Introduction to Nuclear and Particle Physics - 8.701
Lecture 2
SM processes and Relativistic Kinematics

Bernd Surrow

MIT Massachusetts Institute of Technology

Lecture 2: Standard Model Interactions and Relativistic Kinematics

Overview:

1. Standard Model interactions:

a.) Four forces

b.) QED

c.) QCD

d.) Weak interactions

2. Relativistic kinematics:

a.) Lorentz transformation

b.) Consequences of Lorentz transf.

c.) Four vectors

d.) Energy + momentum

e.) Conservation laws

1. Standard Model Interactions:

a.) Four forces:

As far as we know, there are just four fundamental forces in nature:

strong, electromagnetic, weak and gravitational

The Standard Model refers to three of these fundamental forces!

Properties:

	strong	electromag.	weak
Mediator	gluon ($m=0$)	γ ($m=0$)	$\sim 80\,GeV$ $W^{\pm}$, Z^0 $\sim 90\,GeV$
Acts on	color charge	el. charge	flavor
Particles experiencing it	gluons, quarks	charged part.	leptons, quarks
Range	$c\Delta t = \frac{\hbar}{m_\pi c} \sim 1F$	∞	$c\Delta t = \frac{\hbar}{m_W c} \sim 10^{-3}F$
Typical life time	$10^{-23}\,s$	$10^{-20} - 10^{-16}\,s$	$10^{-12}\,s$
Typical cross-section	$10\,mb$ $\pi p \to \pi p$	$10^{-3}\,mb$ $\gamma p \to p \pi^0$	$10^{-14}\,mb$ $\nu p \to \nu p$
Typical coupling α_i	1	10^{-2}	10^{-6}

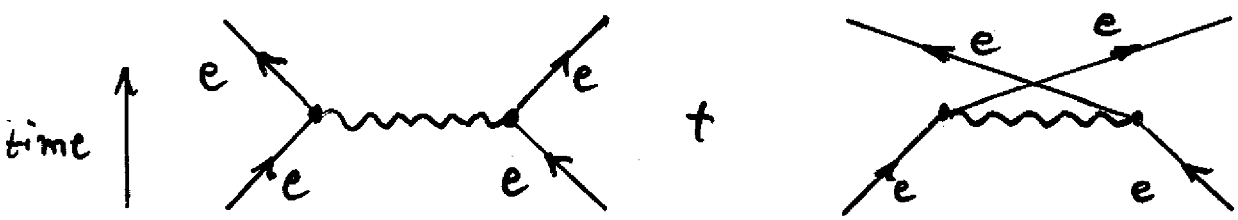
6.) QED

All electromagnetic phenomena are ultimately reducible to the following elementary coupling:

U(1) gauge theory
→ 1 charge /
1 boson (γ)

- two elements: charged particle, photon
- Meaning: charged particle e enters, emits (or absorbs) a photon γ and exits

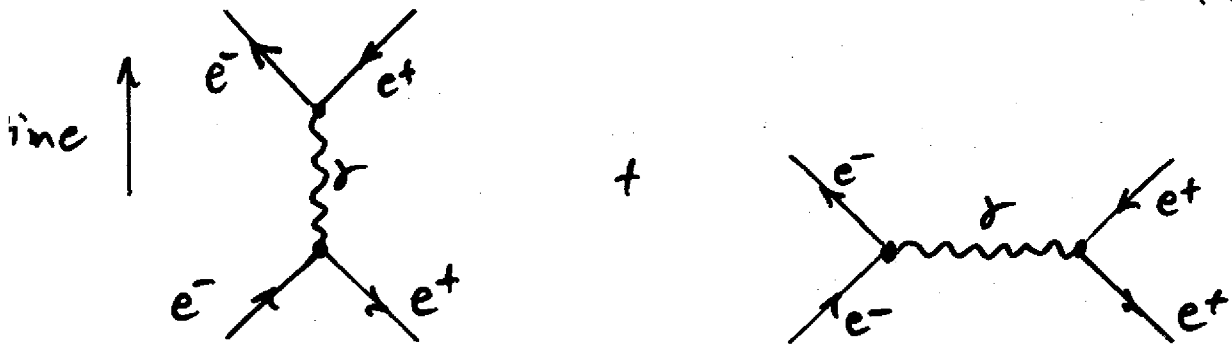
1. Example of a complete process: Møller scattering
$e + e \to e + e$



Meaning: Interaction of two electrons which is mediated by a photon

classical case: Coulomb repulsion

2. Example of a complete process: Bhabha scattering
$e^- + e^+ \to e^- + e^+$



<u>Rule :</u> Particles which are running "backward in time" are to be interpreted as the corresponding anti-particle!

Bhabha scattering is related to Møller scattering by a general principle which is known as <u>crossing symmetry:</u>

$$A + B \rightarrow C + D$$

<u>Rule :</u> Any particle can be "crossed" over the other side of the equation, provided it turns into its anti-particle:

$$\left.\begin{aligned} &A \rightarrow \bar{B} + C + D \\ &A + \bar{C} \rightarrow \bar{B} + D \\ &\bar{C} + \bar{D} \rightarrow \bar{A} + \bar{B} \end{aligned}\right\}$$

These processes are dynamically allowed, but not necessarily kinematically!

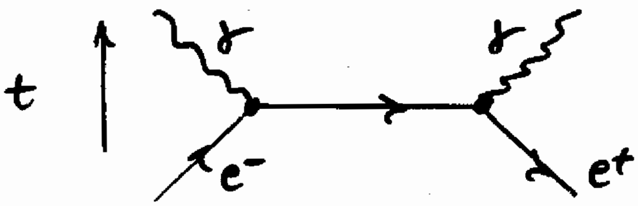
(e.g. if A weighs less than $\bar{B}$, C and D, then this process in kinematically not allowed)

Bhabha scattering and Møller scattering are related by crossing symmetry!

