

#### Problem set # 4

1)  $p$ -forms and their representations in theories with 32 supercharges.

a) Find the number of 1-forms in the massless supergravity multiplet in dimension  $d$  with 32 supercharges for  $d=3, \dots, 11$ .

Compare this number with the dimension of the fundamental representation of  $E_n$ , for  $n=11-d$ .

b) Find the number of 2-forms in the massless supergravity multiplet in dimension  $d$  with 32 supercharges for  $d=6, \dots, 11$ .

Compare this number with the corresponding representation of  $E_n$ . Which representation of  $E_n$  is it for this case?

2) Branes ending on branes

Write down all the possible branes which can end on each of the following branes (You can use dualities such as S and T duality)

a) A  $Dp$ -brane for  $p=1, \dots, 9$ .

b) M5 brane

c) M2 brane

d) NS5 brane in Type IIB

e) NS5 brane in Type IIA

3) Electric and magnetic objects on the brane

Find the electric excitation and the magnetic excitation with respect to the form field (the degree of the form varies: it is a 1-form for the case of a  $Dp$  brane, it is a 2-form for the case of the M5 brane, etc.) living on the brane. In each case, find the ending brane which gives rise to the electrically charged object and the ending brane which gives rise to the magnetically charged object. Do this for the following cases:

a)  $Dp$ -brane for  $p=1, \dots, 9$ .

b) M5 brane

c) M2 brane

d) NS5

4) SYM actions in various dimensions

a) Write down the classical, 16 supercharges, SYM action for a vector multiplet in 10 dimensions for an arbitrary simple gauge group  $G$ . Please include the kinetic term for both the gauge field and the gaugino.

b) By applying dimensional reduction to this theory compute the action for a vector multiplet in 9 dimensions. Find the moduli space of vacua for this theory.

c) Compute the mass of a  $W$  boson for the special case  $G=SU(2)$ . Generalize this computation to  $W$  bosons for a gauge group  $G=SU(n)$ .

d) Apply again dimensional reduction and compute the action in 8 dimensions. What is the moduli space of vacua in this case?

e) Write down the bosonic part of the action for any dimension  $d < 10$ .