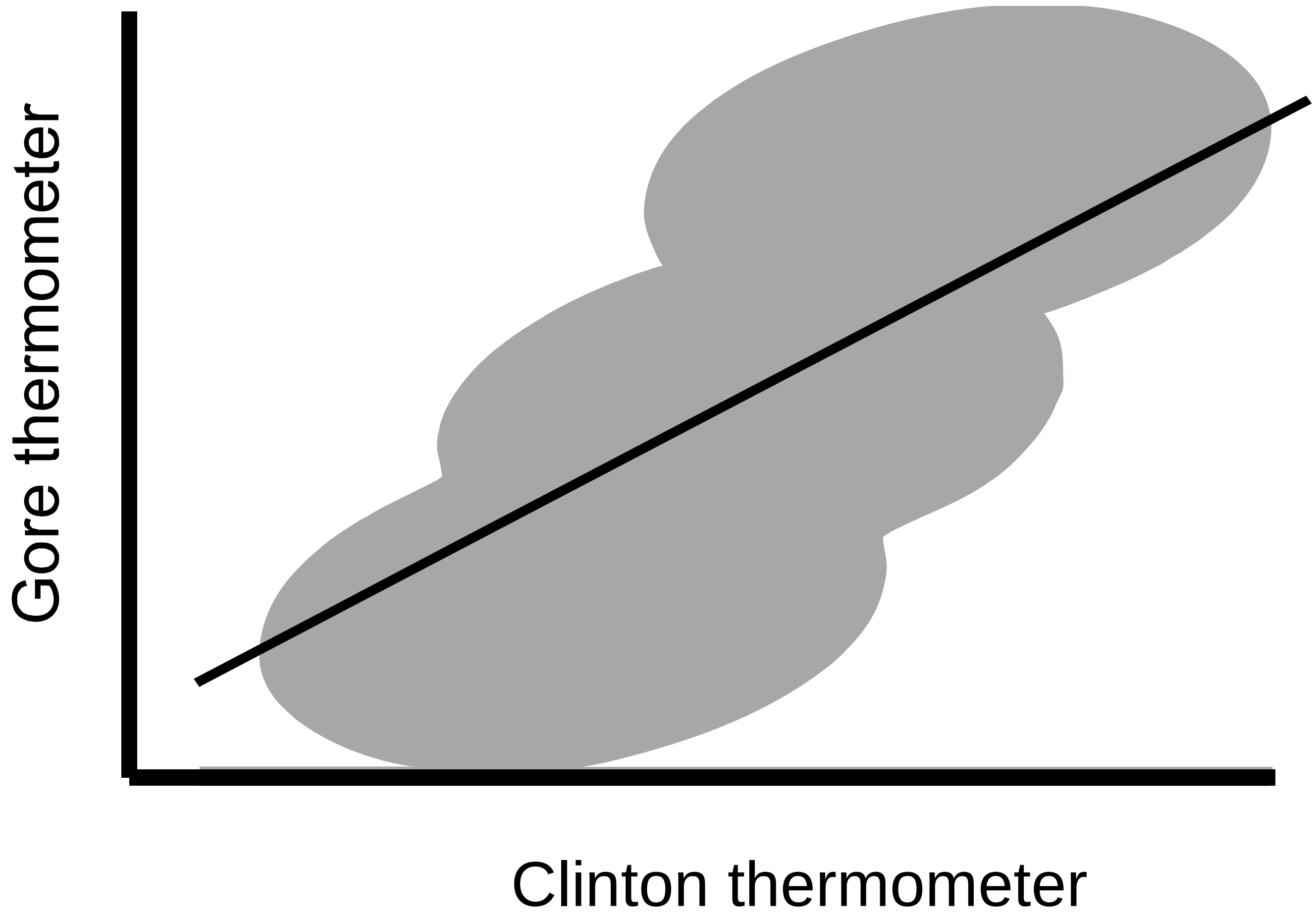
Republican picture



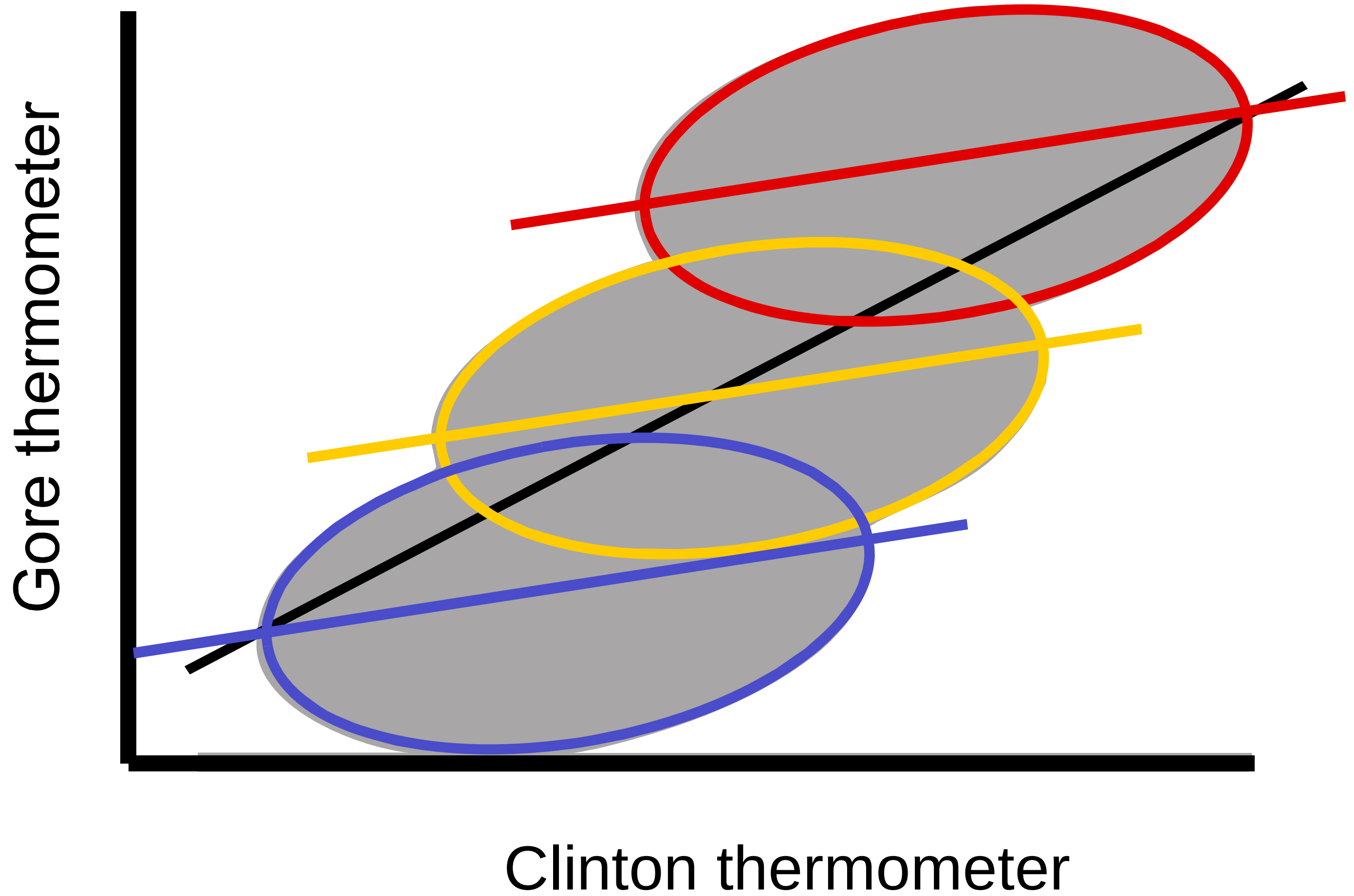
Combined data picture



Combined data picture with regression: bias!