3. Example of complete process:

Pair annihilation

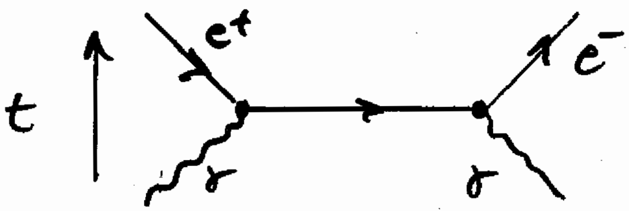
$e^- + e^+ \to \gamma + \gamma$



4. Example of complete process:

Pair production

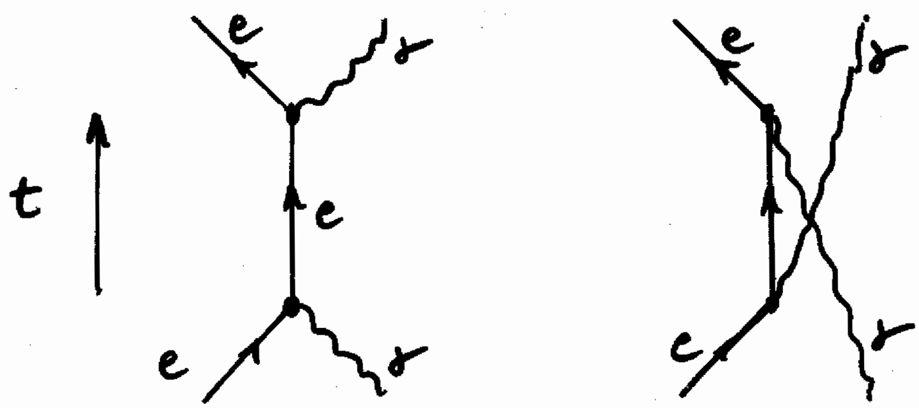
$\gamma + \gamma \to e^- + e^+$



5. Example of complete process:

Compton scattering

$e + \gamma \to e + \gamma$

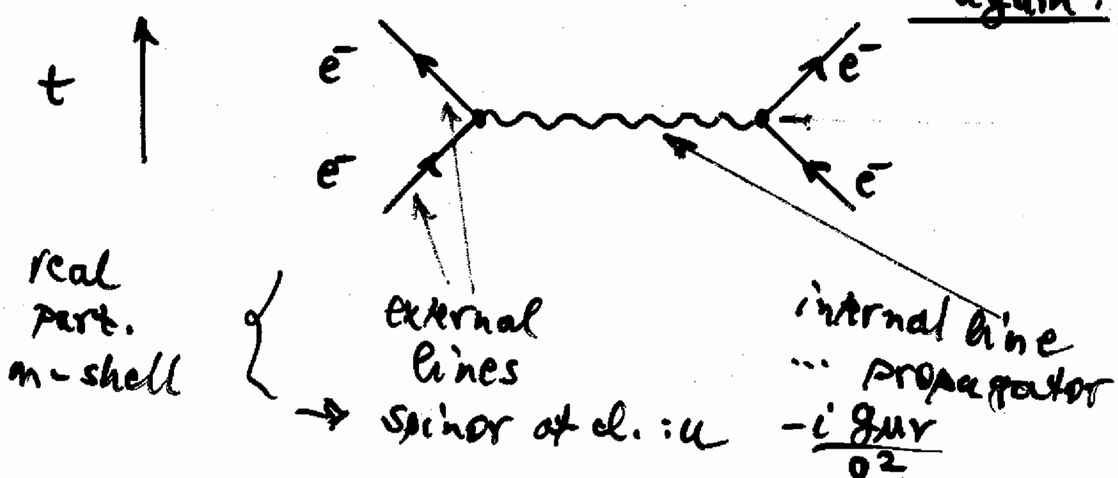


Remarks on Feynman diagrams:

1. Feynman diagrams are purely symbolic, they <u>do not</u> represent particle trajectories.

2. <u>Example</u> : <u>"Components" of a Feynman diagram</u>

<u>again:</u>



vertex factor: $ig_e\gamma^\mu$
$g_e = \sqrt{4\pi\alpha} \propto \sqrt{\alpha}$
$\alpha = \frac{1}{137}$

3. Each Feynman diagram represents a Matrix, $\mathcal{M}$, to account for a particular process

4. Feynman rules enforce conservation of energy and momentum at each vertex, and hence for the diagram as a whole.

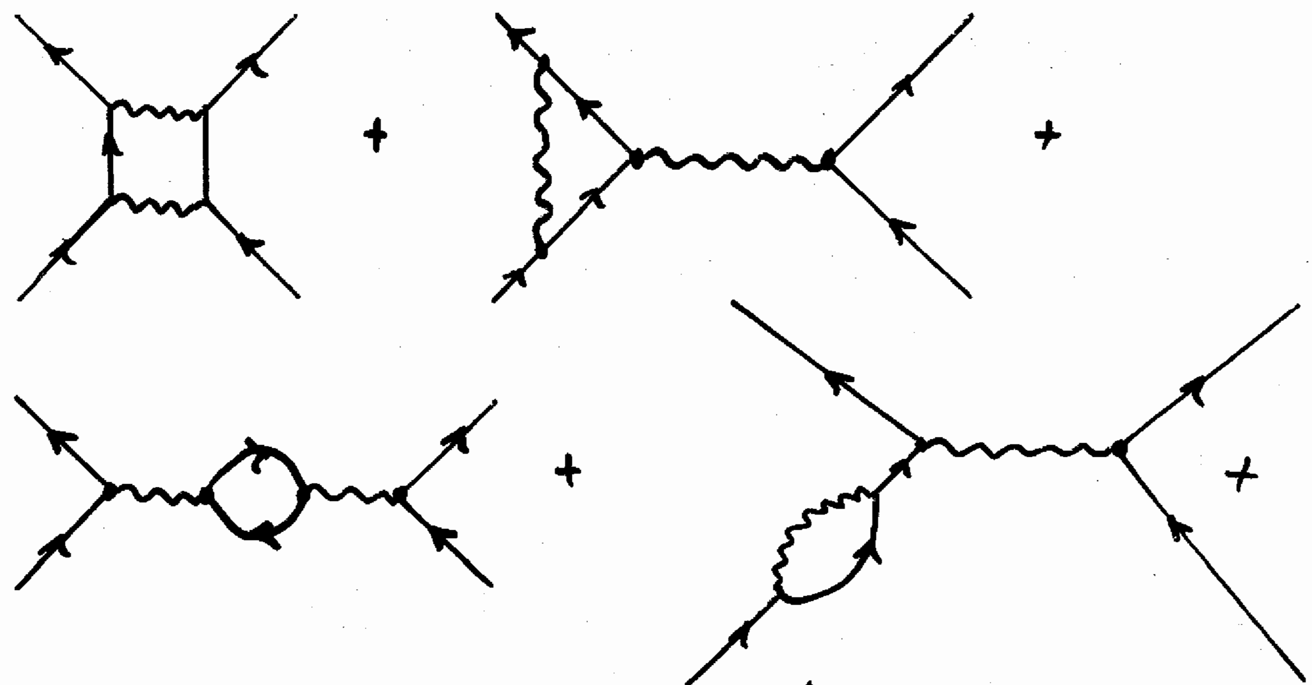
5. <u>Procedure:</u>

a.) Draw all diagrams that have the appropriate external lines
b.) Sum of <u>all diagrams</u> with the given external lines represents the actual physical process.

