

2. Hint for Problem 2(a), relativistic Euler equation

(a) Starting from the stress-energy tensor for a perfect fluid,  $\mathbf{T} = \rho \vec{U} \otimes \vec{U} + P \mathbf{h}$ , where  $\mathbf{h} = \mathbf{g}^{-1} + \vec{U} \otimes \vec{U}$ , using local energy momentum conservation,  $\nabla \cdot \mathbf{T} = 0$ , derive the relativistic Euler equation,

$$(\rho + P) \nabla_{\vec{U}} \vec{U} = -\mathbf{h} \cdot \nabla P . \quad (1)$$

(Note: Because both  $\mathbf{T}$  and  $\mathbf{h}$  are symmetric tensors, there is no ambiguity in the dot products that appear in this problem.)

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It turns out that if you just evaluate  $\nabla \cdot \mathbf{T} = 0$ , you are unlikely to get a very useful answer. The result you get is, technically, an Euler equation, but it does not naturally reduce to the form we are hoping to show. In particular, you won't recover the non-relativistic limit in a natural way.

A more useful form is obtain by separately equating to zero the components of  $\nabla \cdot \mathbf{T}$  parallel to and orthogonal to the fluid's 4-velocity,  $\vec{u}$ . In particular, if we define  $j^\nu \equiv \nabla_\mu T^{\mu\nu}$ , then the equations

$$\begin{aligned} j^\nu u_\nu &= 0 && \text{(Component parallel to } \vec{u}) \\ j^\nu h^\lambda{}_\nu &= 0 && \text{(Component orthogonal to } \vec{u}) \end{aligned}$$

give us useful information. The second equation in particular can be considered the relativistic analog of the usual Euler equation. (It's worth examining the first equation and seeing what it corresponds to.)

Sorry for this confusion! I'm extending the pset due date by one day to make sure everyone gets a chance to take advantage of this hint.