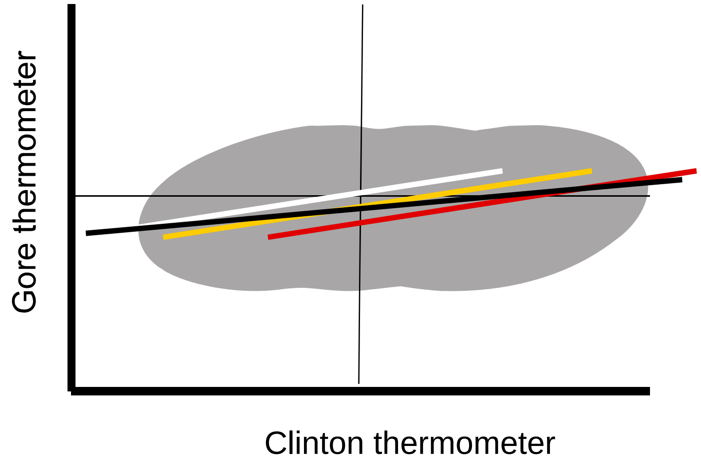


Combined data picture with “true” regression lines overlaid

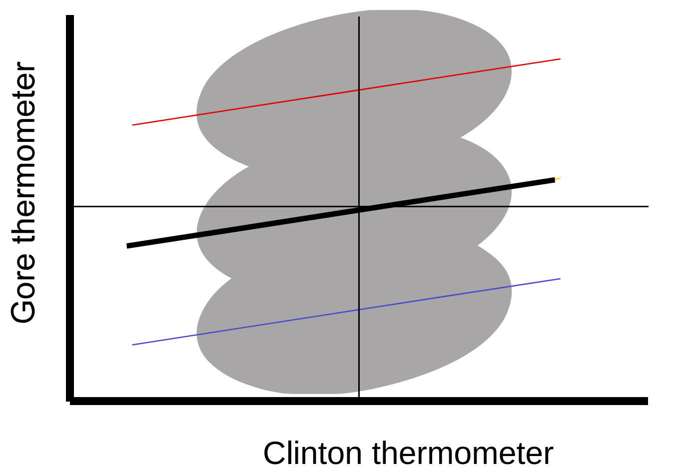


Tempting yet wrong normalizations

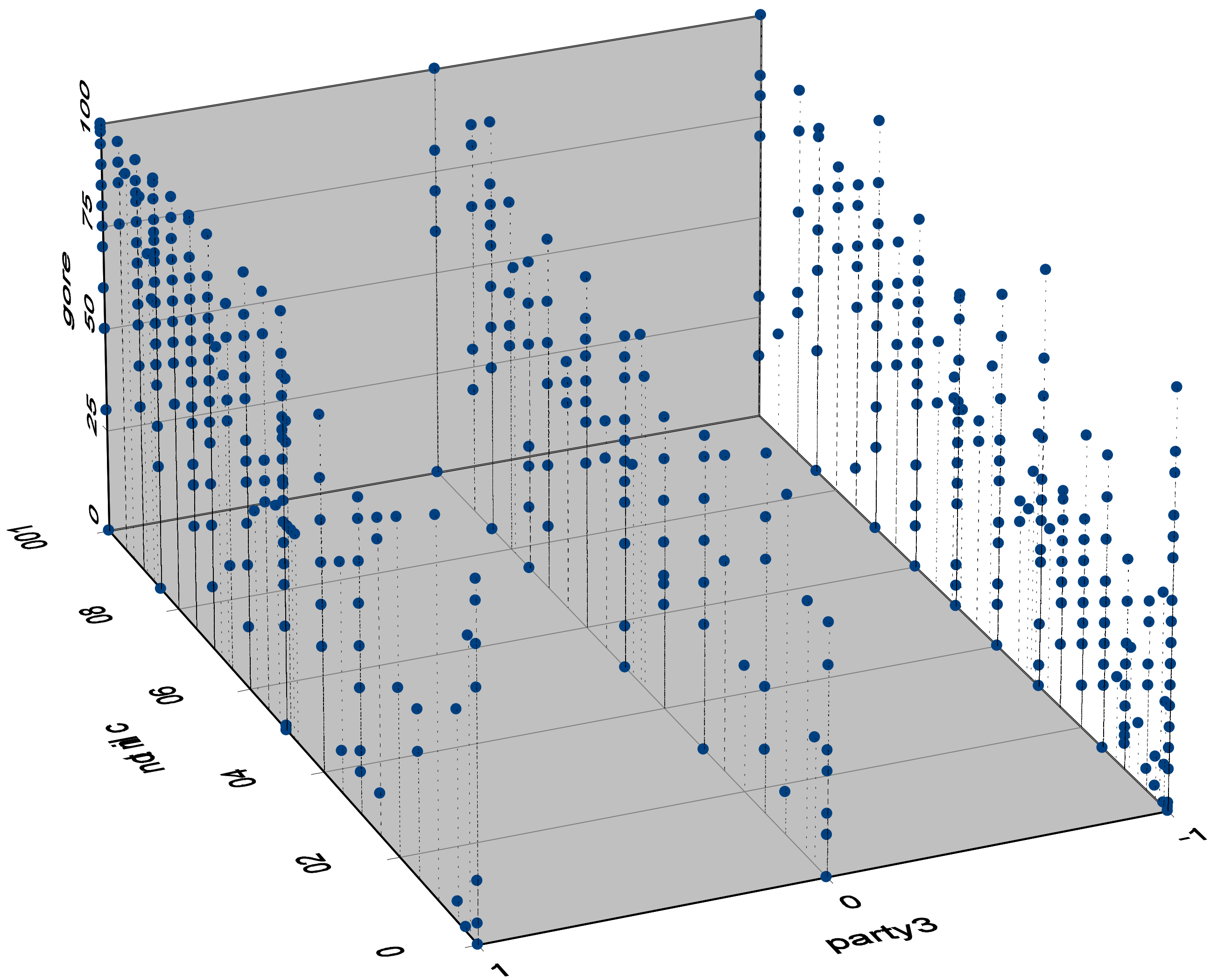
Subtract the Gore therm. from the avg. Gore therm. score



