

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
Department of Physics

Physics 8.901: Astrophysics I

Spring Term 2006

PROBLEM SET 6

**Due:** Thursday, April 13 in class

**Reading:** Finish reading the rest of Chapter 2 (§2.7–2.15) on stellar evolution in Hansen, Kawaler, & Trimble. You can also read more about white dwarf properties, cooling, and crystallization physics in Chapter 10 of HK&T, as well as in Chapters 3–4 of *Black Holes, White Dwarfs, and Neutron Stars* by Shapiro & Teukolsky.

1. **Properties of a white dwarf.** (Carroll & Ostlie, Problem 15.1). The most easily observed white dwarf in the sky is in the 40 Eri triple star system: 40 Eri A is a 4th-magnitude star similar to the Sun, 40 Eri B is a 10th-magnitude white dwarf, and 40 Eri C is an 11th-magnitude red M5 star. This problem deals only with the latter two stars, which are widely separated from 40 Eri A by 400 AU.
  - (a) The period of the 40 Eri B+C “binary” is 247.9 yr. The system’s measured trigonometric parallax is 0.201 arcsec and the true angular extent of the semimajor axis of the reduced mass is 6.89 arcsec. The ratio of the distances of B and C from the center of mass is  $a_B/a_c = 0.37$ . Find the mass of 40 Eri B and 40 Eri C in solar units.
  - (b) The absolute bolometric magnitude of 40 Eri B is 9.6. Determine its luminosity in solar units.
  - (c) The effective temperature of 40 Eri B is 16900 K. Calculate its radius and its average density.
  - (d) Compare the radius and mean density of 40 Eri B with those of the first known white dwarf, Sirius B ( $R = 0.008 R_\odot$ ,  $\rho = 3.0 \times 10^6 \text{ g cm}^{-3}$ ). Which is denser, and why?
2. **White dwarf cooling.** (Carroll & Ostlie, Problem 15.8). Crystallization will occur in a cooling white dwarf when the electrostatic potential between neighboring ions dominates their thermal energy. Their ratio is

$$\Gamma = \frac{Z^2 e^2}{r k T},$$

where  $r$  is the typical distance between neighboring nuclei, taken as the radius of the sphere whose volume is equal to the mean volume per nucleus, so that

$$\frac{4}{3}\pi r^3 = \frac{A m_p}{\rho},$$

where  $A$  is the atomic weight of the nuclei.

- (a) Calculate  $r$  for a  $0.6 M_\odot$  pure carbon white dwarf with radius  $0.012 R_\odot$ .
- (b) Much effort has been spent on precise numerical calculations of  $\Gamma$  to obtain realistic white dwarf cooling curves. The results indicate that  $\Gamma \approx 160$  for the onset of crystallization. Estimate the interior temperature  $T_c$  where this occurs.
- (c) Estimate the luminosity of a pure carbon white dwarf with this interior temperature. For the nondegenerate envelope, assume that hydrogen mass fraction  $X = 0$ , helium fraction  $Y = 0.9$ , and metal fraction  $Z = 0.1$ .
- (d) For roughly how many years could the white dwarf sustain this luminosity using just the latent heat of crystallization of  $kT$  per nucleus released upon crystallization?

3. **Cosmochronology with white dwarfs.** (Adapted from Hansen, Kawaler, & Trimble, Problem 2.9).

In lecture, we discussed how the luminosity function (number per cubic parsec as a function of luminosity) of local white dwarfs can be used to estimate the age of the Galaxy. If low-mass stars had been forming forever into the past, then the luminosity function would rise indefinitely for fainter luminosities. However, for a finite history of star formation, the luminosity function should cut off below some luminosity. From the plot I handed out in lecture (or see Figure 15.10 in Carroll & Ostlie or Figure 2.16 in Hansen, Kawaler, & Trimble), we see that the observed luminosity function for local white dwarfs cuts off for luminosities below  $\approx 10^{-4.4}L_{\odot}$ .

From stellar evolution modeling, the time it takes for a star of mass  $M$  to go from birth on the zero-age main sequence to the end of the planetary nebula phase (when a hot white dwarf is revealed) is

$$\log \tau_{\text{pn}}(\text{yr}) = 9.921 - 3.6648 \log(M/M_{\odot}) + 1.9697 \log^2(M/M_{\odot}) - 0.9369 \log^3(M/M_{\odot}),$$

for ZAMS masses  $0.6 < (M/M_{\odot}) < 10$ . Note that  $M$  in this equation is the *original* ZAMS mass (i.e., before any mass loss). You should assume that the sample is large enough that the WDs at dropoff started out at the maximum mass for stars ending up as WDs. (Why make this assumption?)

