THIN LENS APPROXIMATION

effective 2-D gravitational potential:

$$\psi_{2D} = \frac{D_{LS}}{D_S} \int_{observer}^{source} \frac{2\Phi_{3D}}{c^2} \frac{d\ell}{D_L} \quad \text{(dimensionless)}$$

$$\begin{pmatrix} \text{time} \\ \text{delay} \end{pmatrix} = \begin{pmatrix} \text{geometric} \\ \text{delay} \end{pmatrix} + \begin{pmatrix} \text{gravitational} \\ \text{delay} \end{pmatrix}$$

$$\tau = \frac{1 + z_L}{c} \frac{D_L D_S}{D_{LS}} \left[\frac{1}{2} (\vec{\theta} - \vec{\beta})^2 - \psi_{2D}(\vec{\theta}) \right]$$

lens equation ($\vec{\theta} \equiv$ image pos$^{\text{n}}$; $\vec{\beta} \equiv$ source pos$^{\text{n}}$):

$$\vec{\nabla}\tau = 0 \quad \Rightarrow \quad \vec{\theta} - \vec{\beta} - \vec{\nabla}\psi_{2D} = 0$$

inverse "magnification" matrix [$\vec{\theta} \equiv (x, y)$]:

$$\frac{\partial \vec{\beta}}{\partial \vec{\theta}} = \begin{pmatrix} 1 - \frac{\partial^2 \psi}{\partial x^2} & -\frac{\partial^2 \psi}{\partial x \partial y} \\ -\frac{\partial^2 \psi}{\partial x \partial y} & 1 - \frac{\partial^2 \psi}{\partial y^2} \end{pmatrix}$$

typical galaxy quadrupole potential [$\vec{\theta} \equiv (r, \phi)$]:

$$\psi_{2D} = br[1 + \gamma \cos 2(\phi - \phi_\gamma)]$$

model parameters: b, γ, ϕ_γ, β_x and β_y