<u>Problem:</u> There are infinite many Feynman diagrams!

Example: Møller scattering

Besides the above 2-vertex Feynman diagrams for Møller scattering, we have with 4 vertices:



calculations get very complicated!

higher order diagrams

BUT: Each vertex contributes a factor $\sqrt{\alpha}$!

α: QED finestructure constant (general: QED coupling constant)

Higher order terms are suppressed since $\alpha \ll 1$ (perturbation theory)!

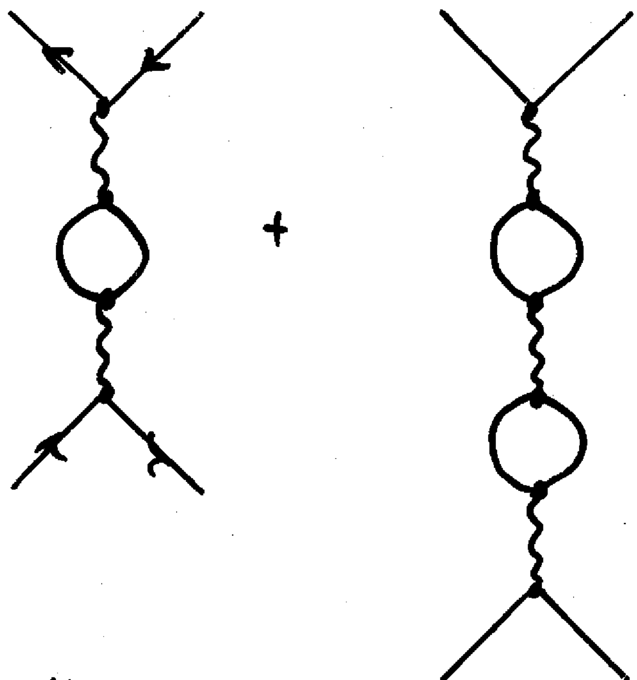
What matters "mainly" is the leading order contribution!

However: There are many examples where measurements are so precise that higher order terms have to be taken into account! $\longrightarrow$ TEST OF QED

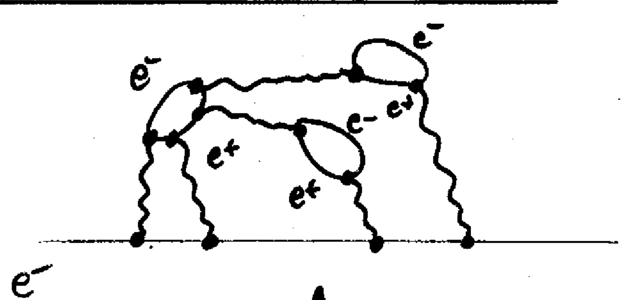
example: Measurement of anom. magn. moment of muon!

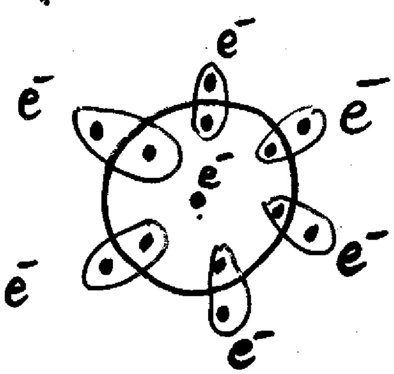
QED vacuum polarization:

In QED, the vacuum behaves like a dielectric, creating e^-/e^+ pairs out of the vacuum:

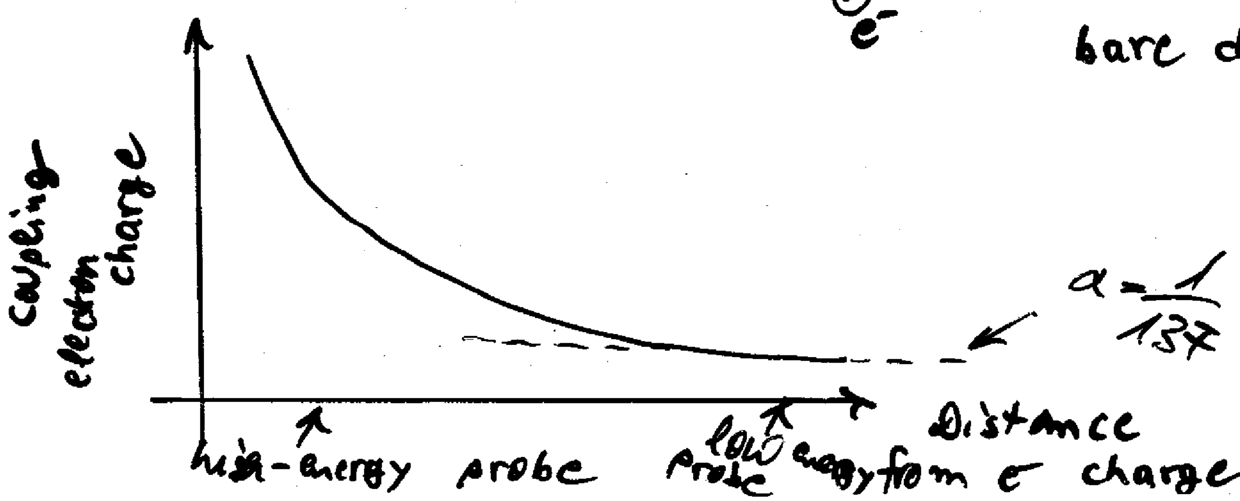


or view it like this:





"bare" electron charge is screened eff. charge << bare charge

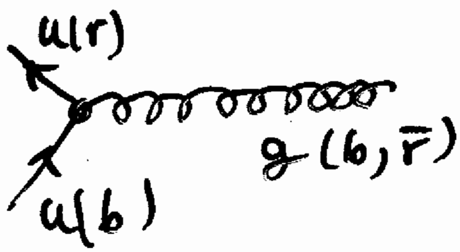


c.) QCD :

In Quantum - Chromo - Dynamics : SU(3) gauge theory
→ 3 charges / 8 bosons (gluons)

color plays the role of charge and the fundamental process is: quark → quark + gluon

- Fundamental vertex :



color is always conserved:
→ gluon is carrying away the difference

- Types of gluons : 9 "possibilities"

$r\bar{r},\ r\bar{b},\ r\bar{g},\ b\bar{r},\ b\bar{b},\ b\bar{g},\ g\bar{r},\ g\bar{r},\ g\bar{b},\ g\bar{g}$

color octet

$|1\rangle = (1/\sqrt{2})(r\bar{b} + b\bar{r})$ $\qquad$ $|5\rangle = -i\,(r\bar{g} - g\bar{r})/\sqrt{2}$

$|2\rangle = -1/\sqrt{2}\,(r\bar{b} - b\bar{r})$ $\qquad$ $|6\rangle = (b\bar{g} + g\bar{b})/\sqrt{2}$

$|3\rangle = (r\bar{r} - b\bar{b})/\sqrt{2}$ $\qquad$ $|7\rangle = -i\,(b\bar{g} - g\bar{b})/\sqrt{2}$

$|4\rangle = (r\bar{g} + g\bar{r})/\sqrt{2}$ $\qquad$ $|8\rangle = (r\bar{r} + b\bar{b} - 2\,g\bar{g})/\sqrt{6}$

color singlet

$|9\rangle = (r\bar{r} + b\bar{b} + g\bar{g})\,/\,\sqrt{3}$ "no net color"

... like a photon ...

