1.3 Extragalactic empiricism

Before we can discuss galaxies quantitatively, we must first introduce some new terms. The *specific intensity* $I_\nu(\vec{\theta})$ is a measure of energy flux incident on a unit surface area per unit solid angle and frequency (erg/s/cm^2/sterad/Hz). Then the *specific flux* on a given surface element is the specific intensity integrated over all solid angle:

$$f_\nu = \int I_\nu d\Omega. \tag{1.21}$$

For the purpose of the present discussion, the units of specific flux are ergs/s/cm^2/Hz. The *specific luminosity* L_ν at a given frequency ν of an object at a distance D is then

$$L_\nu = 4\pi D^2 f_\nu. \tag{1.22}$$
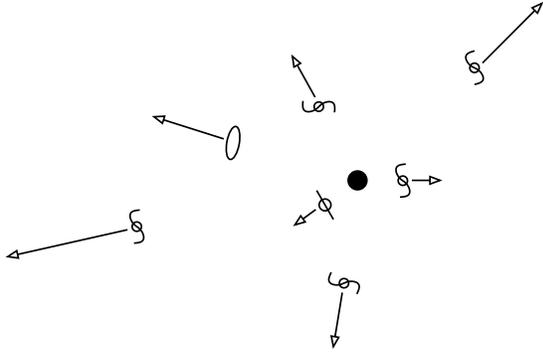
(Continued on next page.)

Figure 1.11: The observed velocities of nearby galaxies are proportional to their distances, permitting distances to be estimated from Doppler shifts.

Astronomers often use *Pogson quantified magnitudes*, a logarithmic system to describe the incident flux in a certain frequency bandpass. There are many different variants, but the one that will appeal most to physicists is the "absolute" system with

$$m_{\text{AB}} \equiv -2.5 \log f_\nu - 48.60 \quad . \tag{1.23}$$

The zero points of the various magnitude systems are arbitrary [2], but they all try, to some extent, to preserve Ptolemy's scheme of 2000 years ago. The surface brightness μ (in dimensionless units of magnitude per square arcsecond) is then

$$\mu \equiv -2.5 \log I_\nu - 48.60 - 2.5 \log(206265)^2 \quad . \tag{1.24}$$

Observers will immediately recognize the last quantity in parentheses as the number of arcseconds in a radian (and the number of astronomical units in a parsec). As a matter of convention, the visible and near infrared portions of the electromagnetic spectrum are split up into a number spectral bands that have their origin in recipes for the glass that was used (by Harold Johnson) to construct filters. In order of increasing wavelength they are called U, B, V, R, I, J, H, and K where the first five letters stand for "ultraviolet", "blue", "visual", "red", and "infrared". When giving the luminosity or brightness of a source, one typically specifies a wavelength. For example, the background magnitude of the night sky in the B filter is roughly $\mu_{\text{B}} \sim 22.05$ $\text{B}_{\text{mag}}/\text{arcsec}^2$, corresponding to a luminosity surface density of $100 L_\odot/\text{pc}^2$.

Empirically, it is found that many galaxies obey simple "laws" that give the surface brightness as a function of radius from the galactic center. The best-known of these empirical relations are the *Freeman disk* and the *deVaucouleurs spheroid*:

$$I(r) = \begin{cases} I_0 \exp\left(\frac{r}{r_s}\right) & : \quad \text{Freeman disk} \\ I_0 \exp\left[-7.67\left(\frac{r}{r_e}\right)^{1/4}\right] & : \quad \text{deVaucouleurs spheroid.} \end{cases} \tag{1.25}$$

For disks, the central intensity is such that at $r = 0$, the surface brightness is $\mu_0 \sim 21.5$ $\text{B}_{\text{mag}}/\text{arcsec}^2$. For spheroids, the effective radius, r_e, is defined so that half the light lies interior to it. At $r = r_e$ the surface brightness is typically $\mu_e \sim 22.5$ $\text{B}_{\text{mag}}/\text{arcsec}^2$. These are both very close to the surface brightness of the background night sky, which is made up primarily of zodiacal light and terrestrial airglow. This makes it relatively difficult to detect low surface brightness (LSB) galaxies, and those given to fretting (or quibbling) might worry that we were missing a large fraction of the universe

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[2]This particular definition gives a magnitude for the star Vega of $m_{\text{AB}} \simeq 0$ at $5556 \mathring{A}$
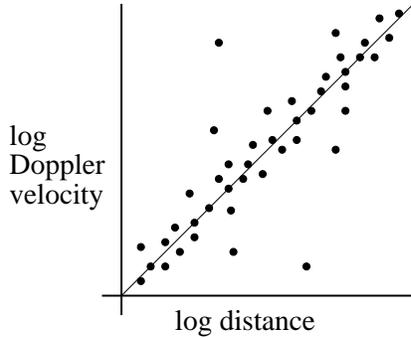
Figure 1.12: A schematic plot of redshift versus distance for a sample of galaxies. The slope is unity, implying a linear relation. Notice that a few points lie far from the mean. Real data is never pretty.

in failing to find such galaxies. Recent work by Blanton et al. using the Sloan Digital Sky Survey indicate that at most $\sim 5\%$ of the total luminosity density of the Universe might be lurking in galaxies too faint to detect against the surface brightness of the night sky.