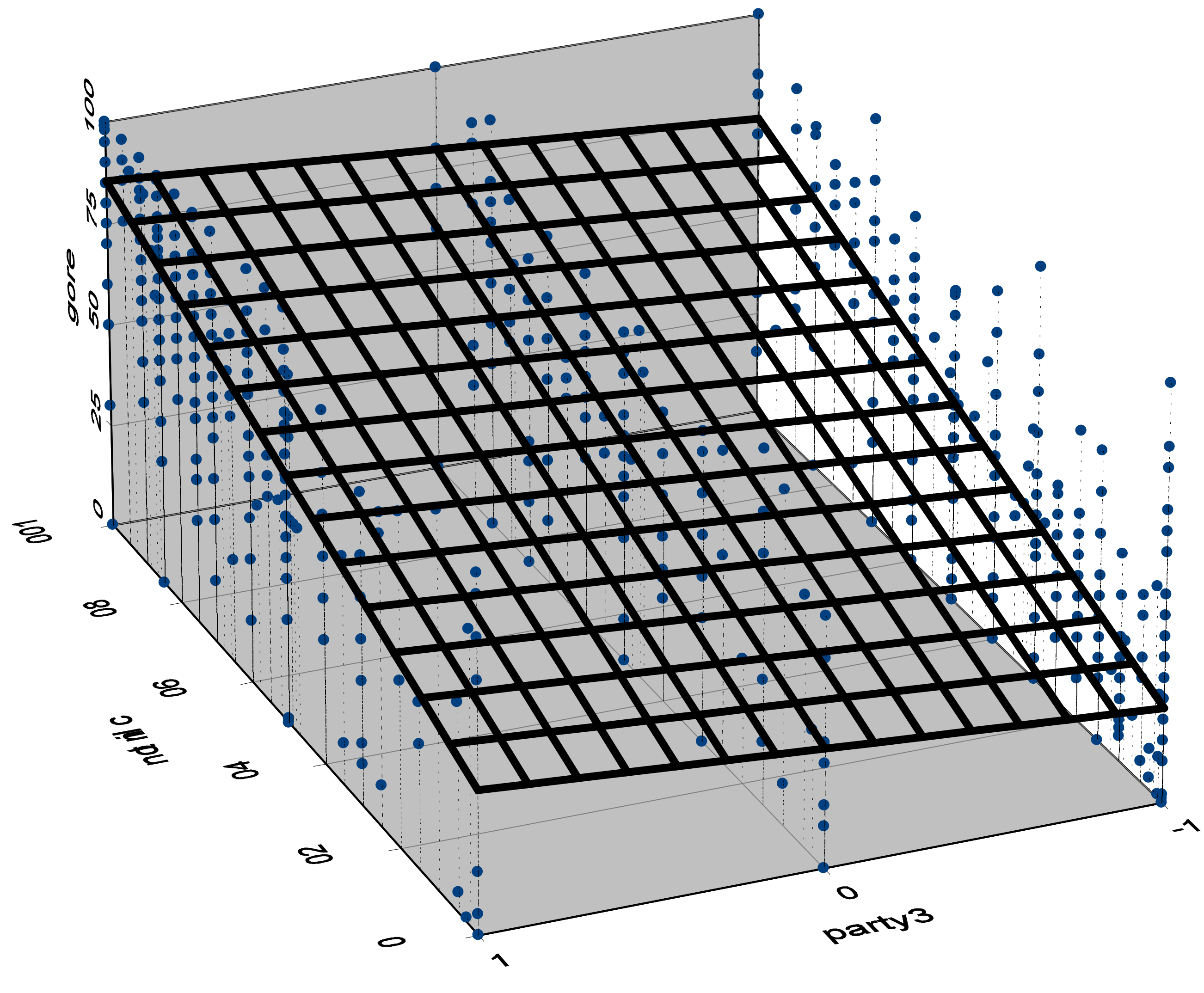
Subtract the Clinton therm. from the avg. Clinton therm. score