• Note:

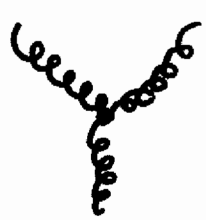
Confinement requires that all naturally occurring particles be color singlets. $|9\rangle$ is a color singlet state. If it exists as a mediator, it should also occur as a free particle

$\longrightarrow$ As such a mediator: Exchange between color singlet particles, e.g. a proton and neutron $\Rightarrow$ long-range force with strong coupling.

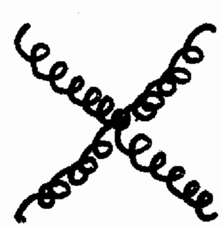
However: Strong force is of very short range. So: Experiments tell us that there are only 8 gluons: color octet : SU(3)

Because gluons themselves carry color (in contrast to the photon which is electrically neutral): Gluons couple directly to one another:

3 gluon vertices:



4 gluon vertices:



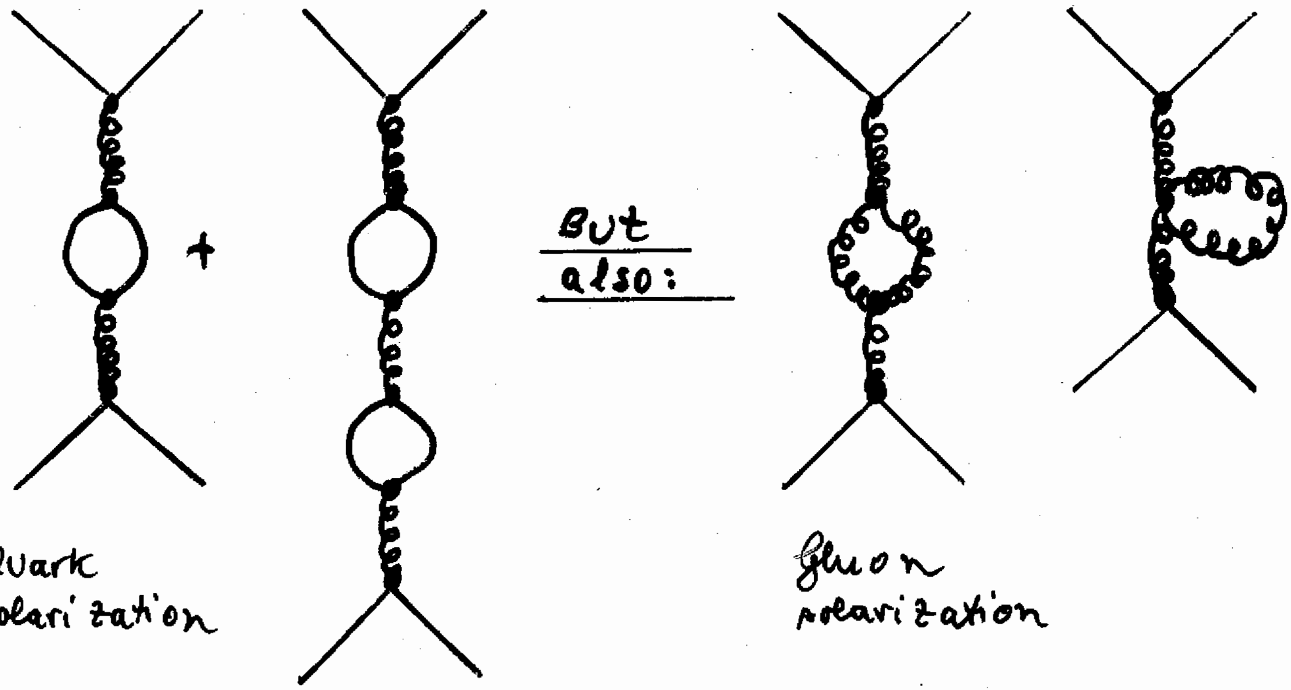
Calculations: Apply QCD Feynman rules to calculate various processes!

$\longrightarrow$ How does the QCD coupling constant behave compared to the QED coupling constant?

Clear difference between QED and QCD:

→ self-coupling of gluons which is absent in case of QED.

QCD vacuum polarization diagrams:



Quark polarization

Gluon polarization

→ Behavior is opposite for α_s:

Quark polarization: α_s is large at short distances

Gluon polarization: α_s is small at short distances

A priori not clear, who wins!

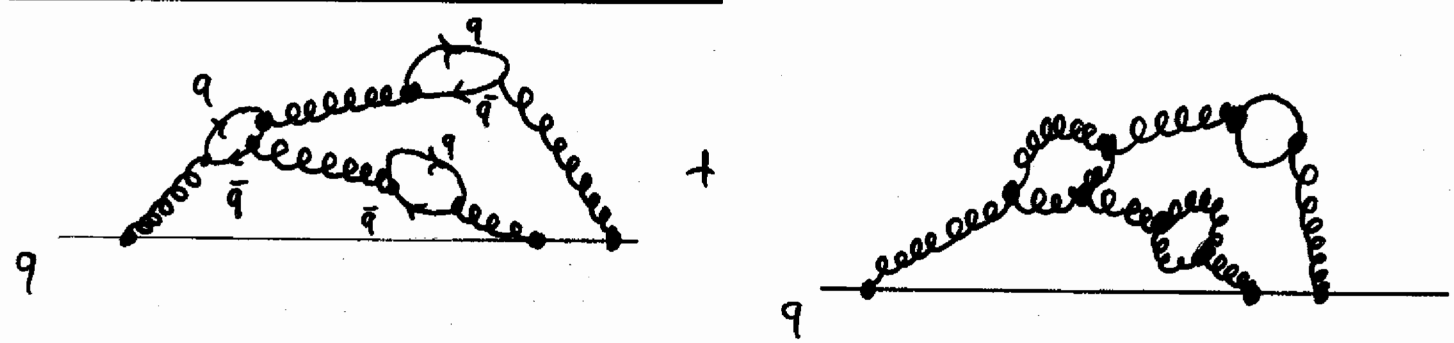
The winner depends on the relative number of flavors (f) and colors (n):

