Chapter 2: Deriving AdS/CFT

MIT OpenCourseWare Lecture Notes

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Lecture 14

Important equations for this lecture from the previous ones:

1. The DBI effective action describing the perturbative dynamics of a Dp-brane (as $F_{\alpha\beta}$ and $\partial_\alpha \Phi^a$ have very small derivatives), equation (12) in lecture 13:

$$S_{Dp} = -T_p \int d^{p+1}x \sqrt{-\left|G_{\alpha\beta} + 2\pi\alpha' F_{\alpha\beta}\right|} \;\;,\;\; G_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu} = g_{\alpha\beta} + \partial_\alpha \Phi^a \partial_\beta \Phi^a \tag{1}$$

$G_{\alpha\beta}$ is the induced metric on the Dp-brane.

2. The tension of a D-brane, as mentioned in lecture 13, can be found (at least from the lowest order of perturbation) by studying the stringy worldsheet of annulus topology.

3. The gravitational constant, equation (15) in lecture 12:

$$G_N \sim \kappa_0^2 \sim g_s^2 \tag{2}$$

2.1.3: D-BRANES (cont.)

Turning of the $F_{\alpha\beta}$ configuration, from the DBI effective action one arrives at the generalization of the Nambu-Goto action for higher dimensional objects:

$$S_{Dp} = -T_p \int d^{p+1}x \sqrt{-\left|G_{\alpha\beta}\right|} \tag{3}$$

D-branes are dynamical objects, with their dynamics governed perturbatively by open string excitations on them (among which massless modes are the most special ones, the lightest non-tachyonic degrees of freedom), and D-branes' states should be included in the complete stringy Hilbert space. The modes describing the D-branes motion appear to be the massless modes in the D-brane world volume can be understood from the fact that the underlying D-dimensional Poincare symmetry of the spacetime in which a Dp-brane is embedded is translational invariant, so any $\Phi^a = \text{const}$ should be an allowed solution (there's no potential generated from Φ^a but its derivatives). The field Φ^a can be viewed as the Goldstone modes for breaking the translational symmetries mentioned above.

Strings of annulus topology can be used to described the interaction between the 2 Dp-branes, which gives a scattering amplitude $\sim G_N T_p^2$ (G_N is the gravitational constant of the effective gravity theory and each T_p comes from the vertex weight associated with closed string emission from the D-brane (read-off from the DBI effective action). This topology can be viewed (open/closed duality) as an open string loop of amplitude $\sim g_s^0$, and by matching one arrives at $T_p \sim g_s^{-1}$ (as $G_N \sim g_s^2$). For a general, the worldsheet topology (can be more complicated or simpler) of the string interaction between 2 Dp-branes can still be seen as the loop of open string (open string's vacuum diagrams, worldsheets with at least 1 boundary as each T_p associated with 1 Dp-branes which gives 1 boundary), and the mass of D-brane can be shown to be exactly the vacuum energy of all open string living on it. The leading diagram has the disk topology with the Euler characteristic $\chi = 1$, hence $\sim g_s^{-\chi} = g_s^{-1}$, expectedly. From dimensional analysis:

$$T_p = \frac{C_p}{g_s \alpha'^{\frac{p+1}{2}}} \tag{4}$$

The duality between the open string and closed string channel can be considered to by the stringy origin of holographic duality.

The relation between open string coupling g_o (3 open strings interaction) and closed string coupling $g_c = g_s$ (3 closed strings interaction) can be seen schematically from the fact that adding a boundary on a band topology is the same as having a closed string insertion $\sim g_c$ or 2 interactions of weight g_o, hence:

$$g_o \sim g_c^{1/2} \sim g_s^{1/2} \tag{5}$$

To be precised, the relation between $g_c = g_s$ and g_o is fixed by the consistency (open/closed duality) of the interaction in a string theory with both closed and open strings (for example, with the existence of a single Dp-brane in the theory).