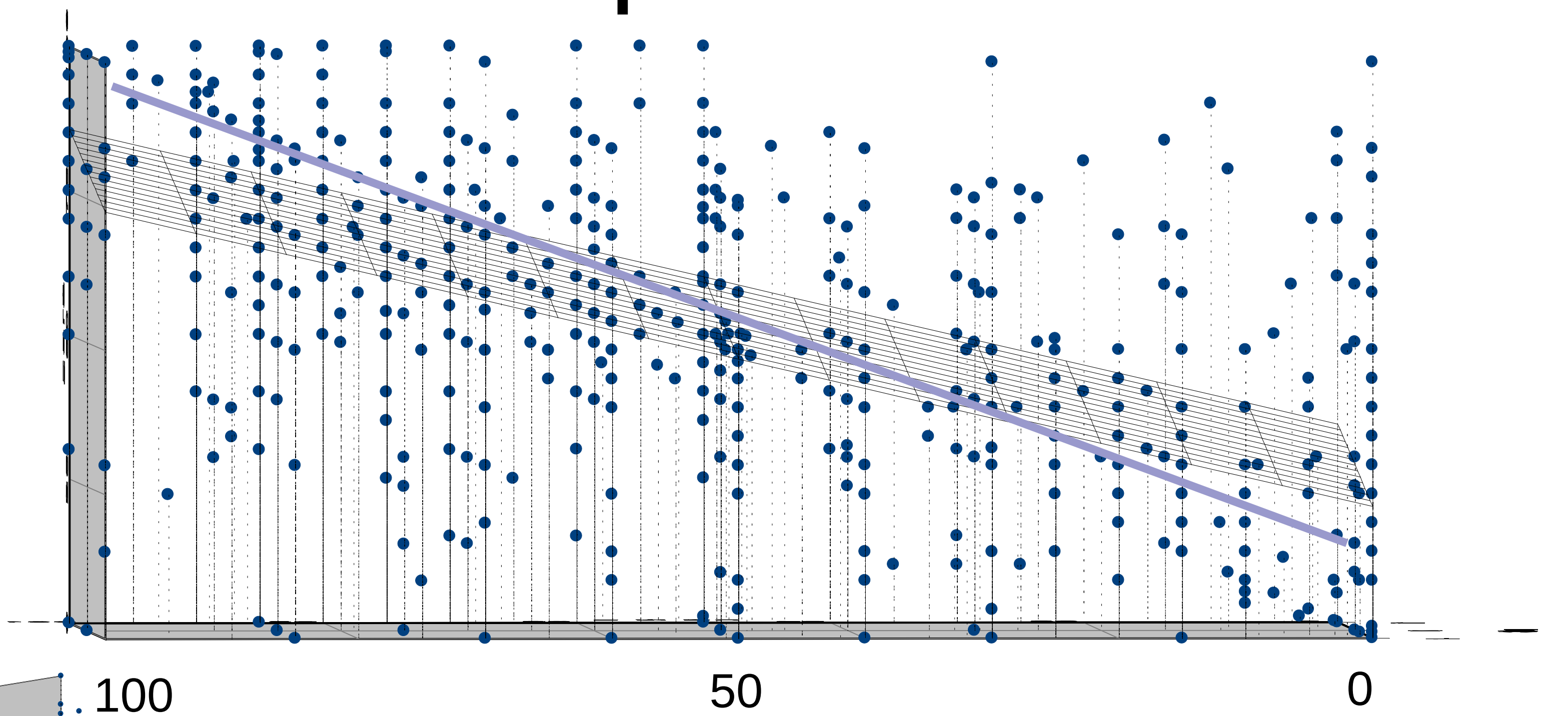
3D Relationship

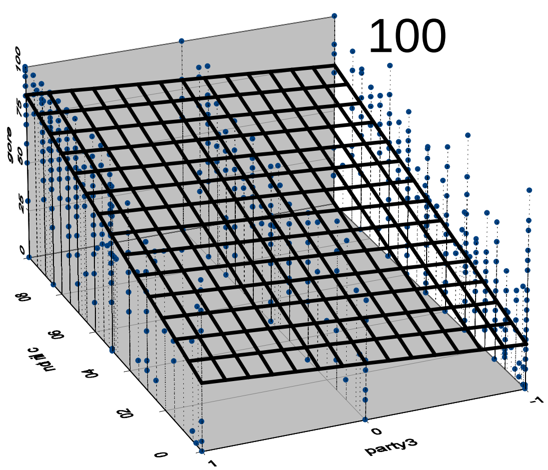


3D Linear Relationship

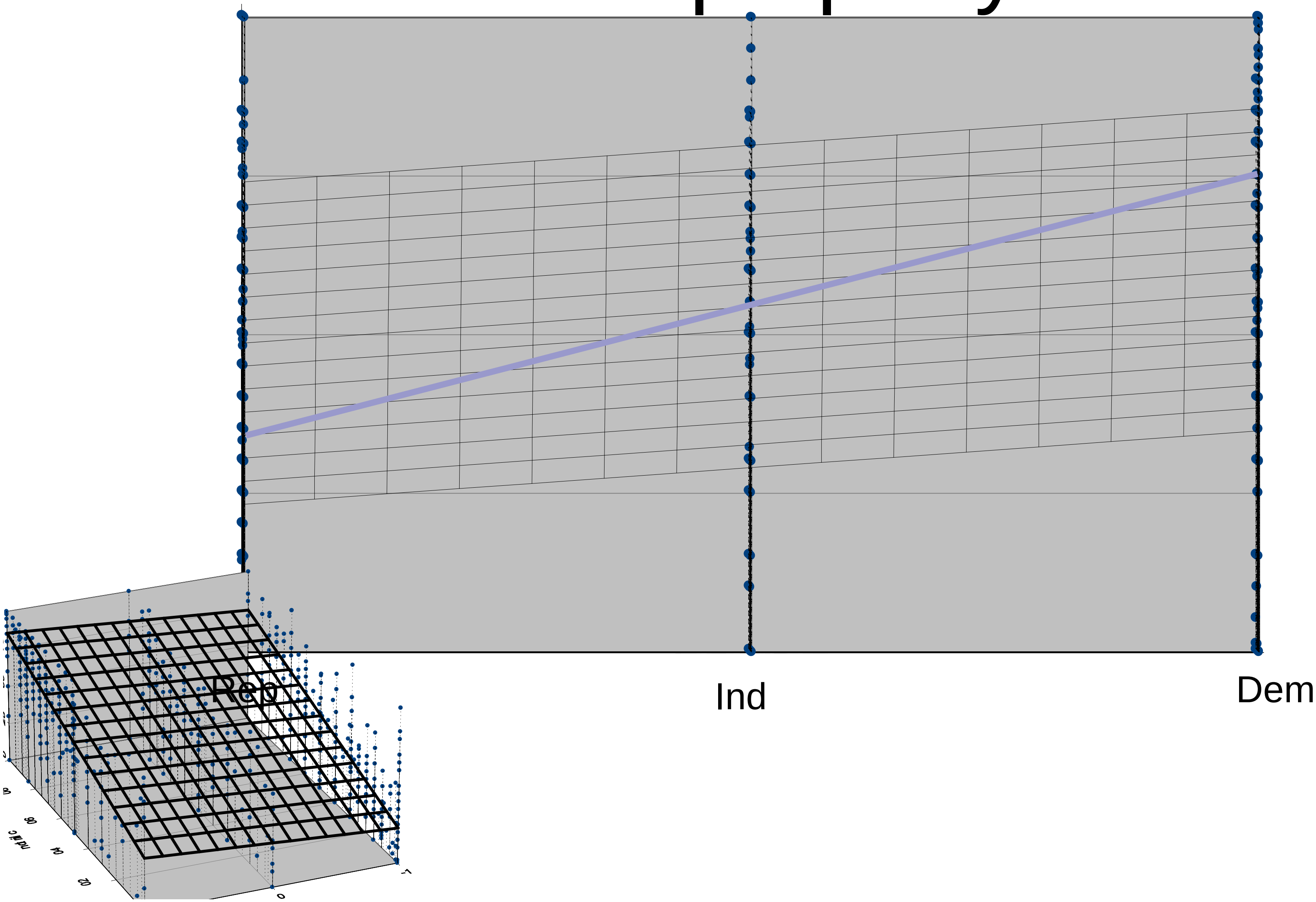


3D Relationship: Clinton





3D Relationship: party



The Linear Relationship between Three Variables

Gore thermometer

Clinton thermometer

Party ID

$$Y_i = \beta_0 + \beta_1 X_{1,i} + \beta_2 X_{2,i} + \varepsilon_i$$

The method of least squares (again)