In addition to these scaling laws that describe the surface brightness of a single galaxy, there are also a number of relationships that describe how galaxies of different luminosities are distributed throughout the Universe. Unfortunately, intrinsic luminosities cannot be determined without knowing the distance to a galaxy. Distances are difficult to measure. This would be a serious problem were it not for the Hubble relation, which tells us (to first order) that distance is proportional to Doppler shift, $D = v/H_0$ (see Figure 1.12). The Hubble constant, H_0, was poorly known until recently.

In the absence of a definitive measure of H_0, it was absorbed into many results, giving an overall map of the universe with a sliding scale. An example is *Schechter's Law*, an empirical expression for the "luminosity function" [# galaxies/unit volume with luminosity between L and $L + dL$],

$$\phi(L)dL = \phi^\star \left(\frac{L}{L^\star}\right)^\alpha \exp\left(-\frac{L}{L^\star}\right) d(L/L^\star). \tag{1.26}$$

Schechter's law works best for galaxies in the range $0.003 < L/L^\star < 3$ where $L^\star (\simeq 10^{10} L_\odot h^{-2})$ is determined from a fit giving $m_B^\star = -19.75 + 5\log h$, $\phi^\star = 2.0 \times 10^{-2} h^3 \mathrm{Mpc}^{-3}$, and $\alpha = -1.09$. In each of these parameters, h is the dimensionless expansion factor of the Universe as a fraction of the canonical value for the Hubble constant[3]:

$$h \equiv \frac{H_0}{100 \text{ km/s/Mpc}} = \frac{v/D}{100 \text{ km/s/Mpc}}. \tag{1.27}$$

The luminosity function $\phi(L)$ has a form often seen in physical processes, a power law dependence with an exponential cutoff, shown schematically in Figure 1.13. Although $\phi(L)$ diverges for faint galaxies ($L \to 0$), the integrated luminosity is finite and actually analytic:

$$\int \phi(L)LdL = \Gamma(\alpha + 2)\phi^\star L^\star = (1 + \alpha)!\phi^\star L^\star. \tag{1.28}$$

Given the difficulty of measuring absolute distances and magnitudes of galaxies, it is useful to establish that one or another observable property of galaxies is correlated (the more strongly the better) with the intrinsic luminosity of a galaxy. The *Tully-Fisher Law* for spiral galaxies is one such

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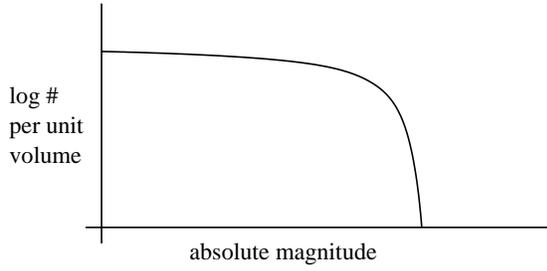[3]The best measurements today give $h \simeq 0.7$, or equivalently, $H_0 \simeq (14\mathrm{Gyr})^{-1}$

Figure 1.13: A schematic representation of the luminosity function for galaxies, cutting off sharply at the bright end and diverging slowly at the faint end.

empirical relation, between the circular velocity v_c and the total luminosity, L, of a galaxy, with

$$v_c \simeq 250 \text{ km/s} \left(\frac{L}{L^\star}\right)^{1/4} \tag{1.29}$$

and the *Faber-Jackson Law* gives a similar relationship for the line-of-sight velocity dispersion σ_{los} for elliptical galaxie, :

$$\sigma_{\text{los}} \sim 220 \text{ km/s} \left(\frac{L}{L^\star}\right)^{1/4}. \tag{1.30}$$

The *fundamental plane* relation for ellipticals is an extension of the Faber-Jackson relation to include a dependence upon surface brightness. It is usually expressed in terms of deVaucouleurs' effective radius r_e rather than luminosity,

$$r_e \propto \sigma_e^{1.49\pm0.05} I_e^{-0.75\pm0.01} \quad . \tag{1.31}$$

Here effective radius, r_e, depends upon distance, but the velocity dispersion and intensity do not.

Another two techinques for estimating galaxy distances involve the luminosity functions for globular clusters and planetary nebulae within galaxies. Yet another technique (the surface brightness fluctuation method) involves measuring the graininess of the observed surface brightnesses of elliptical galaxies due to the finite numbers of stars in each square arcsecond. The best technique requires patience and luck – observing a supernova within a galaxy.