Consider there are multiple coincidental Dp-branes in spacetime, a N Dp-brane configuration. For N=2 (brane 1 and brane 2), by labeling the end points position, there are 4 types of possible open strings with exactly the same open string spectrum: 1-1, 1-2, 2-1 and 2-2. The open string excitations are described by the state $|\Psi; IJ\rangle$, with $I, J = 1, 2$, which means they can be represented by a 2×2 matrix and the fields can be written as $(A_\alpha)^I_J$ and $(\Phi^a)^I_J$. For a general N, the generalization is straight forward with the matrix $N \times N$. The open string interactions:

1. From the worldsheet point of view, the open string interacts by joining their ends, therefore in I,J indices the scattering amplitude can simply be the matrix product.

2. The interactions have the following symmetry: associate each brane by a phase $e^{i\theta_I}$, the $\sigma = 0$ end is multiply by $e^{i\theta_I}$ while the $\sigma = \pi$ end is multiply by $e^{-i\theta_I}$ then $(\Phi^a)^I_I$ is invariant (this is expected, for a single D-brane configuration). From that symmetry, it is expected that $(...)^I_J = (...)^{J\dagger}_I$, and indeed the symmetry is the demonstration for the physically meaning of directional labelling of open strings to be represented by the mathematical matrix complex conjugate.

2. Since D-branes of the same kind are indistinguishable from one another, one has the freedom to reshuffle their indices with a transformation U:

$$|Psi; IJ\rangle \to |\Psi'; IJ\rangle = U_{IK} U^\dagger_{JL} |\Psi; KL\rangle \quad , \quad \Psi \to \Psi' = U\Psi U^\dagger \tag{6}$$

Unitarity requires that the transformation should be unitary as $U \in U(N)$, hence each open string excitation transforms under the adjoint representation of the $U(N)$ symmetry. On the worldsheet point of view, this $U(N)$ is a global symmetry, but in spacetime (the worldvolume of the Dp-branes), this must become a gauge symmetry, and $(A_\alpha)^I_J$ is expected to be the corresponding gauge bosons so that at low energies the vector fields must be described by a Yang-Mills theory. This can be confirmed by the studies of tree-level scattering amplitude and do the matching to read-off the effective action:

$$S = -\frac{1}{g_{YM}^2} \int d^{p+1}x \, \mathrm{Tr} \left(\frac{1}{4} F^{\alpha\beta} F_{\alpha\beta} - \frac{1}{2} D^\alpha \Phi^a D_\alpha \Phi^a + \left[\Phi^a, \Phi^b\right]^2 + ... \right) \tag{7}$$

The gauge strength and the Yang-Mills coupling:

$$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha - \left[A_\alpha, A_\beta\right] \quad , \quad g_{YM}^2 \sim g_o^2 \sim g_s \to g_{YM}^2 = D_p g_s \alpha'^{\frac{p-3}{2}} \tag{8}$$

It should be noted that the action given in equation (7) can also be derived from dimensional reduction of Yang-Mills theory in 26D (bosonic string) or 10D (superstring).

Now, let's consider the seperating the above N Dp-branes configurations, starting with $N = 2$. Then the 1-1 and 2-2 string are the same as before, while 1-2 and 2-1 are different:

$$1-2 \quad : \quad X(0,\tau) = x_0 \quad , X(\pi,\tau) = x_0 + d \quad , \tag{9}$$

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Take a look at the 1-2 string (2-1 string is similar), then the classical solution gives:

$$X(\sigma,\tau) = x_0 + w\sigma + X_R(\tau-\sigma) + X_L(\tau+\sigma) \to w = \frac{d}{\pi} \tag{11}$$

The mass-shell condition is the same as before except for a shift:

$$\Delta M^2 = \left(\frac{d}{2\pi\alpha'}\right)^2 \tag{12}$$

This means the A_α and Φ_a fields for 1-2 and 2-1 strings become massive (shorten the final expression by the string tension T_s, which gives a very intuitive result):

$$M = \frac{d}{2\pi\alpha'} = dT_s \tag{13}$$

The gauge symmetry is now broken, from $U(2)$ to $U(1) \otimes U(1)$, and the separation of branes from the low effective energy (QFT's) point of view is nothing but the Higgs mechanism. The generalization to N Dp-branes with k coincident of $n_{1,2,\ldots,k}$ Dp-branes gives rise to the symmetry breaking:

$$U(N) \to \bigotimes_{i=1}^{k} U(n_i) \quad , \quad N = \sum_{i=1}^{k} n_i \tag{14}$$

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