Pick $\beta_0, \beta_1,$ and β_2 to minimize

$$\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2 \text{ or}$$

$$\sum_{i=1}^{n}(Y_i - \beta_0 - \beta_1 X_i - \beta_2 X_2)^2$$

The Slope Coefficients

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(\overline{Y} - Y_i)(\overline{X}_1 - X_{1,i})}{\sum_{i=1}^{n}(\overline{X}_1 - X_{1,i})^2} - \hat{\beta}_2 \frac{\sum_{i=1}^{n}(\overline{X}_1 - X_{1,i})(\overline{X}_2 - X_{2,i})}{\sum_{i=1}^{n}(\overline{X}_1 - X_{1,i})^2} \text{ and}$$

$$\hat{\beta}_2 = \frac{\sum_{i=1}^{n}(\overline{Y} - Y_i)(\overline{X}_2 - X_{1,i})}{\sum_{i=1}^{n}(\overline{X}_2 - X_{2,i})^2} - \hat{\beta}_1 \frac{\sum_{i=1}^{n}(\overline{X}_1 - X_{1,i})(\overline{X}_2 - X_{2,i})}{\sum_{i=1}^{n}(\overline{X}_2 - X_{2,i})^2}$$

$$\hat{\beta}_0 = \overline{Y} - \hat{\beta}_1 \overline{X}_1 - \hat{\beta}_2 \overline{X}_2$$

X_1 is Clinton thermometer, X_2 is PID, and Y is Gore thermometer

The Slope Coefficients More Simply

$$\hat{\beta}_1 = \frac{\text{cov}(X_1, Y)}{\text{var}(X_1)} - \hat{\beta}_2 \frac{\text{cov}(X_1, X_2)}{\text{var}(X_1)} \text{ and}$$

$$\hat{\beta}_2 = \frac{\text{cov}(X_2, Y)}{\text{var}(X_2)} - \hat{\beta}_1 \frac{\text{cov}(X_1, X_2)}{\text{var}(X_2)}$$

X_1 is Clinton thermometer, X_2 is PID, and Y is Gore thermometer

The Matrix form

y_1
y_2
…
y_n

1	$x_{1,1}$	$x_{2,1}$	…	$x_{k,1}$
1	$x_{1,2}$	$x_{2,2}$	…	$x_{k,2}$
1	…	…	…	…
1	$x_{1,n}$	$x_{2,n}$	…	$x_{k,n}$

$$\beta = (X'X)^{-1} X'y$$

Multivariate slope coefficients

Clinton effect (on Gore) in bivariate (*B*) regression

Are Gore and Party ID related?

Bivariate estimate: $\hat{\beta}_1^B = \dfrac{\text{cov}(X_1, Y)}{\text{var}(X_1)}$ vs.

Multivariate estimate: $\hat{\beta}_1^M = \dfrac{\text{cov}(X_1, Y)}{\text{var}(X_1)} - \hat{\beta}_2^M \dfrac{\text{cov}(X_1, X_2)}{\text{var}(X_1)}$

Clinton effect (on Gore) in multivariate (*M*) regression

Are Clinton and Party ID related?

When does $\hat{\beta}_1^B = \hat{\beta}_1^M$? Obviously, when $\hat{\beta}_2^M \dfrac{\text{cov}(X_1, X_2)}{\text{var}(X_1)} = 0$

X_1 is Clinton thermometer, X_2 is PID, and Y is Gore thermometer

The Output

```
. reg gore clinton party3

      Source |       SS       df       MS              Number of obs =    1745
-------------+------------------------------           F(  2,  1742) = 1048.04
       Model |  629261.91     2  314630.955           Prob > F      =  0.0000
    Residual |  522964.934  1742  300.209492           R-squared     =  0.5461
-------------+------------------------------           Adj R-squared =  0.5456
       Total |  1152226.84  1744   660.68053           Root MSE      =  17.327

------------------------------------------------------------------------------
        gore |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     clinton |   .5122875   .0175952    29.12   0.000     .4777776    .5467975
      party3 |   5.770523   .5594846    10.31   0.000     4.673191    6.867856
       _cons |    28.6299   1.025472    27.92   0.000     26.61862    30.64119
------------------------------------------------------------------------------
```

Interpretation of `clinton` effect: *Holding constant party identification*, a one-point increase in the Clinton feeling thermometer is associated with a .51 increase in the Gore thermometer.