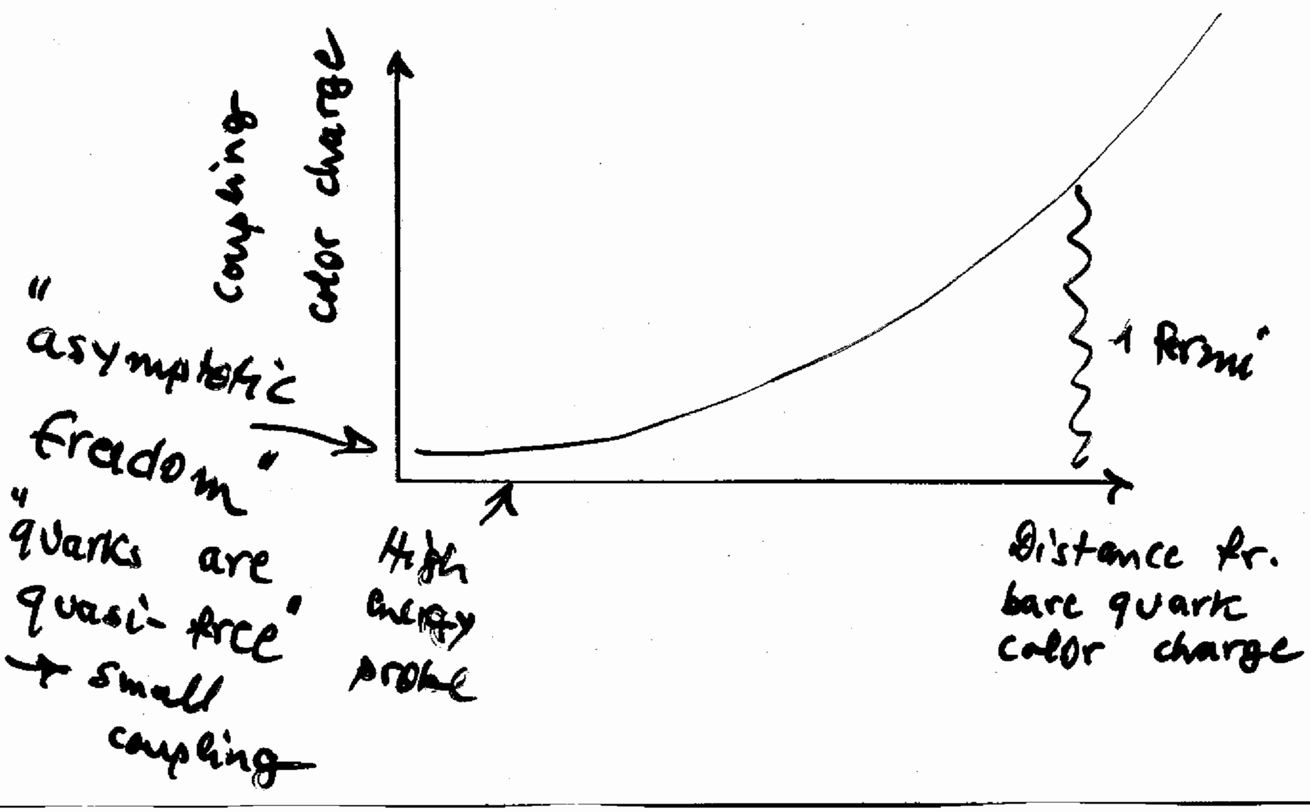
critical parameter: $\boxed{a \equiv 2f - 11n}$

Standard Model: $f = 6$, $n = 3 \rightarrow a = -21$

$\longrightarrow$ QCD coupling constant decreases at short distances!

Another "last picture":





d.) Weak interactions:

SU(2) gauge theory: 3 mediators: Z^0; $W^\pm$

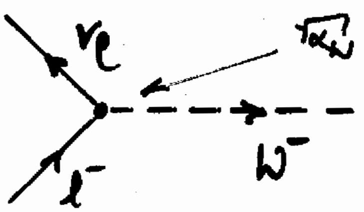
- Quarks and leptons take part in weak interactions
- 2 types of interactions:

a.) charged current: $W^\pm$

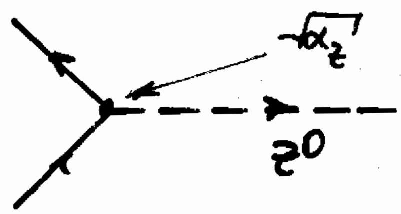
b.) neutral current: Z^0

1. leptons:

charged vertex:

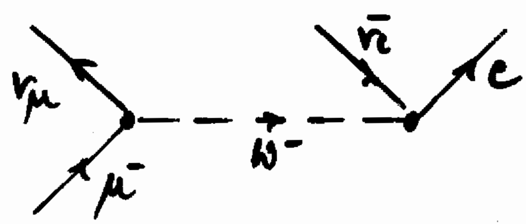


neutral vertex:

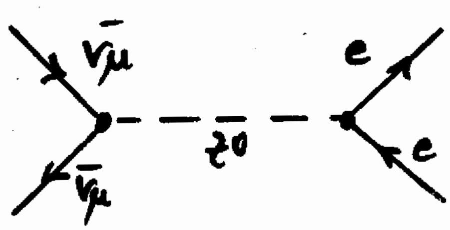


example:

$\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e$



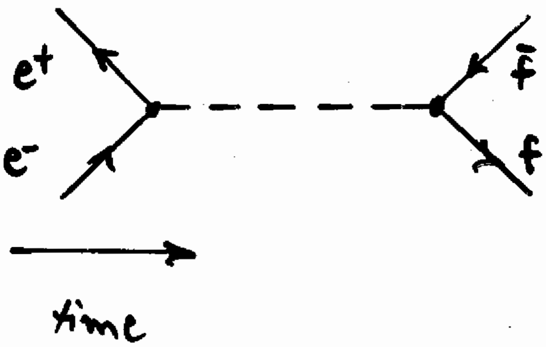
$\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$



First "picture" of neutral weak process discovered at CERN in 1973.

The Glashow-Weinberg-Salam (GWS) model includes neutral weak processes as an essential ingredient. Their existence was confirmed experimentally at CERN in 1973.

