Massachusetts Institute of Technology
Department of Physics

Physics 8.942 Fall 2001

Problem Set #7
Due in class Tuesday, November 6, 2001.

1. **Galaxy Number Counts**

The differential number counts of galaxies have been measured to faint magnitudes in the Hubble Deep Field (Williams et al 1996, AJ 112, 1335). At $B = 29$, the HDF results give $dN/dm = 10^{5.54}$ galaxies per magnitude and per square degree.

a) Assuming isotropy, convert the observed number counts to the total counts over the whole sky $-dN/d\ln S$ where S is the flux in the HST blue bandpass. (See Peacock Chapter 13 for the relation between flux and magnitude.) Extrapolate this to get $N(S)$, the total number of galaxies in the universe visible to us with $B \leq 29$. (You may approximate the source counts assuming a constant logarithmic slope $d\ln N/d\ln S = -\beta$.)

b) Let us naively assume that the galaxy luminosity function does not evolve and that there is no K-correction (Peacock p. 395). Suppose that the blue luminosity function is a Schechter function with $\alpha = -1$, $\phi^* = 0.015h^3$ Mpc^{-3}, and $M^* = -19.50 + 5\log_{10} h$. At $B = 29$, what is the luminosity distance (in h^{-1} Mpc) for a galaxy of luminosity L^*? Call this number d^*_{29}.

c) Show that the differential counts at $B = 29$ in a flat universe with $\alpha = -1$ are

$$-\frac{dN}{d\ln S} = 4\pi\left(\frac{c}{H_0}\right)^3 \phi^* \int_0^{H_0\tau_0} \exp\left[-(d_L/d^*_{29})^2\right] x^2\, dx \qquad (1)$$

where $x = H_0\chi/c$ and the luminosity distance is a function of $\chi(z)$ ($d_L \approx \chi$ for $x \ll 1$).

d) Show that for an Einstein-de Sitter universe, $H_0 d_L/c = 4x/(2-x)^2$. Then numerically integrate equation (1) to get the predicted differential source counts at $B = 29$. Compare with your result of part a) from the HDF measurement.

2. **Gunn-Peterson Effect**

Neutral hydrogen at redshift z absorbs background quasar light at a wavelength 121.6 nm, creating absorption at a wavelength $121.6(1+z)$ nm as measured today at redshift zero. Treating the Lyman alpha line as being infinitely narrow, the absorption cross section per neutral hydrogen atom is

$$\sigma(\nu) = \sigma_\alpha\, \delta(\nu - \nu_\alpha)\ , \quad \sigma_\alpha = \frac{\pi e^2}{m_e c} f_\alpha \tag{2}$$

where $\nu = c/\lambda$ is frequency, $\nu_\alpha = c/(121.6\ \text{nm})$ is the Lyman alpha frequency, $f_\alpha = 0.416$ is the oscillator strength, and cgs units are used in writing σ_α (cf. Peacock eq. 12.36). In this problem we neglect peculiar velocities. The original reference by Gunn & Peterson (1965, ApJ, 142 1633) is a very readable guide to this problem (though beware it has some typos).

a) Derive the absorption optical depth at observed frequency ν_0 due to neutral hydrogen at $1 + z = \nu_\alpha/\nu_0$. Your result should depend on σ_α, c, ν_α, $n_{\rm HI}$, and H (the Hubble parameter at z). Compare your result with Peacock eq. (12.41) for a Friedmann universe.

b) The measured optical depth versus frequency is not uniform but varies (the Lyman alpha forest) with $n_{\rm HI}$ along the line of sight. The mean optical depth is unity at $z \approx 3$. Assuming $\Omega_{\rm B} h^2 = 0.019$, $h = 0.65$, $Y = 0.24$ (with helium neutral), and the flat ΛCDM model with $\Omega_\Lambda = 0.65$, what is the mean neutral fraction $1 - x_e$ of hydrogen atoms at $z = 3$? Compare with Peacock p. 364.

c) In equilibrium, hydrogen atoms are photoionized and recombine at equal rates, implying

$$n_{\rm HI}\beta = n_e n_p \alpha^{(2)}(T) \quad \text{or} \quad \frac{1 - x_e}{x_e^2} = \frac{n_{\rm H}\alpha^{(2)}(T)}{\beta} \tag{3}$$

where $n_{\rm H} = (1 - Y)\rho_{\rm B}/m_{\rm H}$ and

$$\beta = \int_{\nu_{\rm L}}^{\infty} 4\pi J_\nu \sigma_i(\nu)\, \frac{d\nu}{h\nu} \approx 3 \times 10^{-12}\, J_{-21}\ \text{s}^{-1}\ ;$$

$$\alpha^{(2)} = 2.06 \times 10^{-11} T^{-1/2} \phi_2(T)\ \text{cm}^3\,\text{s}^{-1}\ , \quad \phi_2(T) = 0.448 \ln\left(1 + \frac{h\nu_{\rm L}}{kT}\right)\ . \tag{4}$$

Here, $h\nu_L = 13.6$ eV is the ionization energy of hydrogen, σ_i is the ionization cross-section, T is the temperature in Kelvin, and J_{-21} is the mean intensity at frequencies just above $\nu_{\rm L}$ (in units of 10^{-21} erg cm^{-2} s^{-1} Hz^{-1} ster^{-1}). The $\alpha^{(2)}$ excludes recombinations directly to the ground state (Case B recombination). For the ΛCDM model, assuming the gas temperature is 10^4 K, what J_{-21} is needed to ionize the gas to the level determined in part b)?