Separate regressions

	(1)	(2)	(3)
Intercept	23.1	55.9	28.6
Clinton	0.62	--	0.51
Party	--	15.7	5.8

$$\hat{\beta}_1 = \frac{\text{cov}(X_1, Y)}{\text{var}(X_1)} - \hat{\beta}_2 \frac{\text{cov}(X_1, X_2)}{\text{var}(X_1)} \text{ and}$$

$$\hat{\beta}_2 = \frac{\text{cov}(X_2, Y)}{\text{var}(X_2)} - \hat{\beta}_1 \frac{\text{cov}(X_1, X_2)}{\text{var}(X_2)}$$

Why did the Clinton Coefficient change from 0.62 to 0.51

```
. corr gore clinton party, cov
(obs=1745)

             |     gore  clinton   party3
-------------+---------------------------
        gore |  660.681
     clinton |  549.993  883.182
      party3 |  13.7008   16.905    .8735
```

The Calculations

$$\hat{\beta}_1^B = \frac{\text{cov}(gore, clinton)}{\text{var}(clinton)} = \frac{549.993}{883.182} = 0.6227$$

$$\hat{\beta}_1^M = \frac{\text{cov}(gore, clinton)}{\text{var}(clinton)} - \hat{\beta}_2^M \frac{\text{cov}(clinton, party)}{\text{var}(clinton)}$$

$$= \frac{549.993}{883.182} - 5.7705 \frac{16.905}{883.182}$$

$$= 0.6227 - 0.1105$$

$$= 0.5122$$

```
. corr gore clinton party,cov
(obs=1745)

             |     gore   clinton    party3
-------------+---------------------------
        gore |  660.681
     clinton |  549.993   883.182
      party3 |  13.7008    16.905     .8735
```

Another way of thinking about this

Rewrite

$$\hat{\beta}_1^M = \frac{\text{cov}(gore, clinton)}{\text{var}(clinton)} - \hat{\beta}_2^M \frac{\text{cov}(clinton, party)}{\text{var}(clinton)}$$

as

$$\frac{\text{cov}(gore, clinton)}{\text{var}(clinton)} = \hat{\beta}_1^M + \hat{\beta}_2^M \frac{\text{cov}(clinton, party)}{\text{var}(clinton)}$$

Total effect = Direct effect + indirect effect

The Total Effect of the Clinton thermometer on the Gore thermometer (.61) can be Broken down into a direct effect of .51, plus an indirect effect (though party) of .11

Drinking and Greek Life Example

- Why is there a correlation between living in a fraternity/sorority house and drinking?
  - Greek organizations often emphasize social gatherings that have alcohol. The effect is being in the Greek organization itself, not the house.
  - There's something about the House environment itself.

Dependent variable: Times Drinking in Past 30 Days

C8. When did you last have a drink (that is more than just a few sips)?

- ◯ I have never had a drink ⟶ Skip to C22 (page 10)
- ◯ Not in the past year ⟶ Skip to C22 (page 10)
- ◯ More than 30 days ago, but in the past year ⟶ Skip to C17 (page 8)
- ◯ More than a week ago, but in the past 30 days ⟶ Go to C9
- ◯ Within the last week ⟶ Go to C9

C9. On how many occasions have you had a drink of alcohol in the past 30 days? (Choose one answer.)

- 1 ◯ Did not drink in the last 30 days
- 2 ◯ 1 to 2 occasions
- 3 ◯ 3 to 5 occasions
- 4 ◯ 6 to 9 occasions
- 5 ◯ 10 to 19 occasions
- 6 ◯ 20 to 39 occasions
- 7 ◯ 40 or more occasions

```
. infix age 10-11 residence 16 greek 24 screen 102
timespast30 103 howmuchpast30 104 gpa 278-279 studying 281
timeshs 325 howmuchhs 326 socializing 283 stwgt_99 475-493
weight99 494-512 using da3818.dat,clear
(14138 observations read)

. recode  timespast30 timeshs (1=0) (2=1.5) (3=4) (4=7.5)
(5=14.5) (6=29.5) (7=45)
(timespast30: 6571 changes made)
(timeshs: 10272 changes made)

. replace timespast30=0 if screen<=3
(4631 real changes made)
```

```
. tab timespast30
```

timespast30	Freq.	Percent	Cum.
0	4,652	33.37	33.37
1.5	2,737	19.64	53.01
4	2,653	19.03	72.04
7.5	1,854	13.30	85.34
14.5	1,648	11.82	97.17
29.5	350	2.51	99.68
45	45	0.32	100.00
Total	13,939	100.00	

Key explanatory variables

- Live in fraternity/sorority house
  - Indicator variable (dummy variable)
  - Coded 1 if live in, 0 otherwise
- Member of fraternity/sorority
  - Indicator variable (dummy variable)
  - Coded 1 if member, 0 otherwise