<u>Production of Z^0 at LEP:</u> max. $E_{e^-} + E_{e^+}$ at m_{Z^0}

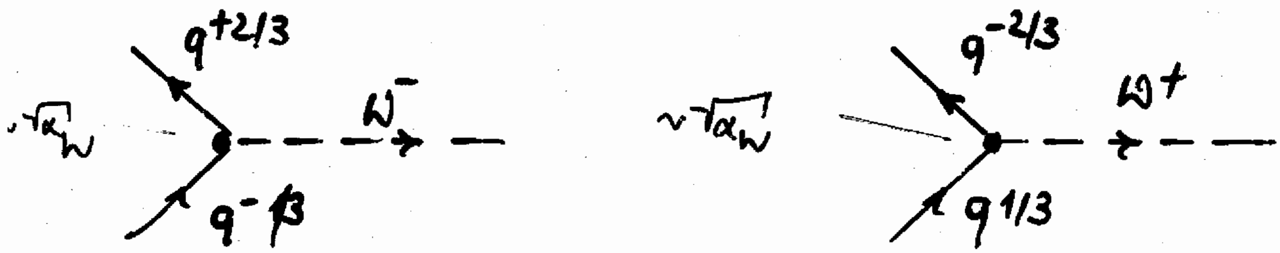


$$\begin{pmatrix} e^+ & \mu^+ & \tau^+ & \bar{\nu}_e & \bar{\nu}_\mu & \bar{\nu}_\tau & \bar{u} & \bar{d} & \bar{s} & \bar{c} & \bar{b} \\ e^- & \mu^- & \tau^- & \nu_e & \nu_\mu & \nu_\tau & u & d & s & c & b \end{pmatrix}$$

. . established <u>3 families</u> in nature!

2. Quarks :

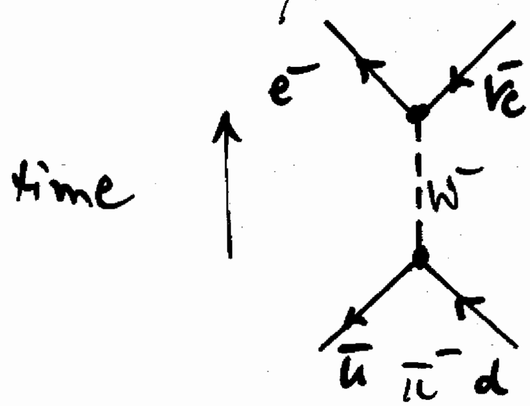
Fundamental vertex : $W^{\pm}$



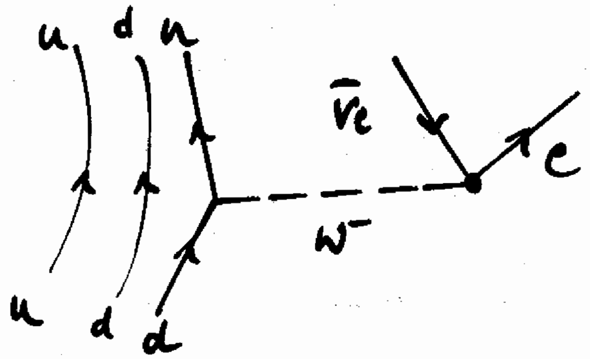
Note: A quark of charge $-1/3$ (d, s, b) converts into the corresponding quark with charge $+2/3$ (u, c, t) with the emission of W^- (vice versa for W^+).

Flavor is <u>not conserved</u> in weak interactions! Since the quark flavor changes at a weak vertex ($W^{\pm}$), as a quark color changes at a strong vertex, weak interactions are sometimes called <u>"flavordyn."</u>

example : $\pi^- \to e^- + \bar{\nu}_e$

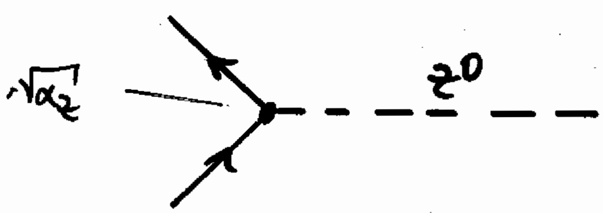


Beta-decay of the neutron:



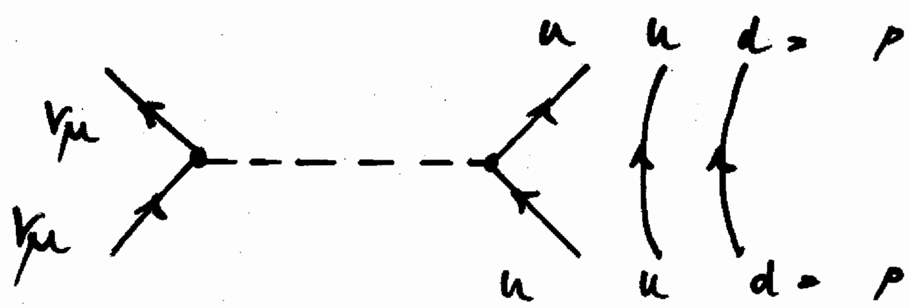
$n \to p + e^- + \bar{\nu}_e$

Fundamental vertex : Z^0



note : quark flavor is not changed !

Example :



$\nu_\mu + p \to \nu_\mu + p$

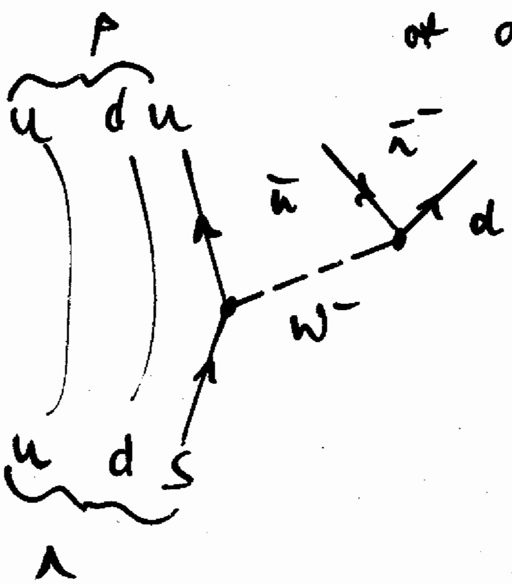
- <u>Flavor changing reactions:</u> $W^{\pm}$