Once the star has reached the end of the planetary nebula phase, it will have the mass it will retain for its white dwarf lifetime. The time in years for a white dwarf of mass  $M_{\text{wd}}$  to cool to some given luminosity  $L$  is

$$\tau_{\text{wd}} = 8.8 \times 10^6 \left(\frac{A}{12}\right)^{-1} \left(\frac{M_{\text{wd}}}{M_{\odot}}\right)^{5/7} \left(\frac{\mu}{2}\right)^{-2/7} \left(\frac{L}{L_{\odot}}\right)^{-5/7} \text{ yr},$$

where  $A$  is the nuclear mass number and  $\mu$  is the mean molecular weight. Assume pure carbon white dwarfs ( $A = \mu = 12$ ) and a “standard” white dwarf mass. Use the observed cut-off luminosity and the above expressions for  $\tau_{\text{pn}}$  and  $\tau_{\text{wd}}$  to estimate the age of the Galactic disk (based on our local neighborhood).

Having done this part of the problem, also estimate the age of the Galactic halo, assuming that the observed cutoff there corresponds to  $T_{\text{eff}} = 4000$  K and assuming a standard mass and radius for the white dwarfs.

4. **Mass loss from winds in hot luminous stars.** (Adapted from Hansen, Kawaler, & Trimble, Problem 2.13).

The mass loss rate  $\dot{M}$  in a stellar wind from a hot, massive star of mass  $M$ , radius  $R$ , and luminosity  $L$  obeys the semi-empirical relation

$$\log(\dot{M} v_{\infty} R^{1/2}) = -1.37 + 2.07 \log(L/10^6),$$

where  $M$ ,  $R$ , and  $L$  are measured in solar units,  $\dot{M}$  is measured in  $M_{\odot} \text{ yr}^{-1}$ , and  $v_{\infty}$  is the terminal velocity of the wind (far from the star) in  $\text{km s}^{-1}$ . This terminal velocity is found to be roughly proportional to the escape velocity  $v_{\text{esc}}$  at the stellar surface,

$$v_{\text{esc}} = \sqrt{2(1 - \Gamma_e)GM/R},$$

with all quantities measured in physical (e.g., cgs) units. The factor of  $(1 - \Gamma_e)$  arises from the levitating effect of radiation pressure at the photosphere, which effectively lowers the escape velocity. For stars with  $T \gtrsim 2.1 \times 10^4$  K, it turns out that  $v_{\infty} \approx v_{\text{esc}}/2.6$ .

- (a) Neglecting radiation pressure (i.e., setting  $\Gamma_e = 0$ ), make a plot of  $\log \dot{M}$  (in units of  $M_\odot \text{ yr}^{-1}$ ) versus  $M$  (in solar masses) for luminosities of  $10^5 L_\odot$ ,  $3 \times 10^5 L_\odot$ ,  $10^6 L_\odot$ , and  $2 \times 10^6 L_\odot$ .
- (b) Now consider the effects of radiation pressure, by using

$$\Gamma_e = \frac{\kappa L}{4\pi c G M},$$

with all quantities in cgs units, and using the electron scattering opacity for the winds of hot stars with  $\kappa = 0.3 \text{ cm}^2 \text{ g}^{-1}$ . Remake your plot of  $\log \dot{M}$  versus  $M$ , including this radiation pressure. What is the effect on the mass loss rates?

5. **Mass loss from winds in cool luminous stars.** (Adapted from Carroll & Ostlie, Problems 13.4–13.5).

The mass loss rate in a wind from a cool, luminous star with luminosity  $L$ , radius  $R$ , and surface gravity  $g$  can be parametrized by the Reimers wind law,

$$\dot{M} = 4 \times 10^{-13} \eta \frac{L}{gR} M_\odot \text{ yr}^{-1},$$

where  $L$ ,  $R$ , and  $g$  are in solar units ( $g_\odot = 2.74 \times 10^4 \text{ cm s}^{-2}$ ), and  $\eta$  is a dimensionless free parameter of order unity.

- (a) Estimate the mass loss rate of a  $1 M_\odot$  AGB star with a luminosity of  $7000 L_\odot$  and a temperature of 3000 K.
- (b) Assuming (incorrectly) that  $L$ ,  $R$ , and  $\eta$  do not change with time, derive an expression for the stellar mass as a function of time. Let the initial mass be  $M_0$  at  $t = 0$ .
- (c) Using  $L = 7000 L_\odot$ ,  $R = 310 R_\odot$ ,  $M_0 = 1 M_\odot$ , and  $\eta = 1$ , plot the stellar mass as a function of time in units of  $10^5 \text{ yr}$ .
- (d) How long does it take the star in the previous part to be reduced to the mass of a  $0.6 M_\odot$  degenerate C-O core?