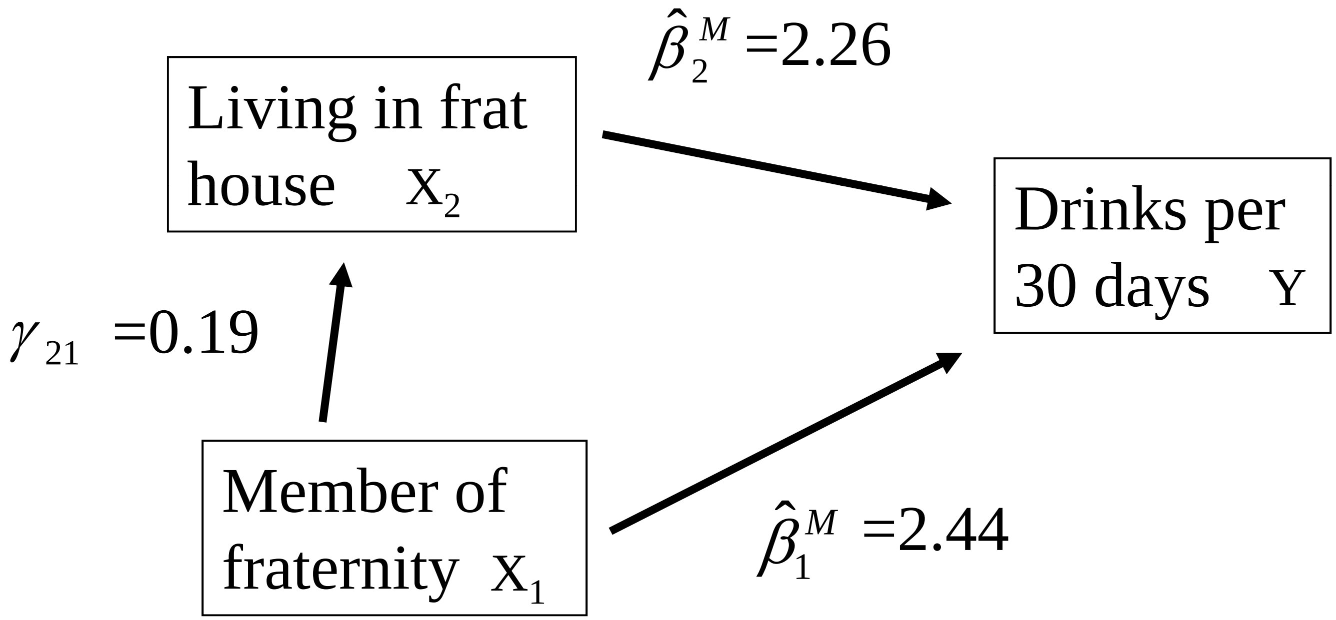
Three Regressions

Dependent variable: number of times drinking in past 30 days			
Live in frat/sor house (indicator variable)	4.44 (0.35)	---	2.26 (0.38)
Member of frat/sor (indicator variable)	---	2.88 (0.16)	2.44 (0.18)
Intercept	4.54 (0.56)	4.27 (0.059)	4.27 (0.059)
S.E.R.	6.49	6.44	6.44
R2	.011	.023	.025
N	13,876	13,876	13,876

What is the substantive interpretation of the coefficients?

Note: Standard errors in parentheses. Corr. Between living in frat/sor house and being a member of a Greek organization is .42

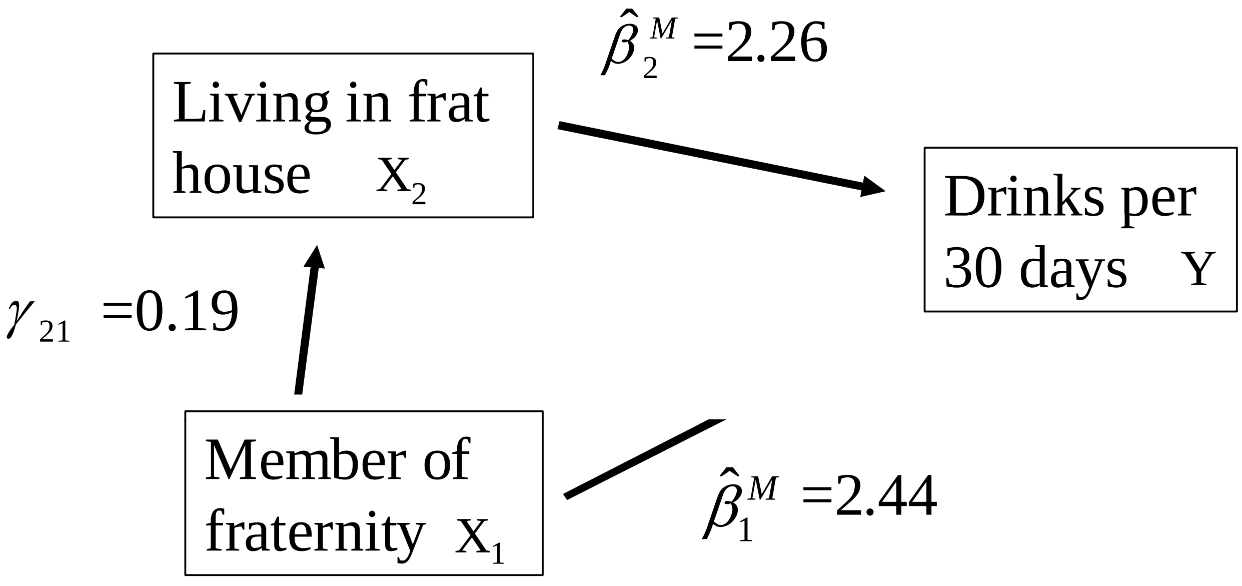
The Picture


$\hat{\beta}_2^M = 2.26$
Living in frat house X_2
Drinks per 30 days Y
$\gamma_{21} = 0.19$
Member of fraternity X_1
$\hat{\beta}_1^M = 2.44$

Accounting for the total effect

$$\hat{\beta}_1^B = \hat{\beta}_1^M + \hat{\beta}_2^M \gamma_{21}$$

Total effect = Direct effect + indirect effect



Accounting for the effects of frat house living and Greek membership on drinking

From bivariate regressions

From multiple regressions

From accounting identity: T=D+I

Effect	Total	Direct	Indirect
Member of Greek org.	2.88	2.44 (85%)	0.44 (15%)
Live in frat/ sor. house	4.44	2.26 (51%)	2.18 (49%)