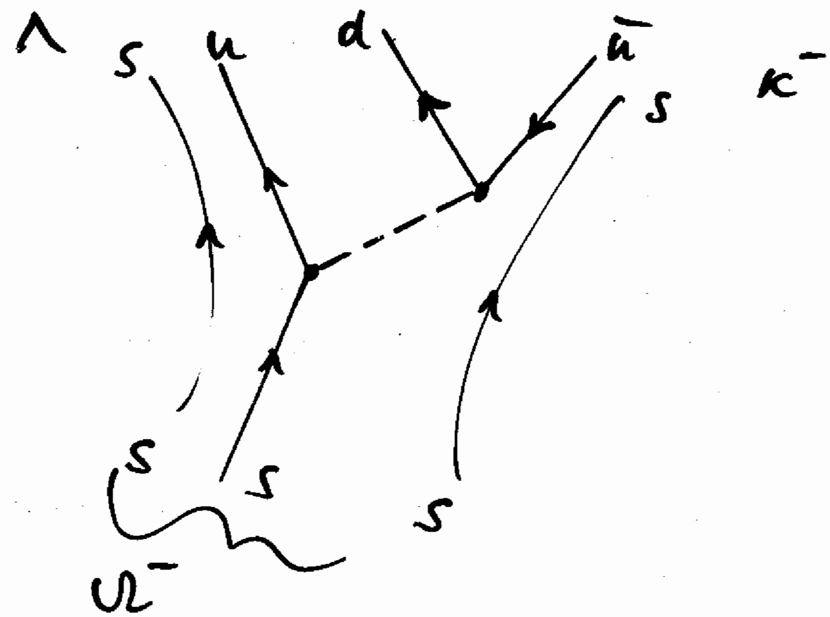
In the spirit of the charged weak coupling with respect to leptons which yield only changes within each family, i.e. : $e^- \leftrightarrow \nu_e$; $\mu^- \leftrightarrow \nu_\mu$; $\tau^- \leftrightarrow \nu_\tau$

<u>one might assume that this also holds for quarks:</u>

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

<u>However:</u> The observed decay $\Lambda \to p + \bar{\pi}^{+}$ or $\Omega^- \to \Lambda + K^-$ involve the conversion of a strange quark into an up-quark:





<u>Note:</u> <u>Flavor changes do occur **not** only within one family!</u>

Therefore:

The flavor eigenstate $|u\rangle$ is not the partner to the flavor eigenstate $|d\rangle$, but to a linear combination of d, s and b:

$$\begin{pmatrix} u \\ d' \end{pmatrix} \quad \begin{pmatrix} c \\ s' \end{pmatrix} \quad \begin{pmatrix} t \\ b' \end{pmatrix}$$

$|d'\rangle$ can be expressed as a linear combination of $|d\rangle$, $|s\rangle$ and $|b\rangle$:

$$\boxed{\begin{pmatrix} |d'\rangle \\ |s'\rangle \\ |b'\rangle \end{pmatrix} = \begin{pmatrix} \underline{V_{ud}} & \underline{V_{us}} & \underline{V_{ub}} \\ \underline{V_{cd}} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & \underline{V_{tb}} \end{pmatrix} \begin{pmatrix} |d\rangle \\ |s\rangle \\ |b\rangle \end{pmatrix}}$$

Note:

The matrix is called after their "inventors": 3 x 3 Kobayashi - Maskawa Matrix! The coefficients are sometimes expressed as cosine and sine values of an angle : Cabibbo angle θ_C

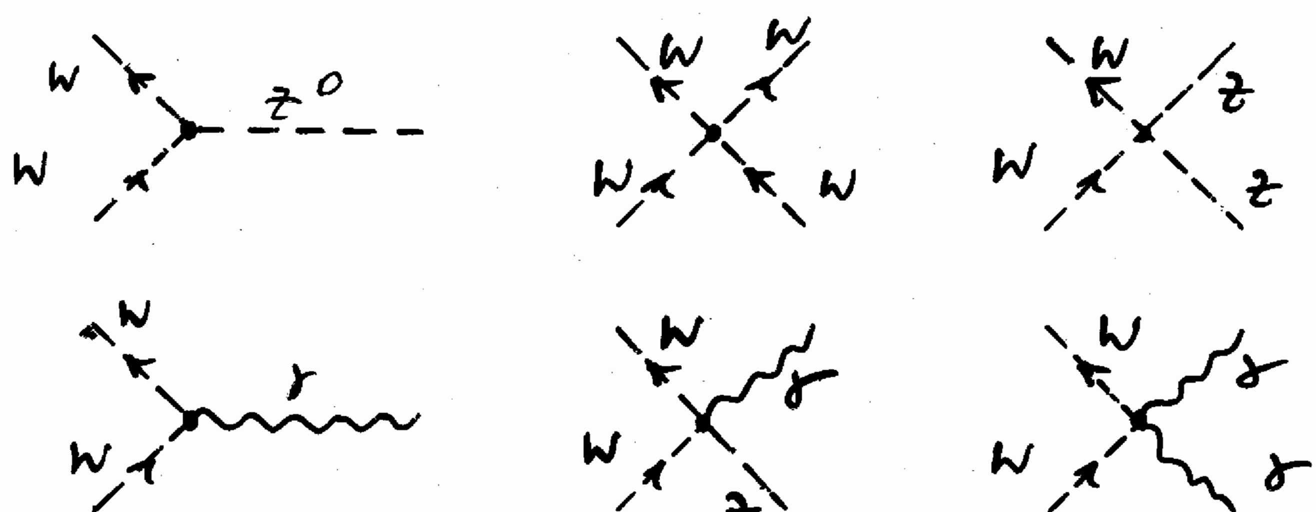
earlier scheme (2 families): $|d'\rangle = |d\rangle \underline{\cos\theta_C} + |s\rangle \underline{\sin\theta_C}$
$|s'\rangle = |d\rangle(-\sin\theta_C) + |s\rangle \underline{\cos\theta_C}$

Experimental data:

$$V_{ij} = \begin{pmatrix} 0{,}9741-0{,}9756 & 0{,}219-0{,}226 & 0{,}0025-0{,}0048 \\ 0{,}219-0{,}226 & 0{,}9732-0{,}9748 & 0{,}038-0{,}044 \\ 0{,}004-0{,}014 & 0{,}037-0{,}044 & 0{,}9990-0{,}9993 \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

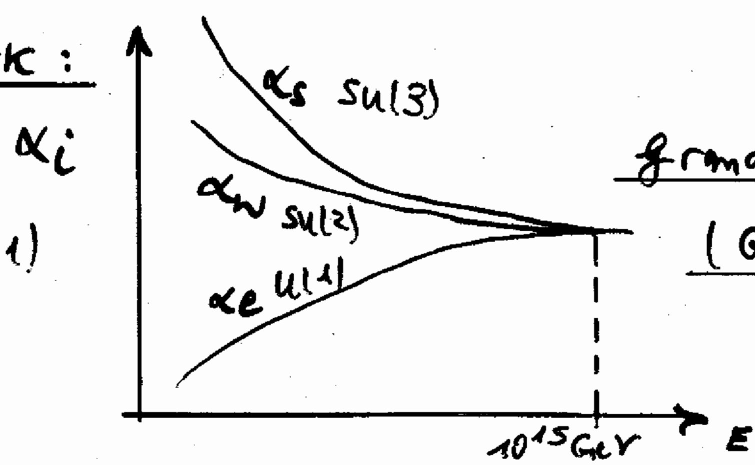
K. Hagiwara et al., Phys. Rev. D 66 (2002) 010001.

Self coupling of weak-bosons and photon:



Last remark:

$SU(3) \otimes SU(2) \otimes U(1)$



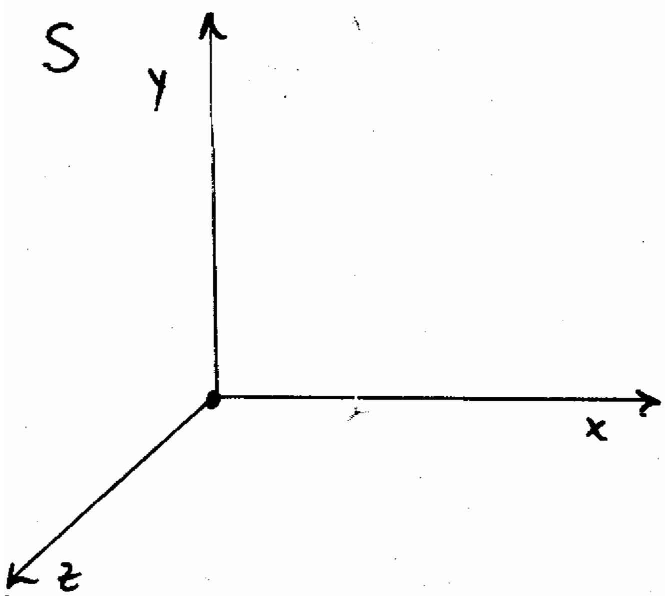
Grand Unified theories

(GUT) "one Force"

2. Relativistic Kinematics:

a.) Lorentz transformations:

given two inertial frames S and S', with S' moving at uniform speed v with respect to S :



Events are related in space-time as follows (Lorentz transformations) :

$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$

$$y' = y \qquad y = y'$$

$$z' = z \qquad z = z'$$

$$t' = \gamma\left(t - \frac{v}{c^2} \cdot x\right) \qquad t = \gamma\left(t' + \frac{v}{c^2} \cdot x'\right)$$

with: $\gamma = 1 / \sqrt{1 - v^2/c^2}$ $\qquad \beta = (v/c)$

ii) Consequences:

1. relativity of simultaneity:

If two events occur at the same time in S, but at different locations, then they do not occur at the same time in S': With $t_A = t_B$:

$$\boxed{t'_A = t'_B + \frac{\gamma v}{c^2}(x_B - x_A)}$$

2. Lorentz contraction:

A moving object is shortened by a factor γ:

$$\boxed{L = L' \cdot \gamma^{-1}}$$

3. Time dilation:

Moving clocks run slow:

$$\boxed{T = T' \cdot \gamma}$$

4. velocity addition:

Particle moves with speed u with respect to S'. What is the speed, u, with respect to S? Note: S' moves with speed v with respect to S:

$$\boxed{u = \frac{u' + v}{1 + (u' \cdot v / c^2)}}$$

if $u' = c \rightarrow u = c$

c) Four vectors:

$$\boxed{x^0 = c \cdot t \ ; \ x^1 = x \ , \ x^2 = y \ \ x^3 = z}$$

Transformation:

compact: $x^{\mu'} = \sum_{\nu=0}^{3} \Lambda^{\mu}_{\nu} \cdot x^{\nu}$

$\mu = 0, 1, 2, 3$

$$x^{0'} = \gamma(x^0 - \beta x^1)$$
$$x^{1'} = \gamma(x^1 - \beta x^0)$$
$$x^{2'} = x^2$$
$$x^{3'} = x^3$$

$$\Lambda = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\beta = v/c$

short :

$$\boxed{x^{\mu'} = \Lambda^{\mu}{}_{\nu} \cdot x^{\nu}}$$

convention by Einstein: | Repeated indices are to be summed!

Invariant:

$$\boxed{I \equiv (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2 = (x^{0'})^2 - (x^{1'})^2 + (x^{2'})^2 + (x^{3'})^2}$$

$\longrightarrow$ same in any inertial system!

Different notation:

$$\boxed{g = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}}$$

$I = g_{\mu\nu} \cdot x^{\mu} \cdot x^{\nu}$ $\qquad$ $g_{\mu\nu}$: Metric tensor

covariant four vector: $\qquad x_{\mu} = g_{\mu\nu} \cdot x^{\nu}$

contravariant four vector: $\qquad x^{\mu} \qquad (I = x_{\mu} x^{\mu})$

With any two four vectors:

$$\boxed{a^\mu \cdot b_\mu = a_\mu \cdot b^\mu = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3}$$

w/ $b_\mu = a_\mu$:

$$\boxed{a^2 = a \cdot a = (a^0)^2 - (\vec{a})^2}$$

$a^2 > 0$: timelike

$a^2 < 0$: space like

$a^2 = 0$: lightlike

$$p_\mu \cdot p^\mu = \frac{E^2}{c^2} - (\vec{p})^2 = m^2 c^2$$

d.) Energy + momentum:

$\vec{\eta} = \gamma \cdot \vec{v}$ (proper velocity): $\eta^\mu = \gamma(c, \overbrace{v_x, v_y, v_z}^{\vec{v}})$

$\vec{p} = m \cdot \vec{\eta} = \gamma \cdot m \vec{v}$; $p^0 = \gamma m c$

$$\boxed{E = \gamma m c^2 = \frac{mc^2}{(1-\beta^2)^{1/2}}}$$

"Relativistic energy"

$$p^\mu = \left(\frac{E}{c}, p_x, p_y, p_z\right)$$

e.) Conservation laws : Analysis of particle reactions :

• Conservation of :

1. Energy and momentum

2. Angular momentum

3. electric charge

4. Color charge

5. Baryon number : $A = 1$ for Baryons ; $A = -1$ for Anti-Baryons $A = 0$ for non Baryons

6. Lepton number :

Particles of each generation (Leptons) are conserved :

L_e, L_μ, L_τ

$L_i = +1$: particle

$L_i = -1$: anti-particle

example : $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$

7. Flavor is conserved in strong and electromag. interactions, but not in weak interactions ($W^\pm$) !